PHYSICS II: EXPERIMENTS

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Foreword

This book is written with a dual purpose in mind. Firstly, it aims to guide the students in the experiments of the elementary physics courses. Secondly, it incorporates the worksheets that the students use during their 2-hour laboratory session.

There are six books to accompany the six elementary physics courses taught at Bogazici University. After renovating our laboratories, replacing most of the equipment, and finally removing the 110-V electrical distribution in the laboratories, it has become necessary to prepare these books. Each book starts with the basic methods for data taking and analysis. These methods include brief descriptions for some of the instruments used in the experiments and the graphical method for fitting the data to a straight line. In the second part of the book, the specific experiments performed in a specific course are explained in detail. The objective of the experiment, a brief theoretical background, apparatus and the procedure for the experiment are given in this part. The worksheets designed to guide the students during the data taking and analysis follows this material for each experiment. Students are expected to perform their experiment and data analysis during the allotted time and then hand in the completed worksheet to the instructor by tearing it out of the book.

We would like to thank the members of the department that made helpful suggestions and supported this project, especially Arsin Arşık and Işın Akyüz who taught these laboratory classes for years. Our teaching assistants and student assistants were very helpful in applying the procedures and developing the worksheets. Of course, the smooth operation of the laboratories and the continuous well being of the equipment would not be possible without the help of our technicians, Erdal Özdemir and Hüseyin Yamak, who took over the job from Okan Ertuna.

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Part I. BASIC METHODS

Introduction

Physics is an experimental science. Physicists try to understand how nature works by making observations, proposing theoretical models and then testing these models through experiments. For example, when you drop an object from the top of a building, you observe that it starts with zero speed and hits the ground with some speed. From this simple observation you may deduce that the speed or the velocity of the object starts from zero and then increases, suggesting a nonzero acceleration.

Usually when we propose a new model we start with the simplest explanation. Assuming that the acceleration of the falling object is constant, we can derive a relationship between the time it takes to reach the ground and the height of the building. Then measuring these quantities many times we try to see whether the proposed relationship is valid. The next question would be to find an explanation for the cause of this motion, namely the Newton's Law. When Newton proposed his law, he derived it from his observations. Similarly, Kepler's laws are also derived from observations. By combining his laws of motion with Kepler's laws, Newton was able to propose the gravitational law of attraction. As you see, it all starts with measuring lengths, speeds, etc. You should understand your instruments very well and carry out the measurements properly. Measuring things correctly is absolutely essential for the success of your experiment.

Every time a new model or law is proposed, you can make some predictions about the outcome of new and untried experiments. You can test the proposed models by comparing the results of these actual experiments with the predictions. If the results disagree with the predictions, then the proposed model is discarded or modified. However, an agreement between the experimental results and the predictions is not sufficient for the acceptance of the specific model. Models are tested continuously to make sure that they are valid. Galilean relativity is modified and turned into the special relativity when we started measuring speeds in the order of the speed of light. Sometimes the modifications may occur before the tests are done. Of course, all physical laws are based on experimental studies. Experimental results always take precedence over theory. Obviously, experiments have to be done carefully and objectively without any bias. Uncertainties and any contributing systematic effects should be studied carefully.

This book is written for the laboratory part of the Introductory Physics courses taken by freshman and sophomore classes at Bogazici University. The first part of the book gives you basic information about statistics and data analysis. A brief theoretical background and a procedure for each experiment are given in the second part.

Experiments are designed to give students an understanding of experimental physics regardless of their major study areas, and also to complement the theoretical part of the course. They will introduce you to the experimental methods in physics. By doing these experiments, you will also be seeing the application of some of the physics laws you will be learning in the accompanying course.

You will learn how to use some basic instruments and interpret the results, to take and analyze data objectively, and to report their results. You will gain experience in data taking and improve your insight into the physics problems. You will be performing the experiments by following the procedures outlined for each experiment, which will help you gain confidence in experimental work. Even though the experiments are designed to be simple, you may have some errors due to systematical effects and so your results may be different from what you would expect theoretically. You will see that there is a difference between real-life physics and the models you are learning in class.

You are required to use the worksheets to report your results. You should include all your calculations and measurements to show that you have completed the experiment fully and carried out the required analysis yourself.

DATA TAKING AND ANALYSIS

Dimensions and Units

A physical quantity has one type of dimension but it may have many units. The dimension of a quantity defines its characteristic. For example, when we say that a quantity has the dimension of length (L), we immediately know that it is a distance between two points and measured in terms of units like meter, foot, etc. This may sound too obvious to talk about, but dimensional analysis will help you find out if there is a mistake in your derivations. Both sides of an equation must have the same dimension. If this is not the case, you may have made an error and you must go back and recheck your calculations. Another use of a dimensional analysis is to determine the form of the empirical equations. For example, if you are trying to determine the relationship between the distance traveled under constant acceleration and the time involved empirically, then you should write the equation as

$$d = kat^n$$

where k is a dimensionless quantity and a is the acceleration. Then, rewriting this expression in terms of the corresponding dimensions:

$$L = \left(LT^{-2}\right)T^n$$

will give us the exponent n as 2 right away. You will be asked to perform dimensional analysis in most of the experiments to help you familiarize with this important part of the experimental work.

Measurement and Instruments

To be a successful experimenter, one has to work in a highly disciplined way. The equipment used in the experiment should be treated properly, since the quality of the data you will obtain will depend on the condition of the equipment used. Also, the equipment has a certain cost and it may be used in the next experiment. Mistreating the equipment may have negative effects on the result of the experiment, too.

In addition to following the procedure for the experiment correctly and patiently, an experimenter should be aware of the dangers in the experiment and pay attention to the warnings. In some cases, eating and drinking in the laboratory may have harmful effects on you because food might be contaminated by the hazardous materials involved in the experiment, such as radioactive materials. Spilled food and drink may also cause malfunctions in the equipment or systematic effects in the measurements.

Measurement is a process in which one tries to determine the amount of a specific quantity in terms of a pre-calibrated unit amount. This comparison is made with the help of an instrument. In a measurement process only the interval where the real value exists can be determined. Smaller interval means better precision of the instrument. The smallest fraction of the pre-calibrated unit amount determines the precision of the instrument.

