Boğaziçi University

Introductory Phys Labs

THE PHYSICAL PENDULUM

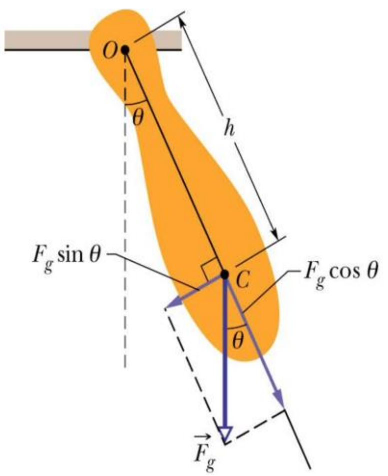
PHYS 102-130

Boğaziçi University, Physics Department

THEORY

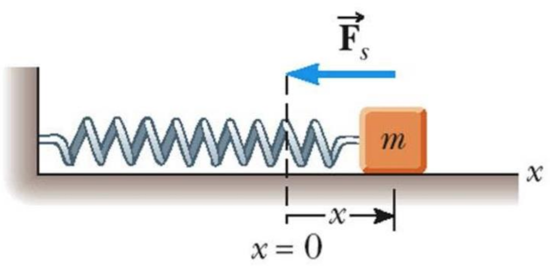
Simple harmonic motion is a special type of periodic motion where the restoring force on the moving object is directly proportional to the object's displacement magnitude and acts towards the object's equilibrium position.

Physical Pendulum



$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

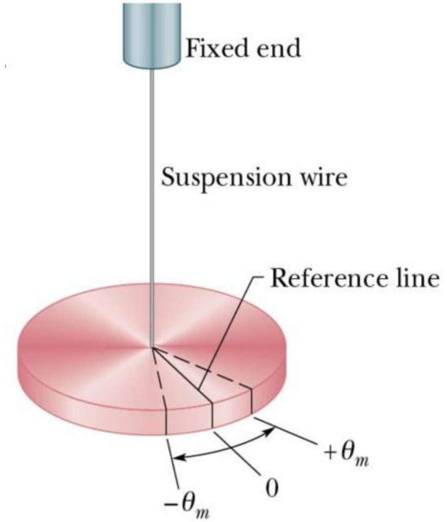
Simple Harmonic Oscillator



$$T = 2\pi\sqrt{\frac{m}{k}}$$

4th experiment

Torsional Pendulum



$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

5th experiment

THE PHYSICAL PENDULUM

A physical pendulum, is any object whose oscillations are similar to those of the simple pendulum, but cannot be modeled as a point mass on a string, and mass distribution must be included into the equation of motion.

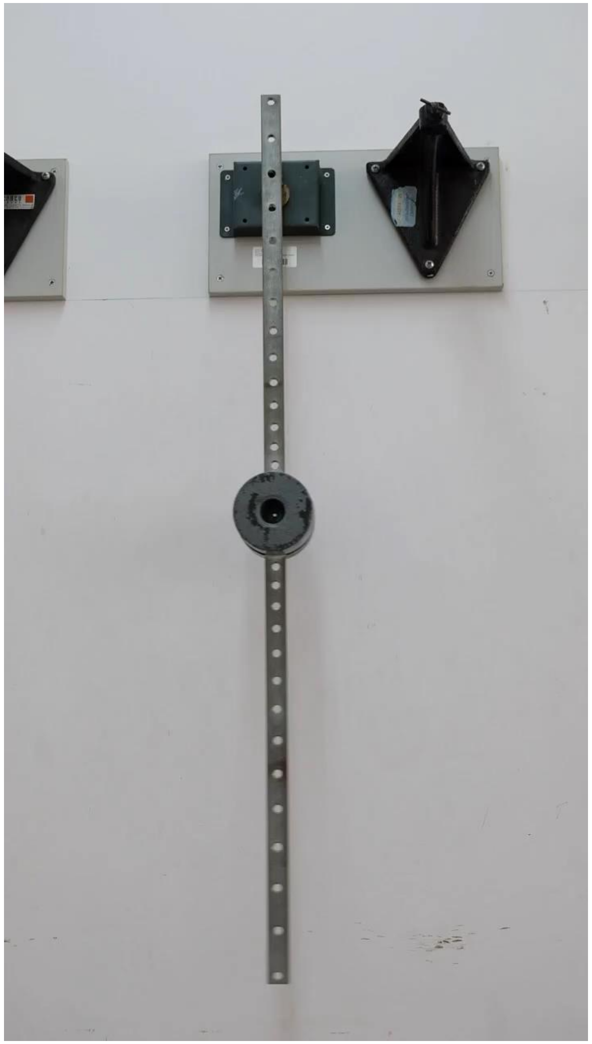
THE PHYSICAL PENDULUM

As for simple pendulum, the restoring force of the physical pendulum is the force of gravity. In the simple pendulum, the force of gravity acts on the center of the pendulum bob. In the case of physical pendulum, the force of gravity acts on the center of mass (CM) of an object.

THE PHYSICAL PENDULUM

- A physical pendulum makes simple harmonic motion if the amplitude is small.

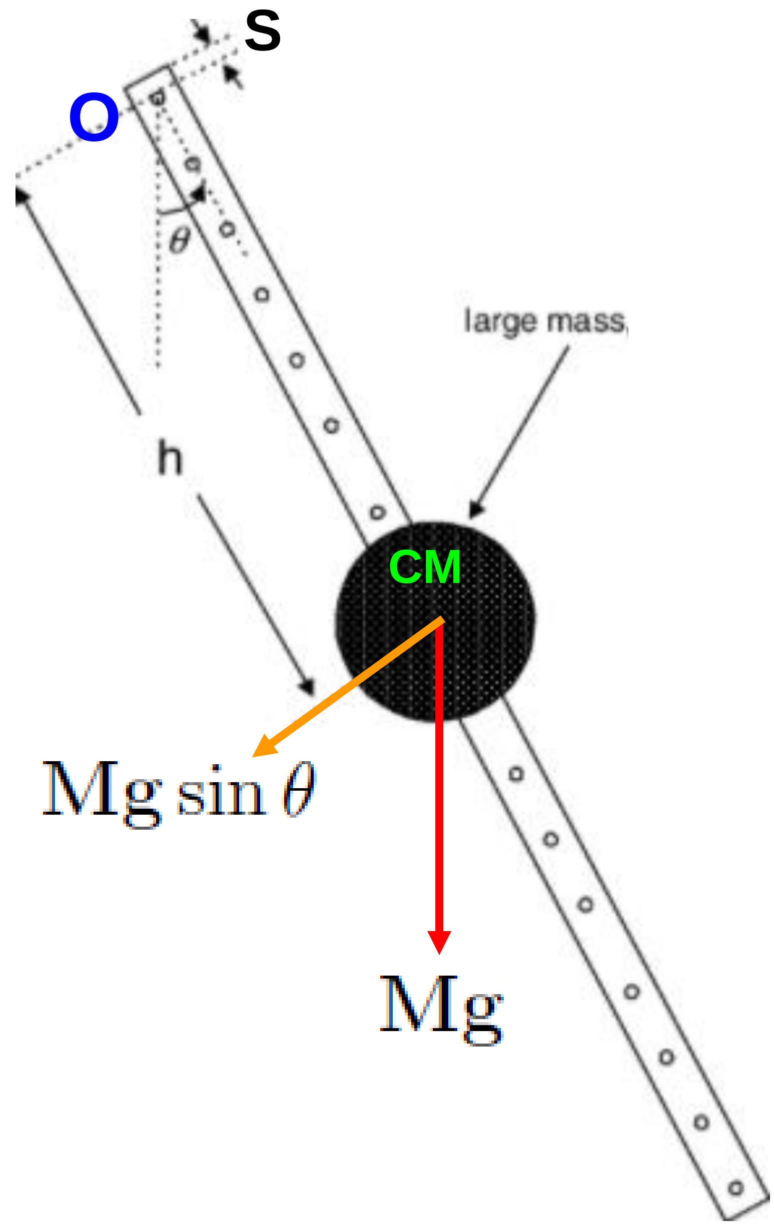
- Gravity acts on the center of mass to rotate the pendulum. The pendulum continues to swing back and forth.



Consider a rigid body that can freely rotate around a horizontal axis through a fixed center of suspension O.

The gravitational force produces a torque about the suspension point O having magnitude:

$$\boxed{\tau = \mathrm{Mgh}\sin\theta}$$



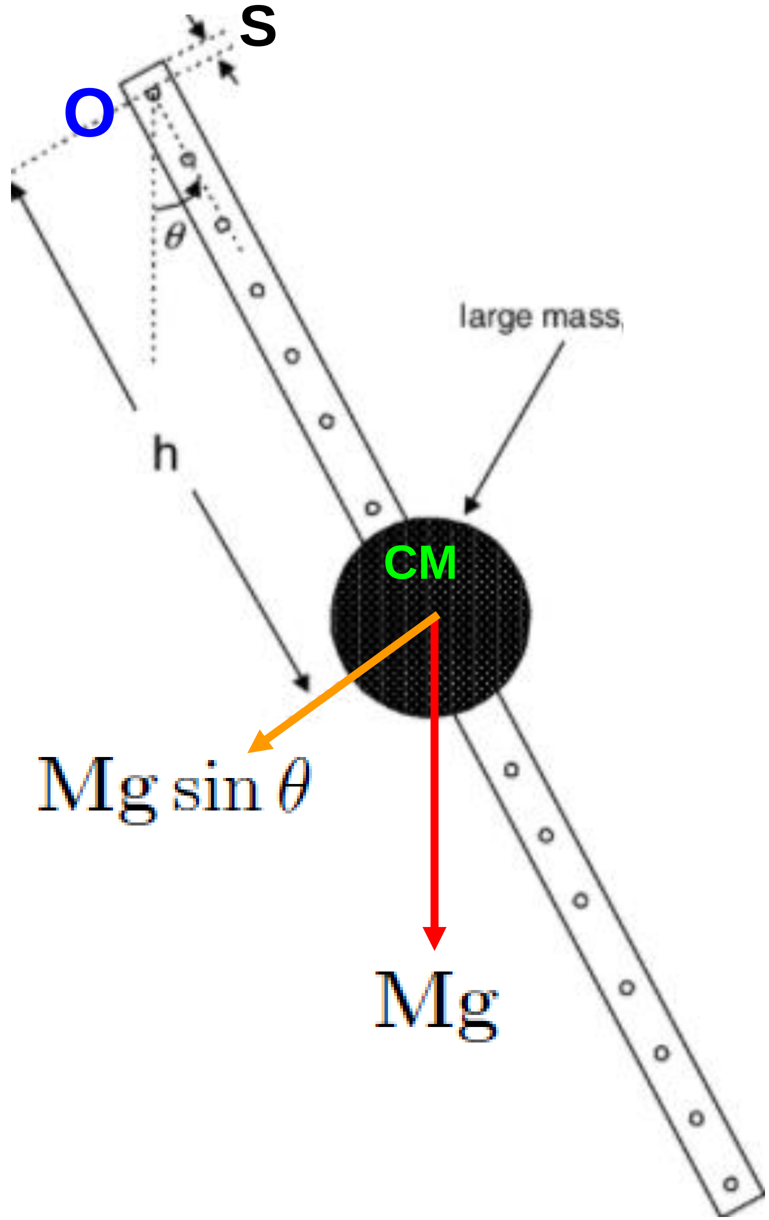
THE PHYSICAL PENDULUM

- Let us use the rotational form of Newton's second law : $\tau = \mathrm{I}\,\alpha$

(I is the moment of inertia about the axis through pivot point O)

(α: angular acceleration)

$$\alpha = \frac{d^2\theta}{dt^2}$$



Equating two torque expressions gives

$$\mathrm{I}\frac{d^2\theta}{dt^2} = -Mgh\sin\theta$$

Now, if we assume θ is small, we have

$$\boxed{\sin\theta \approx \theta}$$

Finally, the equation of motion of the rigid body becomes

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\mathrm{Mgh}}{\mathrm{I}}\theta = 0$$

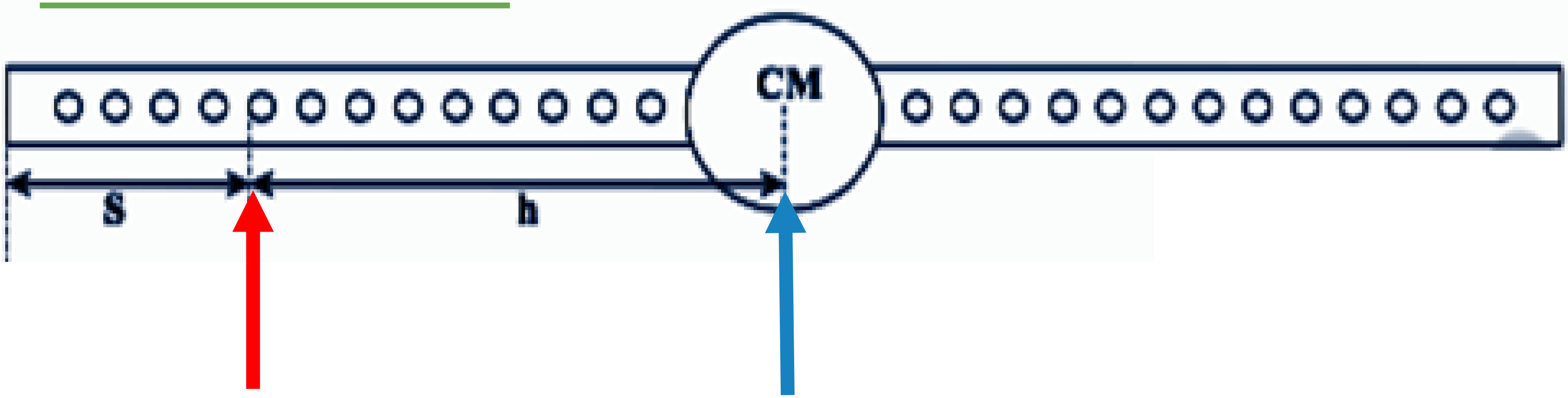
- This is in the form of the equation of Simple Harmonic Motion.
- The period of oscillation is

$$\boxed{\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{Mgh}}}}$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$T = \frac{2\pi}{\omega}$$