You should have a very good knowledge of the instruments you will be using in your measurements to achieve the best possible results from your work. Here we will explain how to use some of the basic instruments you will come across in this course.

Reading analog scales:

You will be using several different types of scales. Examples of these different types of scales are rulers, vernier calipers, micrometers, and instruments with pointers.

The simplest scale is the **meter stick** where you can measure lengths to a millimeter. The precision of a ruler is usually the smallest of its divisions.

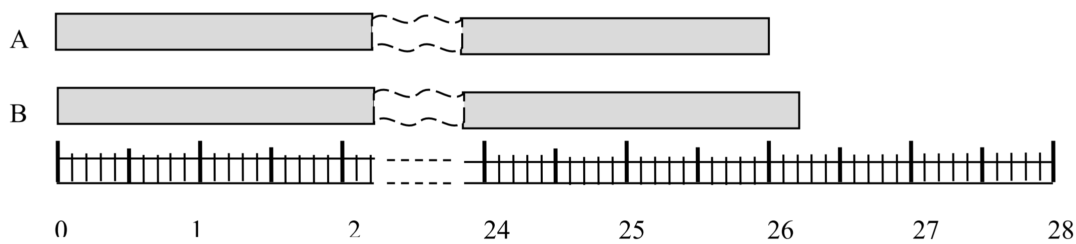


Figure 1. Length measurement by a ruler

In Figure 1, the lengths of object A and B are observed to be around 26 cm. Since we use a ruler with millimeter division the measurement result for the object A should be given as 26.0 cm and B as 26.2 cm. If you report a value more precise than a millimeter when you use a ruler with millimeter division, obviously you are guessing the additional decimal points.

Vernier Calipers (Figure 2) are instruments designed to extend the precision of a simple ruler by one decimal point. When you place an object between the jaws, you may obtain an accurate value by combining readings from the main ruler and the scale on the frame attached to the movable jaw. First, you record the value from the main ruler where the zero line on the frame points to. Then, you look for the lines on the frame and the main ruler that looks like the same line continuing in both scales. The number corresponding to this line on the frame gives you the next digit in the measurement. In Figure 2, the measurement is read as 1.23 cm. The precision of a vernier calipers is the smallest of its divisions, 0.1 mm in this case.

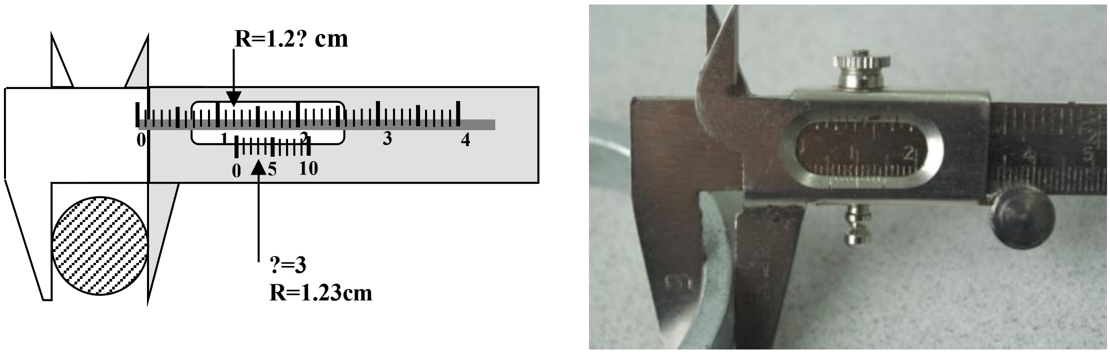
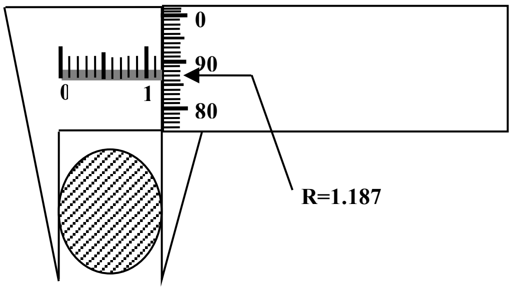


Figure 2. Vernier Calipers.

Micrometer (Figure 3) is similar to the vernier calipers, but it provides an even higher precision. Instead of a movable frame with the next decimal division, the micrometer has a cylindrical scale usually divided into a hundred divisions and moves along the main ruler like a screw by turning the handle. Again the coarse value is obtained from the main ruler and the more precise part of the measurement comes from the scale around the rim of the cylindrical part. Because of its higher precision, it is used mostly to measure the thickness of wires and similar things. In Figure 3, the measurement is read as 1.187 cm. The precision of a micrometer is the smallest of its divisions, 0.01 mm in this case.

Here is an example for the measurement of the radius of a disk where a ruler, a vernier calipers, and a micrometer are used, respectively:

Measurement	Precision	Instrument
$R = (23 \pm 1)mm$	1 mm	Ruler
$R = (23.1 \pm 0.1)mm$	0.1 mm	Vernier calipers
$R = (23.14 \pm 0.01)mm$	0.01 mm	Micrometer



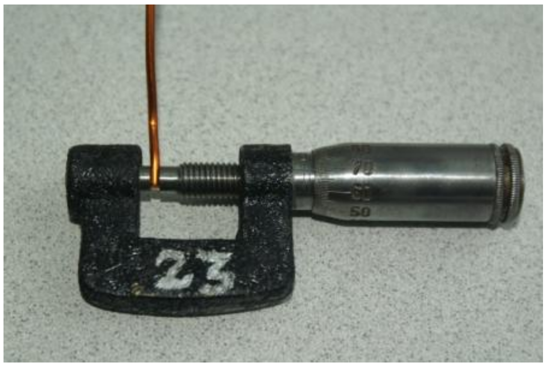


Figure 3. Micrometer.