Moment Of Inertia



suspension point

center of mass

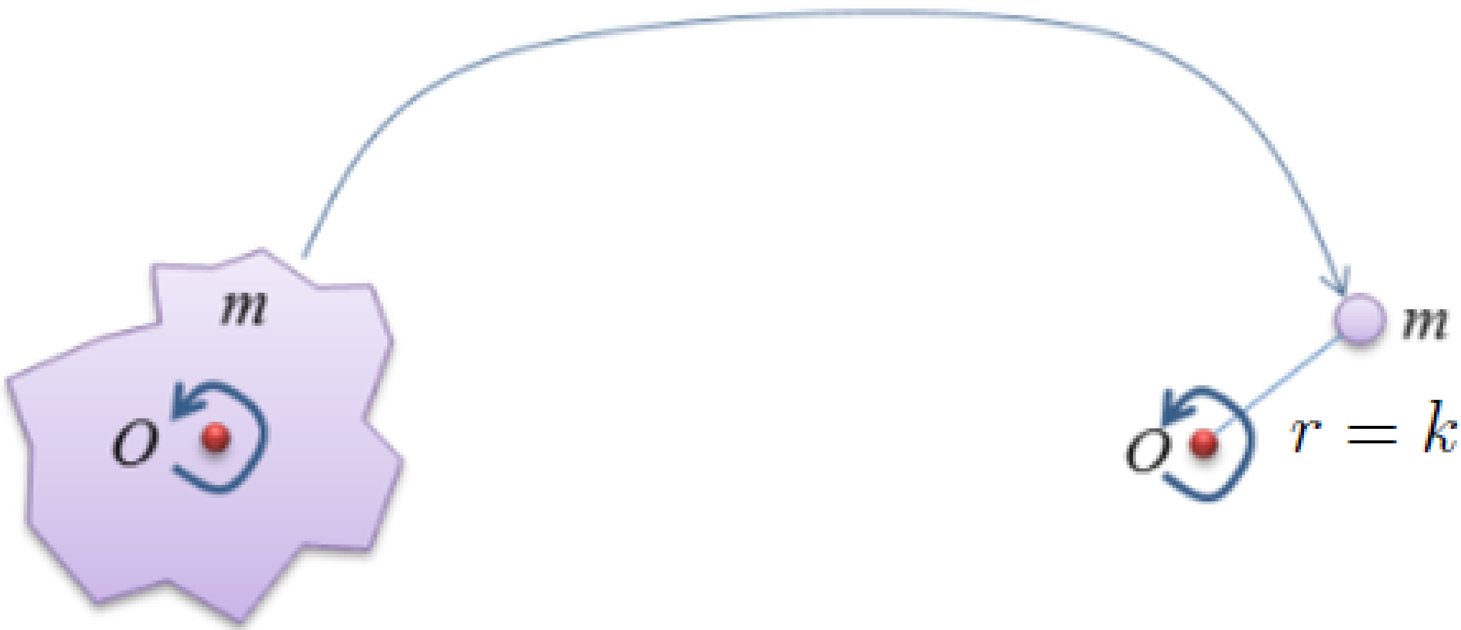
The moment of inertia of a rigid body about any given point can be expressed in terms of the moment of inertia about the center of mass using the parallel axis theorem:

$$\boxed{I = I_{CM} + Mh^2}$$

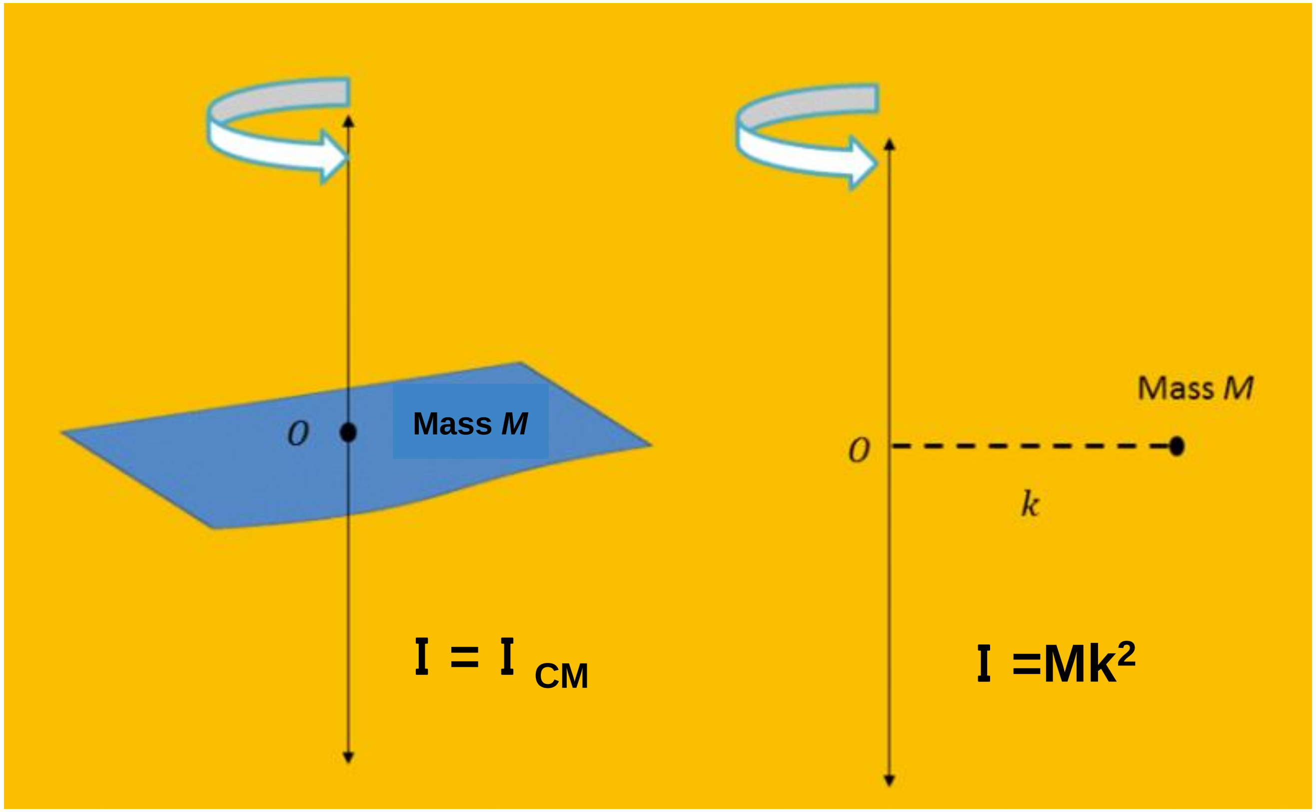
So, what is the moment of inertia about the center of mass?

Radius Of Gyration (k) :

The radius of gyration k is the radial distance from the axis of rotation at which the total mass of a rigid body is assumed to be concentrated while the moment of inertia of the body remains the same.



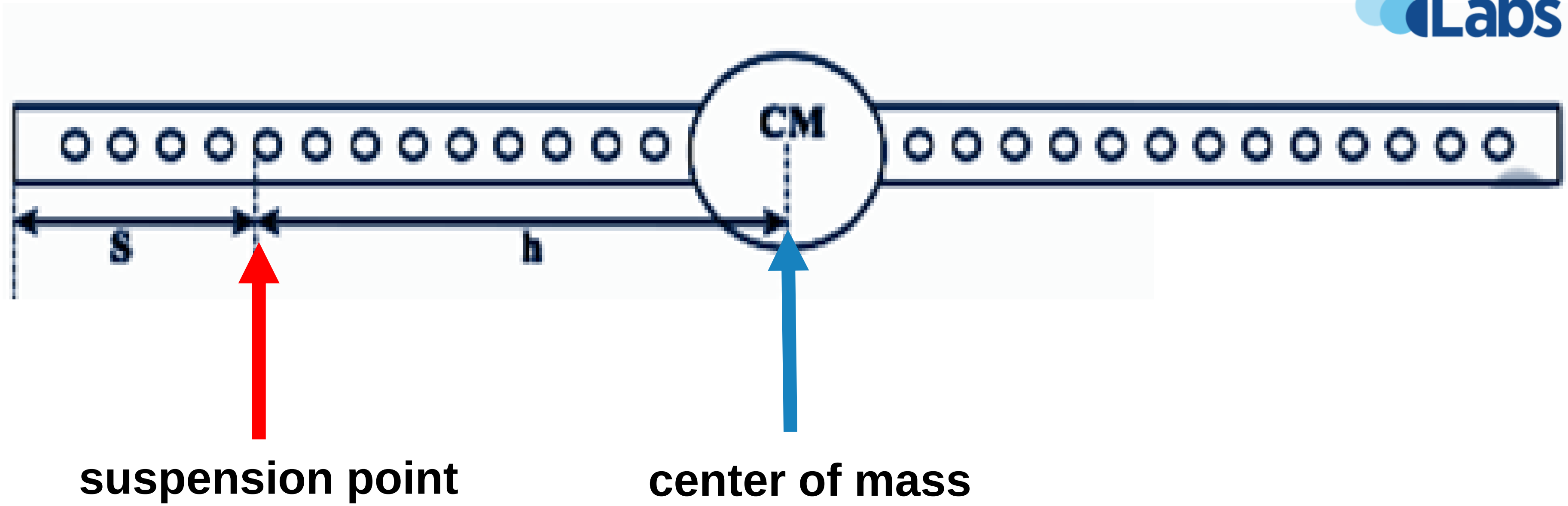
THE PHYSICAL PENDULUM



For a rigid body, the moment of inertia about center of mass can be written in terms of radius of gyration k :

$$\boxed{I_{CM} = Mk^2}$$

THE PHYSICAL PENDULUM

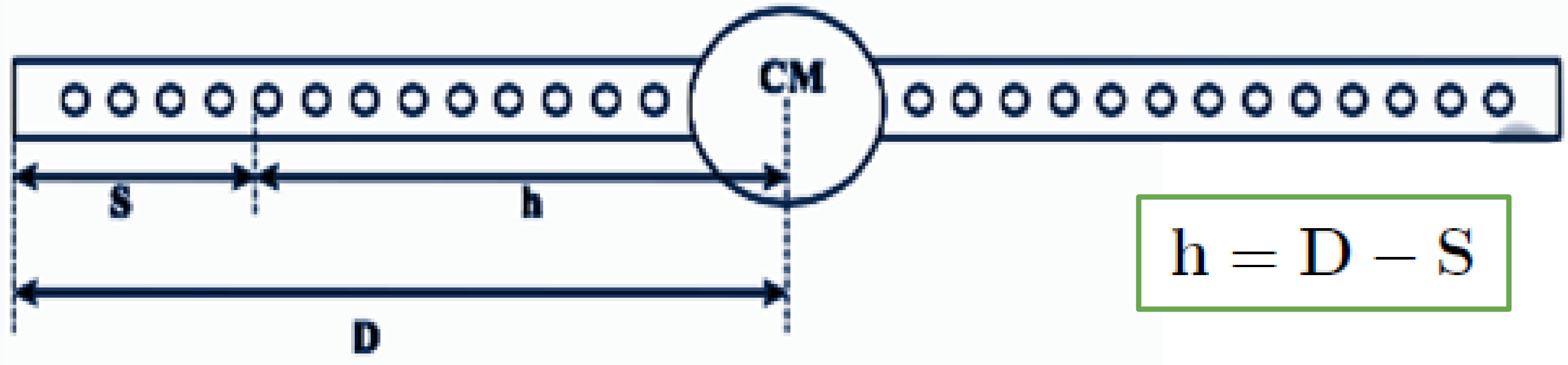


Then, the moment of inertia about the suspension point at a distance h from the center of mass is

$$\boxed{I = M(k^2 + h^2)}$$

$$I = I_{CM} + Mh^2$$

THE PHYSICAL PENDULUM



$$\boxed{h = D - S}$$

<u>Period of Physical Pendulum</u> : **Recall that**

$$\boxed{T = 2\pi\sqrt{\frac{I}{Mgh}}}$$

Combining the period expression with the moment of inertia, we can express the period as

$$\boxed{T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}}$$

THE PHYSICAL PENDULUM

The period of simple pendulum :