Spherometer (Figure 4) is an instrument to determine very small thicknesses and the radius of curvature of a surface. First you should place the spherometer on a level surface to get a calibration reading (CR). You turn the knob at the top until all four legs touch the surface. When the middle leg also touches the surface, the knob will first seem to be free and then tight. The reading at this position will be the calibration reading (CR). Then you should place the spherometer on the curved surface and turn the knob until all four legs again touch the surface. The reading at this position will be the measurement reading (MR). You will read the value from the vertical scale first and then the value on the dial will give you the fraction of a millimeter. Then you can calculate the radius of curvature of the surface as:

$$R = \frac{D}{2} + \frac{A^2}{6D}$$

where $D = |CR\text{-}MR|$ and A is the distance between the outside legs.

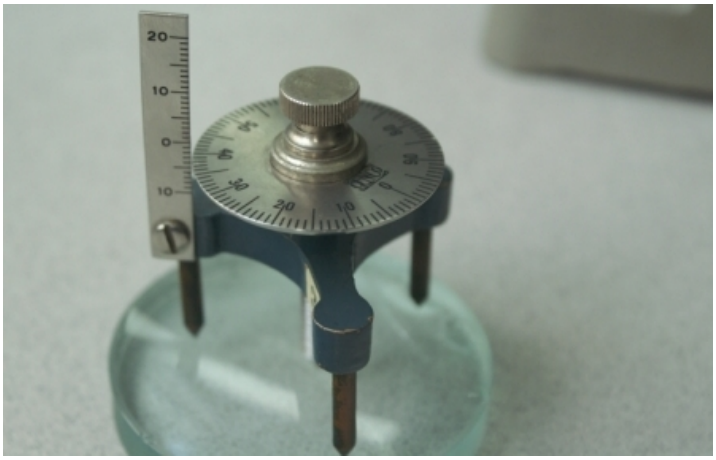


Figure 4. Spherometer.

Instruments with pointers usually have a scale along the path that the pointer moves. Mostly the scales are curved since the pointers move in a circular arc. To avoid the systematic errors introduced by the viewing angle, one should always read the value from the scale where the pointer is projected perpendicularly. You should not read the value by looking at the pointer and the scale sideways or at different angles. You should always look at the scale and the pointer perpendicularly. Usually in most instruments there is a mirror attached to the scale to make sure the readings are done similarly every time when you take a measurement (Figure 5). When you bring the scale and its image on the mirror on top of each other, you will be looking at the pointer and the scale perpendicularly. Then you can record the value that the pointer shows on the scale. Whenever you measure something by such an instrument, you should follow the same procedure.

Figure 5. A voltmeter with a mirror scale.

Data Logger

In some experiments we will be using sensors to measure some quantities like position, angle, angular velocity, temperature, etc. The output of these sensors will be converted into numbers with the help of a data acquisition instrument called DATA LOGGER (Figure 6).

Data Logger is a versatile instrument that takes data using changeable sensors. When you plug a sensor to its receptacle at the top, it recognizes the type of the sensor. When you turn the data logger on with a sensor attached, it will start displaying the default mode for that sensor. Data taking with the data logger is very simple. You can start data taking by pressing the Start/Stop button (7). You may change the display mode by pressing the button on the right with three rectangles (6). To change the default measurement mode, you should press the plus or minus buttons (3 or 4). If there is more than one type of quantity because of the specific sensor you are using, you may select the type by pressing the button with a check mark (5) to turn on the editing mode and then selecting the desired type by using the plus and minus buttons (3 or 4). You will exit from the editing mode by pressing the button with the check mark (5) again. You may edit any of the default settings by using the editing and plus-minus buttons. For a more detailed operation of the instrument you should consult your instructor.

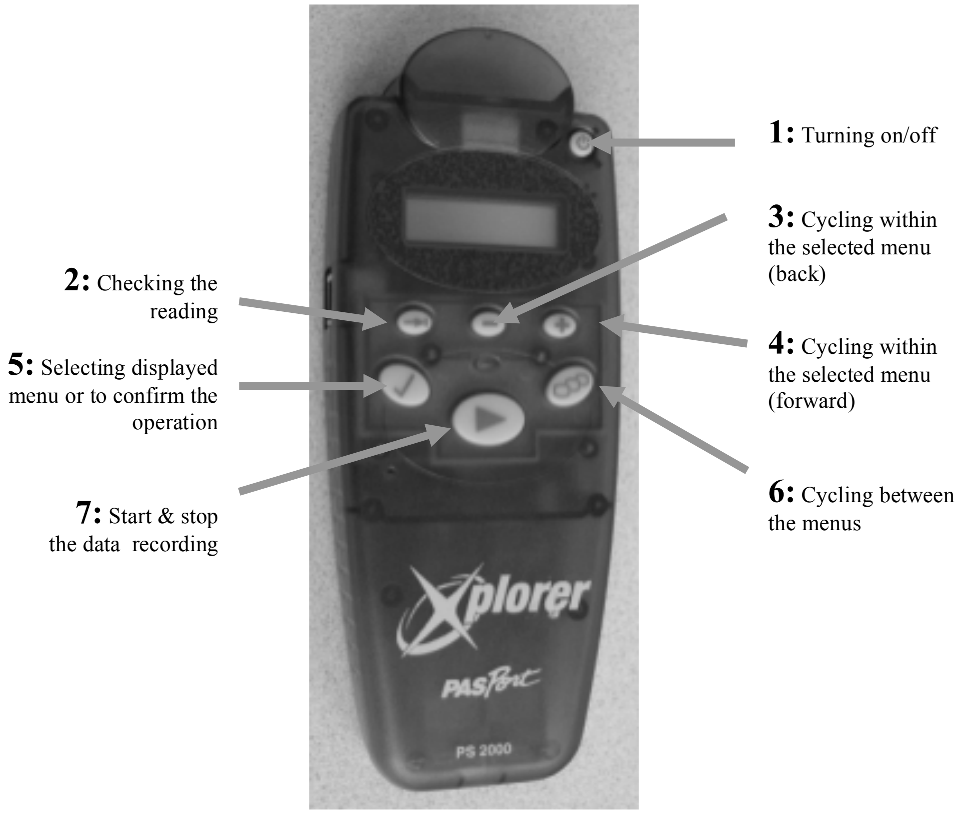


Figure 6. Data Logger.

Basics of Statistics and Data Analysis

Here, you will have an introduction to statistical methods, such as distributions and averages.

All the measurements are done for the purpose of obtaining the value for a specific quantity. However, the value by itself is not enough. Determining the value is half the experiment. The other half is determining the uncertainty. Sometimes, the whole purpose of an experiment may be to determine the uncertainty in the results.
Error and uncertainty are synonymous in experimental physics even though they are two different concepts. Error is the deviation from the true value. Uncertainty, on the other hand, defines an interval where the true value is. Since we do not know the true value, when we say error we actually mean uncertainty. Sometimes the accepted value for a quantity after many experiments is assumed to be the true value.

Sample and parent population

When you carry out an experiment, usually you take data in a finite number of trials. This is our sample population. Imagine that you have infinite amount of time, money, and effort available for the experiment. You repeat the measurement infinite times and obtain a data set that has all possible outcomes of the experiment. This special sample population is called parent population since all possible sample populations can be derived from this infinite set. In principle, experiments are carried out to obtain a very good representation of the parent population, since the parameters that we are trying to measure are those that belong to the parent population. However, since we can only get an approximation for the parent population, values determined from the sample populations are the best estimates.

Mean and Standard deviation

Measuring a quantity usually involves statistical fluctuations around some value. Multiple measurements included in a sample population may have different values. Usually, taking an average cancels the statistical fluctuations to first degree. Hence, the average value or the mean value of a quantity in a sample population is a good estimate for that quantity.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Even though the average value obtained from the sample population is the best estimate, it is still an estimate for the true value. We should have another parameter that tells us how close we are to the true value. The variance of the sample:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

gives an idea about how scattered the data are around the mean value. Variance is in fact a measure of the average deviation from the mean value. Since there might be negative and positive deviations, squares of the deviations are averaged to avoid a null result. Because the variance is the average of the squares, square root of variance is a better quantity that shows the scatter around the mean value. The square root of the variance is called standard deviation:

$$\sigma_s = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

However, the standard deviation calculated this way is just the standard deviation of the sample population. What we need is the standard deviation of the parent population. The best estimate for the standard deviation of the parent population can be shown to be:

$$\sigma_p = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

As the number of measurements, N, becomes large or as the sample population approaches parent population, standard deviation of the sample is almost equal to the standard deviation of the parent population.

Distributions

The probability of obtaining a specific value can be determined by dividing the number of measurements with that value to the total number of measurements in a sample population. Obviously, the probabilities obtained from the parent population are the best estimates. Total probability should be equal to 1 and probabilities should be larger as one gets closer to the mean value. The set of probability values associated with a population is called the probability distribution for that measurement. Probability distributions can be experimental distributions obtained from a measurement or mathematical functions. In physics, the most frequently used mathematical distributions are Binomial, Poisson, Gaussian, and Lorentzian. Gaussian and Poisson distributions are in fact special cases of Binomial distribution. However, in most cases, Gaussian distribution is a good approximation. In fact, all distributions approach Gaussian distribution at the limit (Central Limit Theorem).

Errors

The result of an experiment done for the first time almost always turns out to be wrong because you are not familiar with the setup and may have systematic effects. However, as you continue to take data, you will gain experience in the experiment and learn how to reduce the systematical effects. In addition to that, increasing number of measurements will result in a better estimate for the mean value of the parent population.

Errors in measurements: Statistical and Systematical errors

As mentioned above, error is the deviation between the measured value and the true value. Since we do not know the true value, we cannot determine the error in this sense. On the other hand, uncertainty in our measurement can tell us how close we are to the true value. Assuming that the probability distribution for our measurement is a Gaussian distribution, 68% of all possible measurements can be found within one standard deviation of the mean value. Since most physical distributions can be approximated by a Gaussian, defining the standard deviation as our uncertainty for that measurement will be a reasonable estimate. In some cases, two-standard deviation or two-sigma interval is taken as the uncertainty. However, for our purposes using the standard deviation as the uncertainty would be more than enough. Also, from now on, whenever we use error, we will actually mean uncertainty.

Errors or uncertainties can be classified into two major groups; statistical and systematical.

Statistical Errors

Statistical errors or random errors are caused by statistical fluctuations in the measurements. Even though some unknown phenomenon might be causing these fluctuations, they are mostly random in nature. If the size of the sample population is large enough, then there is equal number of measurements on each side of the mean at about similar distances. Therefore, averaging over such a large number of measurements will smooth the data and cancel the effect of these fluctuations. In fact, as the number of measurements increases, the effect of the random fluctuations on the average will diminish. Taking as much data as possible improves statistical uncertainty.

Systematical Errors

On the other hand, systematical errors are not caused by random fluctuations. One could not reduce systematical errors by taking more data. Systematical errors are caused by various reasons, such as, the miscalibration of the instruments, the incorrect application of the procedure, additional unknown physical effects, or anything that affects the quantity we are measuring. Systematic errors caused by the problems in the measuring instruments are also called instrumental errors. Systematical errors are reduced or avoided by finding and removing the cause.

Example 1: You are trying to measure the length of a pipe. The meter stick you are going to use for this purpose is constructed in such a way that it is missing a millimeter from the beginning. Since both ends of the meter stick are covered by a piece of metal, you do not see that your meter stick is 1 mm short at the beginning. Every time you use this meter stick, your measurement is actually 1 mm longer than it should be. This will be the case if you repeat the measurement a few times or a few million times. This is a systematical error and, since it is caused by a problem in the instrument used, it is considered an instrumental error. Once you know the cause, that is, the shortness of your meter stick, you can either repeat your measurement with a proper meter stick or add 1 mm to every single measurement you have done with that particular meter stick.

Example 2: You might be measuring electrical current with an ammeter that shows a nonzero value even when it is not connected to the circuit. In a moving coil instrument this is possible if the zero adjustment of the pointer is not done well and the pointer always shows a specific value when there is no current. The error caused by this is also an instrumental error.