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Period of Physical Pendulum :

$$T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

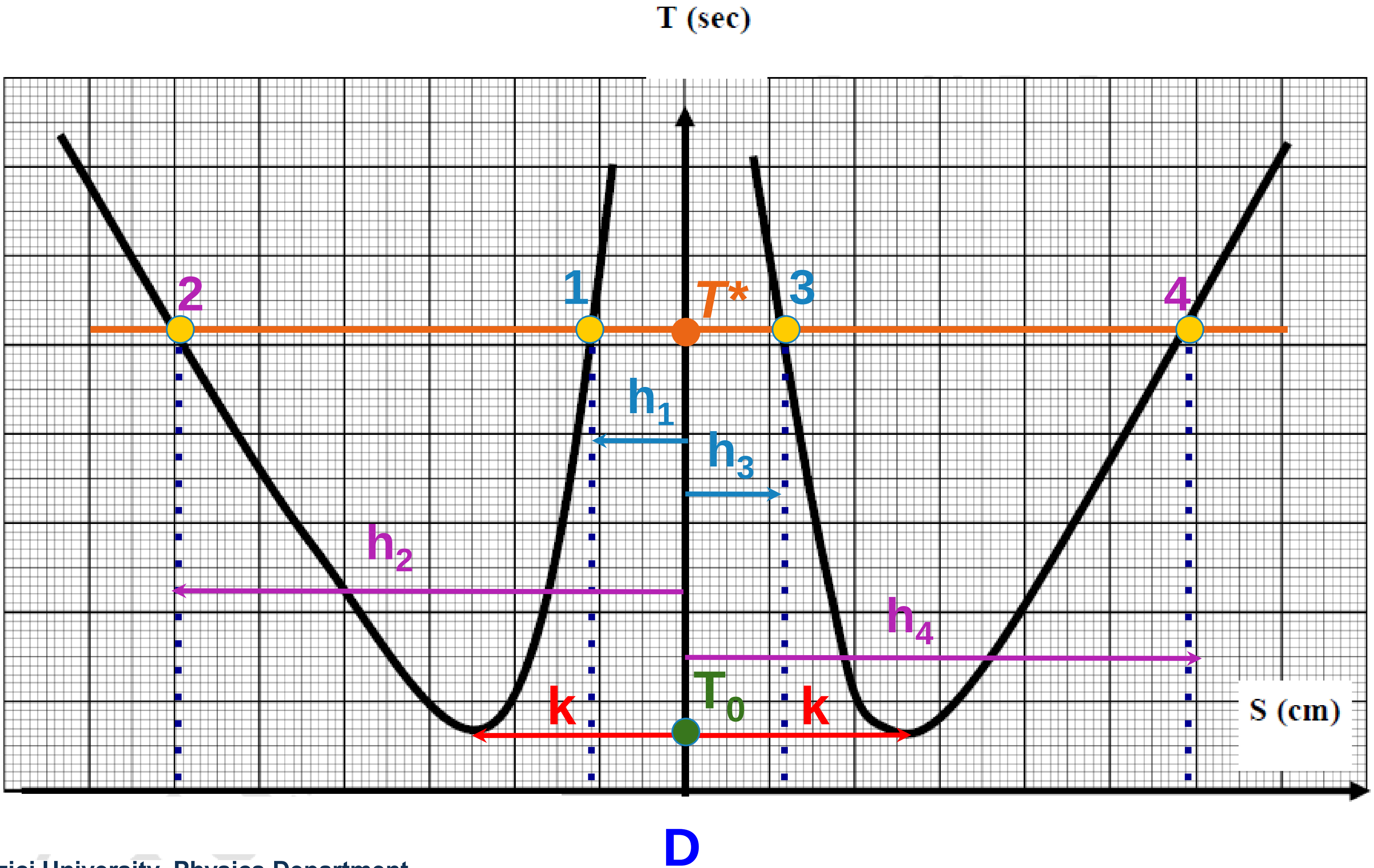
- ★ The physical pendulum is **equivalent to a simple pendulum** with length:

$$L = \frac{k^2 + h^2}{h}$$

THE PHYSICAL PENDULUM

If we plot T vs. S, we see that there are four possible points for a specific period value *T that we can hang the pendulum.**

➔ **Radius of gyration k gives the minimum period (h=0 case).**



For one side of the pendulum, two different distances h_1 and h_2 from the center of mass give the same period:

$$2\pi\sqrt{\frac{k^2+h_1^2}{gh_1}} = 2\pi\sqrt{\frac{k^2+h_2^2}{gh_2}}$$

Solving this equation for the radius of gyration yields

$$\boxed{k^2 = h_1 h_2}$$

Period

$$\boxed{T = 2\pi\sqrt{\frac{h_1+h_2}{g}}}$$

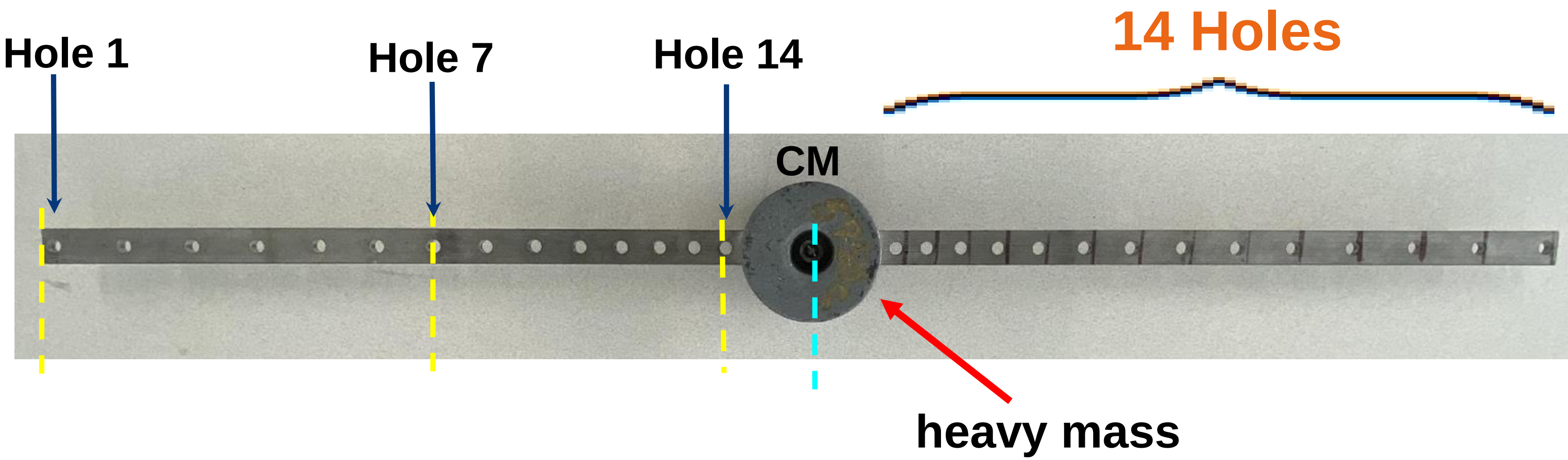
Length of Equivalent Simple Pendulum

$$\boxed{L = h_1 + h_2}$$

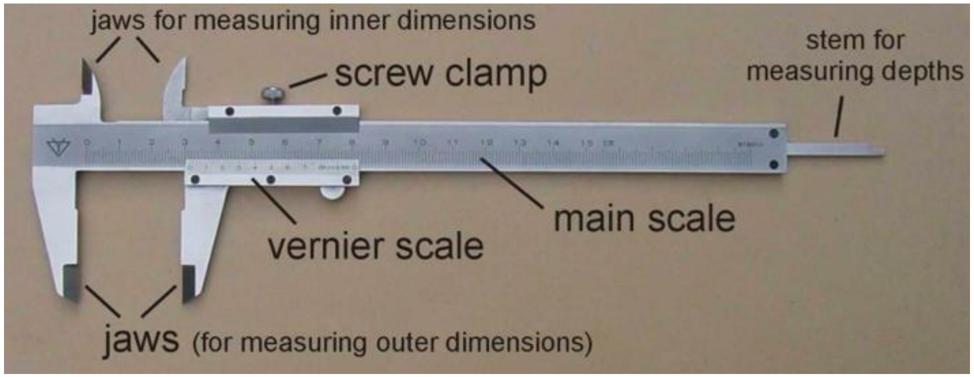
APPARATUS

THE PHYSICAL PENDULUM

Apparatus: **Physical Pendulum, Vernier Calipers, Ruler, Stopwatch**



Vernier Calipers



EXPERIMENT

What to measure : Distance from one end to the center of the pendulum (D), mass of the pendulum (M), distance from one end to each suspension point (S), time for a given number of oscillations for each hole (t)

What to calculate : Period of oscillation (T), radius of gyration (k), moment of inertia (I)

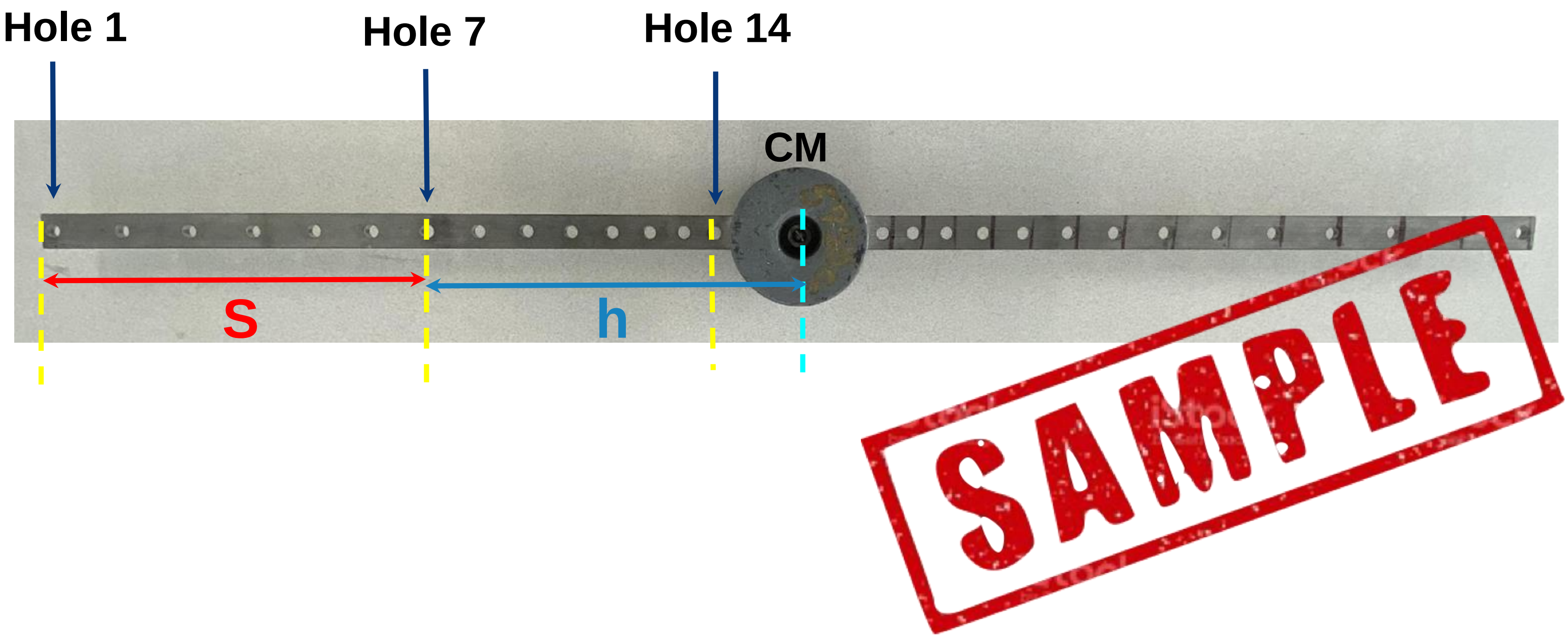
Experimental findings : Gravitational acceleration (g)

(fill in page 71)

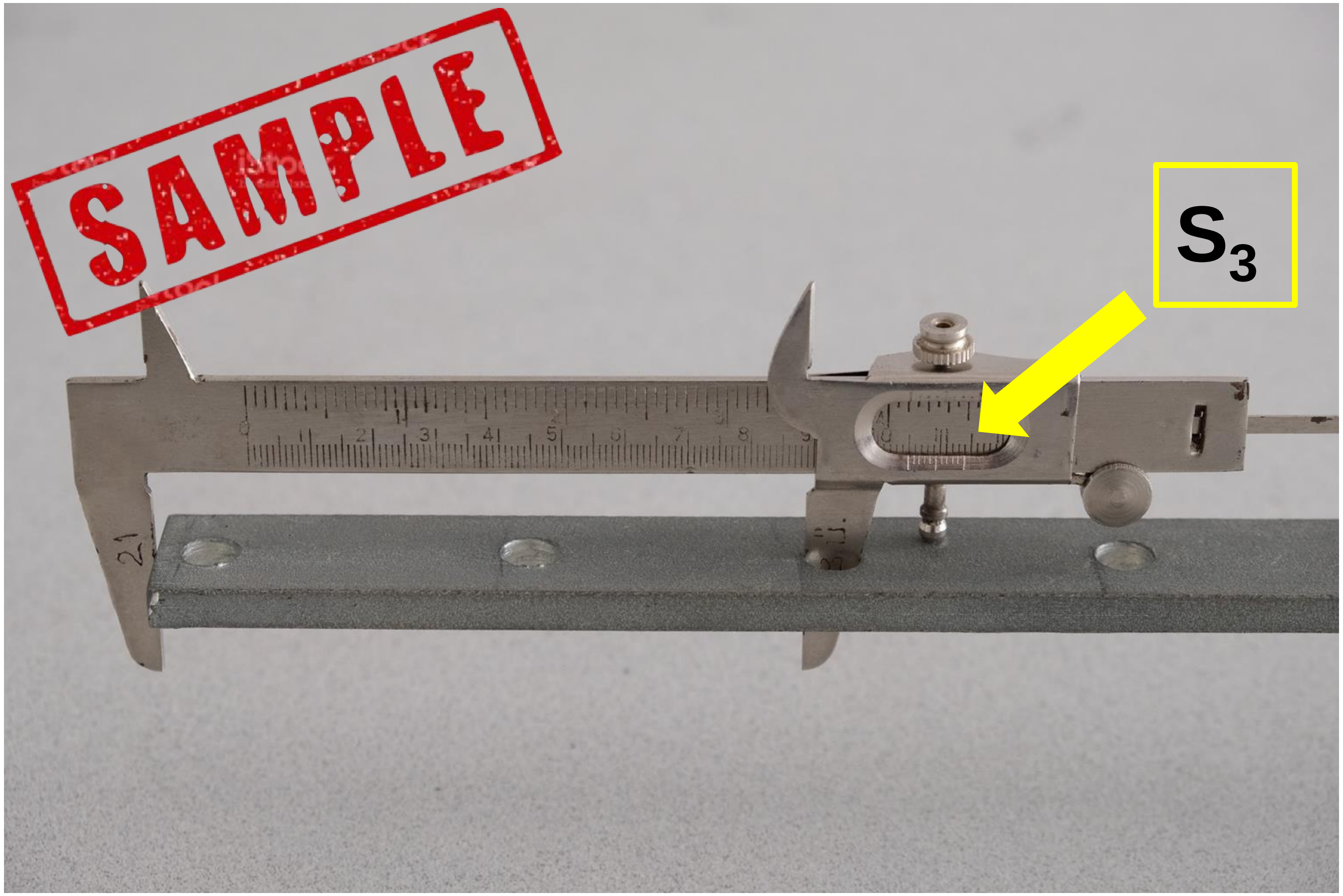
Distance from one end to the center of the pendulum	D	=	. . . .
Mass of of the pendulum	M	=	. . . .
Acceleration due to gravity	g_{TV}	=	. . . .