Example 3: At CERN, the European Research Center for Nuclear and Particle Physics, there is a 28 km long circular tunnel underground. This tunnel was dug about 100 m below the surface. It was very important to point the direction of the digging underground with very high precision. If there were an error, instead of getting a complete circle, one would get a tunnel that is not coming back to the starting point exactly. One of the inputs for the topographical measurements was the direction towards the center of the earth. This could be determined in principle with a plumb bob (or a piece of metal hung on a string) pointing downwards under the influence of gravity. However, when there is a mountain range on one side and a flat terrain on the other side (like the location of the CERN accelerator ring), the direction given by the plumb bob will be slightly off towards the mountainous side. This is a systematic effect in the measurement and since its existence is known, the result can be corrected for this effect.

Once the existence and the cause of a systematic effect are known, it is possible to either change the procedure to avoid it or correct it. However, we may not always be fortunate enough to know if there is a systematic effect in our measurements. Sometimes, there might be unknown factors that affect our experiment. The repetition of the measurement under different conditions, at different locations, and with totally different procedures is the only way to remove the unknown systematic effects. In fact, this is one of the fundamentals of the scientific method.

We should also mention the accuracy and precision of a measurement. The meaning of the word "accuracy" is closeness to the true value. As for "precision," it means a measurement with higher resolution (more significant figures or digits). An instrument may be accurate but not precise or vice versa. For example, a meter stick with millimeter divisions may show the correct value. On the other hand, a meter stick with 0.1 mm division may not show the correct value if it is missing a one-millimeter piece from the beginning of the scale. However, if an instrument is precise, it is usually an expensive and well designed instrument and we expect it to be accurate.

Reporting Errors: Significant figures and error values

As mentioned above, determining the error in an experiment requires almost the same amount of work as determining the value. Sometimes, almost all the effort goes into determining the uncertainty in a measurement.

Using significant figures is a crude but an effective way of reporting the errors. A simple definition for significant figures is the number of digits that one can get from a measuring instrument (but not a calculator!). For example, a digital voltmeter with a four-digit display can only provide voltage values with four digits. All these four digits are significant unless otherwise noted. On the other hand, reporting a six digit value when using an analog voltmeter whose smallest division corresponds to a four-digit reading would be wrong. One could try to estimate the reading to the fraction of the smallest division, but then this estimate would have a large uncertainty.

Significant figures are defined as following:

- Leftmost nonzero digit is the most significant figure.

  Examples: 0.0000**6**520 m
  1234 m
  41.02 m
  126.1 m
  4120 m
  12000 m

- Rightmost nonzero digit is the least significant figure if there is no decimal point.

  Examples: 123**4** m
  41**2**0 m
  1**2**000 m

- If there is a decimal point, rightmost digit is the least significant figure even if it is zero.

  Examples: 0.0000652**0** m
  41.0**2** m
  126.**1** m

Then, the number of significant figures is the number of digits between the most and the least significant figures including them.

Examples:	0.0000**6**52**0** m	4 significant figures
	123**4** m	4 sf
	41.0**2** m	4 sf
	126.**1** m	4 sf
	41**2**0 m	3 sf
	12000 m	2 sf
	1.200**0** x 10^4 m	5 sf

Significant figures of the results of simple operations usually depend on the significant figures of the numbers entering into the arithmetic operations. Multiplication or division of two numbers with different numbers of significant figures should result in a value with a number of significant figures similar to the one with the smallest number of significant figure. For example, if you multiply a three-significant-figure number with a two-significant-figure number, the result should be a two-significant-figure number. On the other hand, when adding or subtracting two numbers, the outcome should have the same number of significant figures as the smallest of the numbers entering into the calculation. If the numbers have decimal points, then the result should have the number of significant figures equal to the smallest number of digits after the decimal point. For example, if three values, two with two significant figures and one with four significant figures after the decimal point, are added or subtracted, the result should have two significant figures after the decimal point.

Example: Two different rulers are used to measure the length of a table. First, a ruler with 1-m length is used. The smallest division in this ruler is one millimeter. Hence, the result from this ruler would be 1.000 m. However, the table is slightly longer than one meter. A second ruler is placed after the first one. The second ruler can measure with a precision of one tenth of a millimeter. Let's assume that it gives a reading of 0.2498 m. To find the total length of the table we should add these two values. The result of the addition will be 1.2498, but it will not have the correct number of significant figures since one has three and the other has four significant figures after the decimal point. The result should have three significant figures after the decimal point. We can get the correct value by rounding off the number to three significant figures after the decimal point and report it as 1.250 m.

More Examples for Addition and Subtraction:

4.122
3.**74**
\+ 0.011
7.873 = 7.**87**   (2 digits after the decimal point)

Examples for Multiplication and Division:

4.782 x **3.05** = 14.5851 = **14.6**   (3 significant figures)

3.728 / 1.6781 = 2.22156 = **2.222**   (4 significant figures)

Rounding off

Sometimes you may have more numbers than the correct number of significant figures. This might happen when you divide two numbers and your calculator may give you as many digits as it has in its display. Then you should reduce the number of digits to the correct number of significant figures by rounding it off. One common mistake is by starting from the rightmost digit and repeatedly rounding off until you reach the correct number of significant figures. However, all the extra digits above and beyond the number of correct significant figures have no significance. Usually you should keep one extra digit in your calculations and then round this extra digit at the end. You should just discard the extra digits other than the one next to the least significant figure. The reasoning behind the rounding off process is to bring the value to the correct number of significant figures without adding or subtracting an amount in a statistical sense. To achieve this you should follow the procedure outlined below:

- If the number on the right is less than 5, discard it.
- If it is more than 5, increase the number on its left by one.
- If the number is exactly five, then you should look at the number on its left.
    - If the number on its left is even then again discard it.
    - If the number on the left of 5 is odd, then you should increase it by one.

This special treatment in the case of 5 is because there are four possibilities below and above five and adding five to any of them will introduce a bias towards that side. Hence, grouping the number on the left into even and odd numbers makes sure that this ninth case is divided into exactly two subsets; five even and five odd numbers. We count zero in this case since it is in the significant part. We do not count zero on the right because it is not significant.

Example: Rounding off 2.4456789 to three significant figures by starting from all the way to the right, namely starting from the number 9, and repeatedly rounding off until three significant figures are left would result in 2.45 but this would be wrong. The correct way of doing this is first dropping all the non-significant figures except one and then rounding it off, that is, after truncation 2.445 is rounded off to 2.44.

More Examples: Round off the given numbers to 4 significant figures:

43.3**7**468	=	43.37	=	43.**4**
43.3**4**468	=	43.34	=	43.**3**
43.3**5**468	=	43.35	=	43.**4**
43.4**5**568	=	43.45	=	43.**4**

If we determine the standard deviation for a specific value, then we can use that as the uncertainty since it gives us a better estimate. In this case, we should still pay attention to the number of significant figures since reporting extra digits is meaningless. For example if you have the average and the standard deviation as 2.567 and 0.1, respectively, then it would be appropriate to report your result as 2.6±0.1.

Weighted Averages

Sometimes we may measure the same quantity in different sessions. As a result we will have different sets of values and uncertainties. By combining all these sets we may achieve a better result with a smaller uncertainty. To calculate the overall average and standard deviation, we can assign weight to each value with the corresponding variance and then calculate the weighted average.

$$\mu = \frac{\sum_{i=1}^{m} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{m} \frac{1}{\sigma_i^2}}$$

Similarly we can also calculate the overall standard deviation.

$$\sigma_{\mu} = \sqrt{\frac{1}{\sum_{i=1}^{m} \frac{1}{\sigma_i^2}}}$$

Error Propagation

If you are measuring a single quantity in an experiment, you can determine the final value by calculating the average and the standard deviation. However, this may not be the case in some experiments. You may be measuring more than one quantity and combining all these quantities to get another quantity. For example, you may be measuring x and y and by combining these to obtain a third quantity z:

$$z = ax + by \qquad \text{or} \qquad z = f(x, y)$$

You could calculate z for every single measurement and find its average and standard deviation. However, a better and more efficient way of doing it is to use the average values of x and y to calculate the average value of z. In order to determine the variance of z, we have to use the square of the differential of z:

$$(dz)^2 = \left(\sum (\frac{\partial f}{\partial x})dx + (\frac{\partial f}{\partial y})dy \right)^2$$

Variance would be simply the sum of the squares of both sides over the whole sample set divided by the number of data points N (or N-1 for the parent population). Then, the general expression for determining the variance of the calculated quantity as a function of the measured quantities would be:

$$\sigma_z^2 = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial x_j} \right)^2 \sigma_j^2 \qquad \text{for } k \text{ number of measured quantities.}$$

Applying this expression to specific cases would give us the corresponding error propagation rule. Some special cases are listed below:

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 \qquad \text{for} \qquad z = ax \pm by$$

$$\frac{\sigma_z^2}{z^2} = \frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2} \qquad \text{for} \qquad z = axy \text{ or } \frac{ax}{y} \text{ or } \frac{ay}{x}$$

$$\frac{\sigma_z^2}{z^2} = b^2 \frac{\sigma_x^2}{x^2} \qquad \text{for} \qquad z = ax^b$$

$$\frac{\sigma_z^2}{z^2} = (b \ln a)^2 \sigma_x^2 \qquad \text{for} \qquad z = a^{bx}$$

$$\frac{\sigma_z^2}{z^2} = b^2 \sigma_x^2 \qquad \text{for} \qquad z = ae^{bx}$$

$$\sigma_z^2 = a^2 \frac{\sigma_x^2}{x^2} \qquad \text{for} \qquad z = a \ln(bx)$$

Multivariable measurements: Fitting procedures

When you are measuring a single quantity or several quantities and then calculating the final quantity using the measured values, all the measurements involve unrelated quantities. There are no relationships between them other than the calculated and measured quantities. However, in some cases you may have to set one or more quantities and measure another quantity determined by the independent variables. This is the case when you have a function relating some quantities to each other. For example, the simplest function would be the linear relationship:

$$y = ax + b$$

where a is called the slope and b the y-intercept. Since we are setting the value of the independent variable x, we assume its uncertainty to be negligible compared to the dependent variable y. Of course, we should be able to determine the uncertainty in y. From such an experiment, usually we have to determine the parameters that define the function; a and b. This can be done by fitting the data to a straight line.

The least squares (or maximum likelihood, or chi-square minimization) method would provide us with the best possible estimates. However, this method involves lengthy calculations and we will not be using it in this course.

We will be using a graphical method that will give us the parameters that we are looking for. It is not as precise as the least squares method and does not give us the uncertainties in the parameters, but it provides answers in a short time that is available to you.

Graphical method is only good for linear cases. However, there are some exceptions to this either by transforming the functions to make them linear or plotting the data on a semi-log or log-log or polar graph paper (Figure 7).$1/r$, $1/r^2$, $y = ax^5$, $y = ae^{-bx}$, are some examples for nonlinear functions that can be transformed to linear expressions. $1/r^n$ type expressions can be linearized by substituting $1/r^n$ with a simple x: $y = A + B/r^n \rightarrow y = A + Bx$ where $x = 1/r^n$. Power functions can be linearized by taking the logarithm of the function: $y = ax^n$ becomes $\log y = \log a + n \log x$ and then through $y' = \log y$, $a' = \log a$, and $x' = \log x$ transformation it becomes $y' = a' + nx'$. Exponential functions can be transformed similar to the power functions by taking the natural logarithm: $y = ae^{-bx}$ becomes $\ln y = \ln a - bx$ and through $y' = \ln y$ and $a' = \ln a$ transformation it becomes $y' = a' - bx$.

Before attempting to obtain the parameters that we are looking for, we have to plot the data on a graph paper. As long as we have linearly dependent quantities or transformed quantities as explained above, we can use regular graph paper.