PROCEDURE:

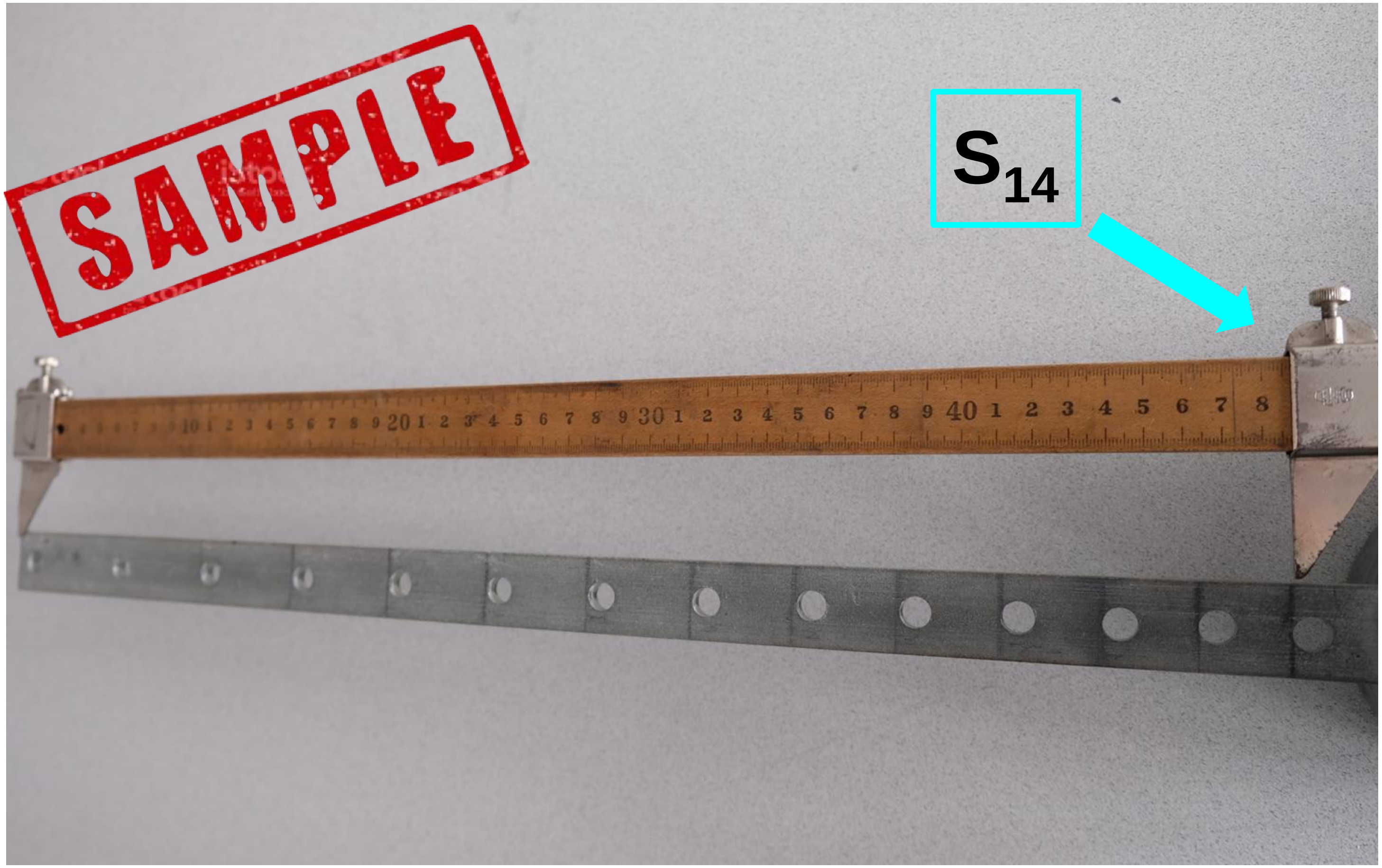
- Measure Distance S from one end to the suspension point for each hole.