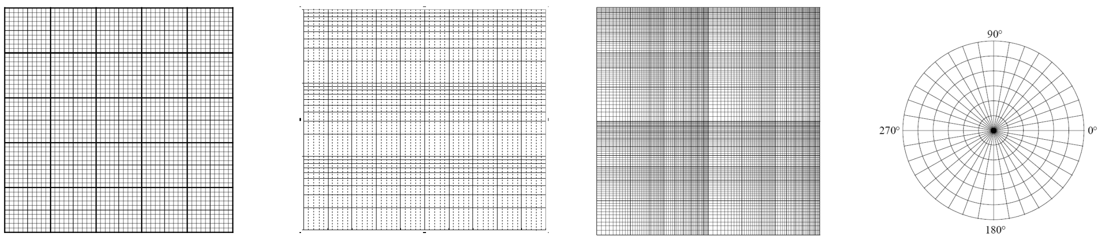


Figure 7: Different types of graph papers: linear, semi-log, log-log, and polar.

You should use as much area of the graph paper as possible when you plot your data. Your graph should not be squeezed to a corner with lots of empty space. To do this, first you should determine the minimum and maximum values for each variable, *x* and *y*, then choose a proper scale value. For example, if you have values ranging from 3 to 110 and your graph paper is 23 centimeters long, then you should choose a scale factor of 1 cm to 5 units of your variable and label your axis from 0 to 115 and marking each big square (usually linear graph papers prepared in cm and millimeter divisions) at increasing multiples of 5. You should choose the other axis in a similar way. When you select a scale factor you should select a factor that is easy to divide by, like 1, 2, 4, 5, 10, etc. Usually scale factors like 3, 4.5, 7.9 etc., are bad choices. Both axis may have different scale factors and may start from a nonzero value. You should clearly label each axis and write down the scale factors. Then you should mark the position corresponding to each data pair with a cross or similar symbols. Usually you should also include the uncertainties as vertical bars above and below the data point whose lengths are determined according to the scale factor. Once you finish marking all your data pairs, then you should try to pass a straight line through all the data points. Usually, this may not be possible since the data points may not fall into a straight line. However, since you know that the relationship is linear there should be a straight line that passes through the data points even though not all of them fall on a line. You should make sure that the straight line passes *through* the data points in a balanced way. An equal number of data points should be below and above the straight line. Then, by picking two points on the line as far apart from each other as possible, you should draw parallel lines to the axes, forming a triangle (Figure 8). The slope is the slope of the straight line. You can calculate the slope as:

$$Slope = \frac{\Delta y}{\Delta x}$$

and read the y-intercept from the graph by finding the point where the straight line crosses the y-axis. You can estimate the uncertainties of the slope and intercept by finding different straight lines that still pass through all the data points in an acceptable manner. The minimum and maximum values obtained from these different trials would give us an idea about the uncertainties. However, obtaining the parameters will be sufficient in this course.

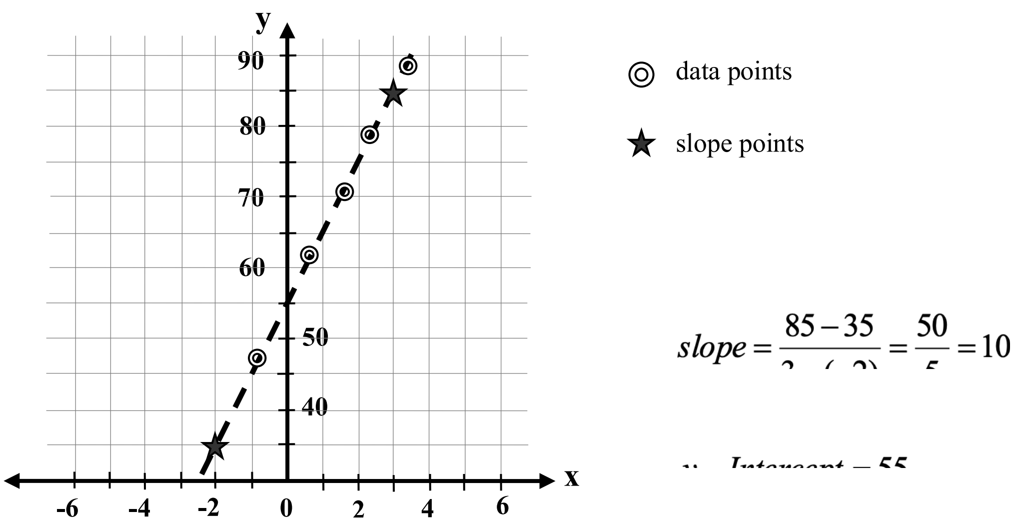


Figure 8: Determining the slope and y-intercept.

Special graph papers, like semi-log and log-log graph papers, are used when you have relationships that can be transformed into linear relationships by taking the base-10 logarithm of both sides. Semi-log graph papers are used if one side of the expression contains powers of ten or single exponential function resulting in a linear variable when you take the base-10 logarithm of both sides.

Logarithmic graph papers are used when you prefer to use the measured values directly without taking the logarithms and still obtaining a linear graph. Each logarithmic axis is divided in such a way that when you use the divisions marked on the paper it will have the same effect as if you first took the logarithm and then plotted on a regular graph paper. Logarithmic graph papers are divided linearly into decades and in each decade is divided logarithmically. There is no zero value in a logarithmic axis. You should plot your data by choosing appropriate scale factors for each axis and then mark the data points directly without taking the logarithms. You should again draw a straight line that will pass through all the data points in a balanced way. The slope of the line would give us the exponent in the relationship. For example, a relationship like $y = ax^n$ would be linearized as $\log y = \log a + n \log x$. If you plot this on a regular graph paper, the slope will be given by $n = (\log y_2 - \log y_1)/(\log x_2 - \log x_1)$ where you will read the logarithms directly from the graph. On the other hand, when you plot your data on a log-log paper, you will be using the measured values directly. When you picked the two points from the straight line that fits the data points best, the slope should be calculated by $n = (\log y_2 - \log y_1)/(\log x_2 - \log x_1)$ where you will calculate the logarithms using the values read from the graph. y-intercept would be directly the value where the straight line crosses the vertical axis at $x = 1$.