- For SMALL distances (S_1, S_2, S_3), use Vernier Calipers

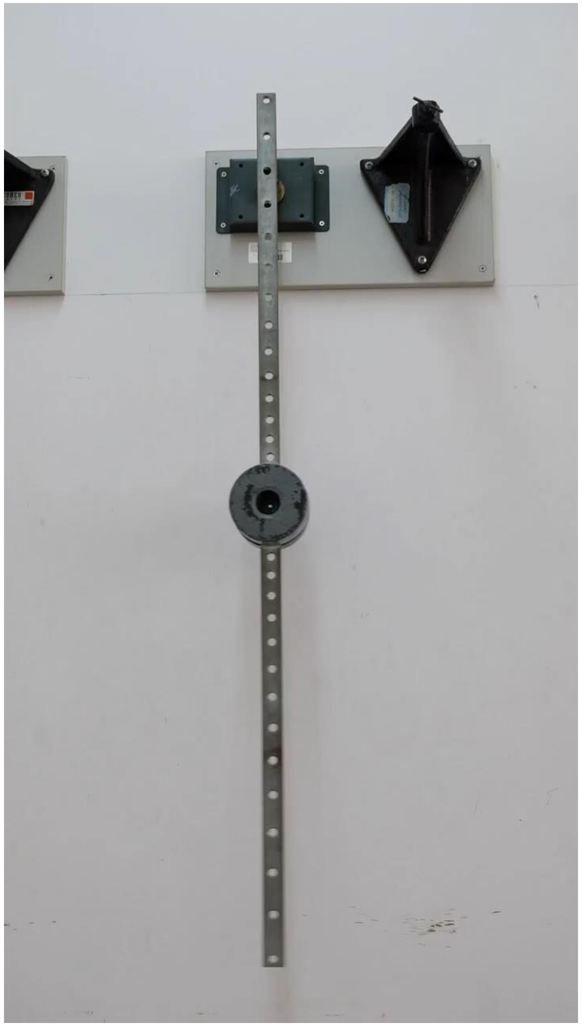


- For LARGER distances (S_4, S_5, ……, S_{14}), use a <u>Ruler</u>



THE PHYSICAL PENDULUM

- Hang the physical pendulum from each hole and make it oscillate.
- Measure time t for the given number of oscillations for each hole using a stopwatch.



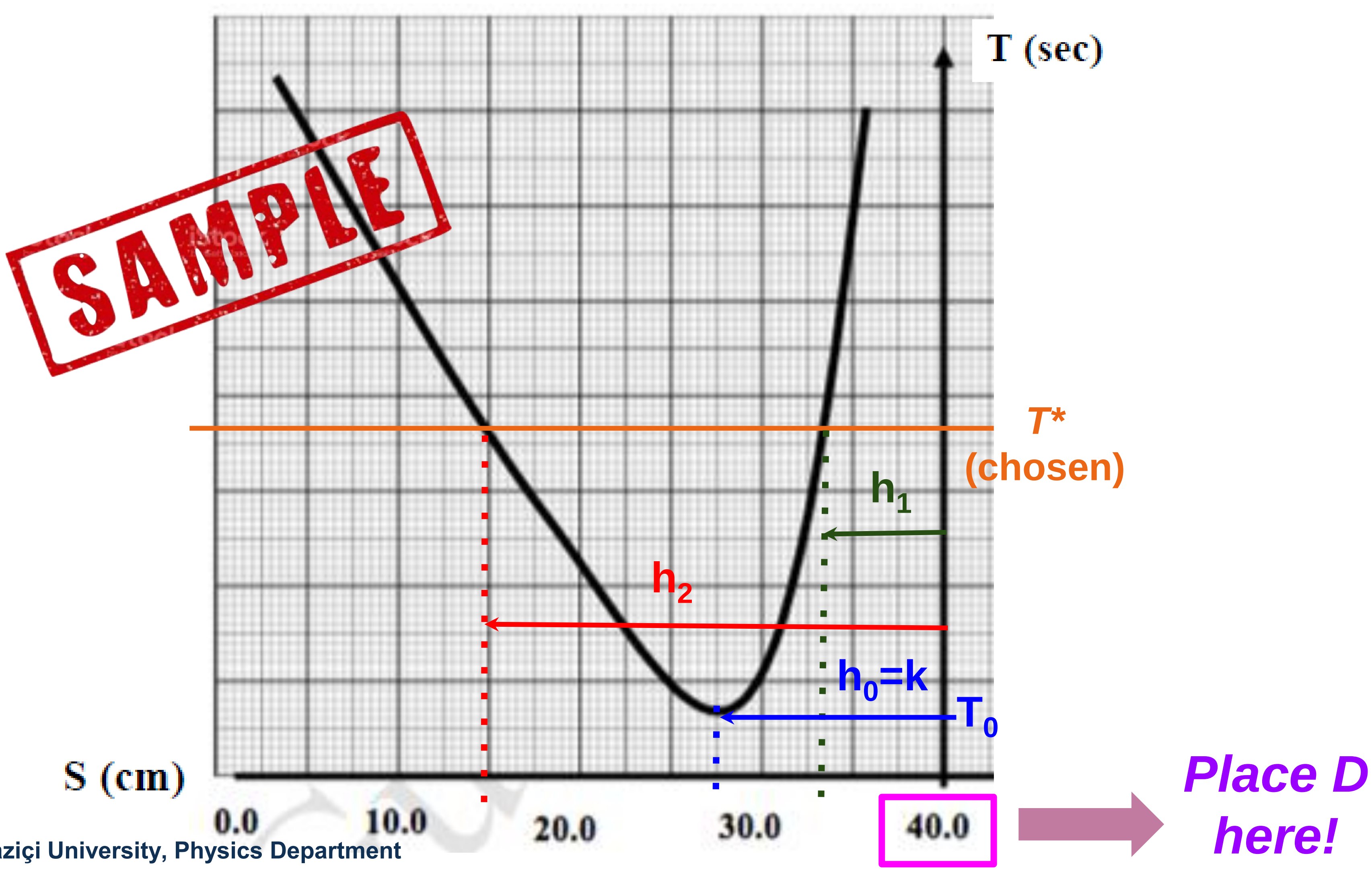
THE PHYSICAL PENDULUM

number of oscillations will be given in DataVideo!

Distance from one end of the pendulum to the suspension point S ( )	Time for n Period t ( )	One Period T ( )

(fill in page 73 for 14 holes)

- Plot S vs. T graph. Choose a period value *T** and draw a horizontal line corresponding to that period.



THE PHYSICAL PENDULUM

READ VALUES FROM THE GRAPH YOU HAVE DRAWN.

(fill in page 77)

Period (any chosen) T = ...

Minimum Period T_0 = ...

Distance from the center to the first suspension point for T, h_1 = ...

Distance from the center to the second suspension point for T, h_2 = ...

For minimum Period: h_o = ...

Radius of Gyration $k = h_o$ = ...

Determine h values by subtracting corresponding S values on the graph from distance D!

PhysLabs

Calculate using h_1 and h_2 you have read from the graph!

(fill in page 77)

Radius of Gyration $k = \sqrt{h_1 h_2}$ = ...

Length of the Equivalent Simple Pendulum L = ...

THE PHYSICAL PENDULUM

Moment of Inertia
about the CM $I_o = I_{CM}$ = ...

. . . .

Moment of Inertia
Corresponding $I_{(\text{for T})}$ = ...
to h_1

(fill in page 79)

. . . .

Moment of Inertia
Corresponding $I_{(\text{for T})}$ = ...
to h_2

. . . .

Acceleration
due to Gravity g_{EV} = ...

Use length of equivalent simple pendulum L and the chosen period T*!

(fill in page 79)

% Error for g $=$ . . . .

Dimensional analysis for the Radius of Gyration, k . .

Dimensional analysis for the moment of Inertia, I: . .