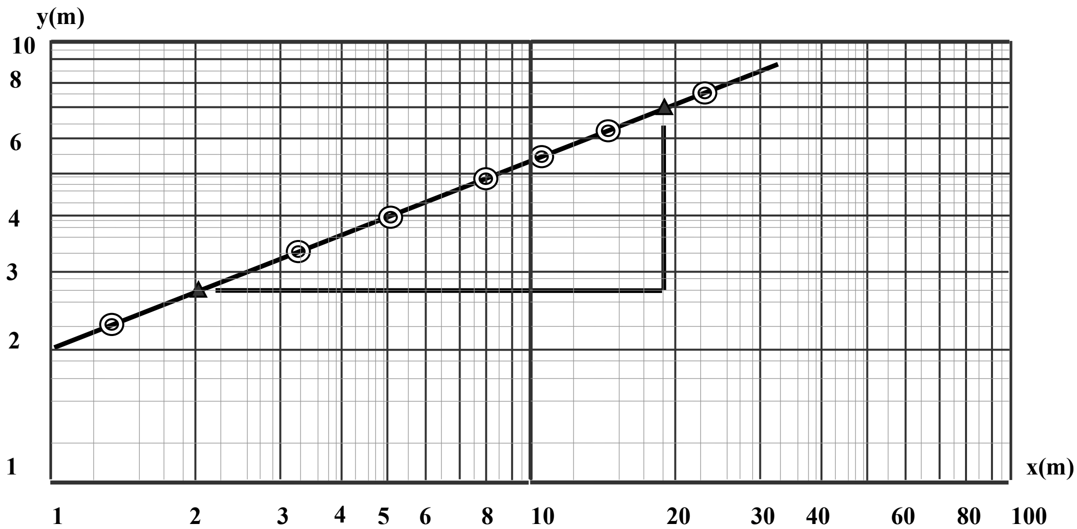


Figure 9: Determining the slope and y-intercept.

slope point 1: (2.0 ; 2.6) and slope point 2 : (18.0 ; 7.0)

$$\text{slope} = \frac{\log(7.0) - \log(2.6)}{\log(18.0) - \log(2.0)} = \frac{0.4301}{0.9542} = 0.4507 \qquad \text{and} \qquad y\text{-intercept} = 2.0.$$

Reports

Obviously, doing an experiment and getting some results are not enough. The results of the experiment should be published so that others working on the same problem will know your results and use them in their calculations or compare with their results. The reports should have all the details so that another experimenter could repeat your measurements and get the same results. However, in an introductory teaching lab there is no need for such extensive reports since the experiments you will be doing are well established and time is limited. You have to include enough details to convince your lab instructor that you have performed the experiment appropriately and analyzed it correctly. The results of your analysis, including the uncertainties in the measurements, should be clearly expressed. The comparisons with the accepted values may also be included if possible.

Part II. EXPERIMENTS

1. *Static Equilibrium of a Rigid Body*

OBJECTIVE : To study the equilibrium conditions of a body when there are forces applied on it.

THEORY : A rigid body is in equilibrium when the total force and the torque acting on it are equal to zero:

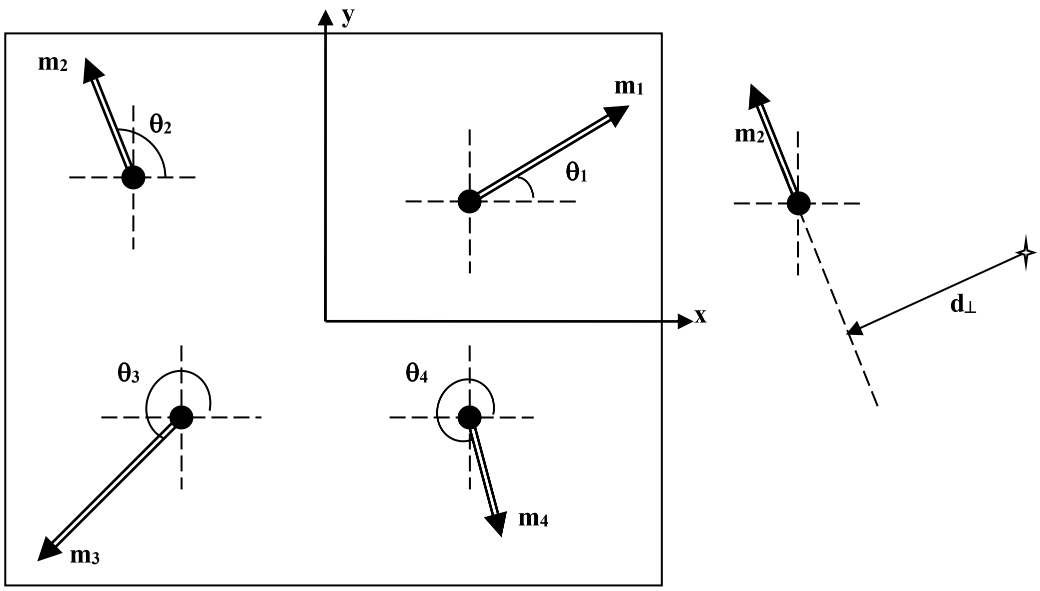
$$\sum \vec{F} = 0, \sum \vec{\tau} = 0$$

or if we write these in component form:

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z = 0$$

$$\sum \tau_x = 0, \qquad \sum \tau_y = 0, \qquad \sum \tau_z = 0.$$

PROCEDURE :



<u>Part 1:</u> Place a piece of paper on the movable disc and replace the center pin. Insert four pegs, by punching through the paper, into four different holes in the disc, and place the strings over the pulleys.

Attach known masses to the free ends of three of the cords. Adjust the angular position and the mass suspended from the fourth cord until the disc is in equilibrium when the pin is removed. With a pencil, mark the positions of the strings and write the magnitude of each force. Indicate the direction of the forces and determine whether the forces are balanced. Choose any point on the data paper and compute the algebraic sum of torques about the chosen point.

Part 2: Repeat the whole procedure by suspending two known and the two unknown masses given to you.

STATIC EQUILIBRIUM OF A RIGID BODY

Name & Surname : **Experiment #:**

Section : **Date :**

DATA: **"TURN IN YOUR DATA SHEETS OTHERWISE YOUR LAB REPORT WILL NOT BE EVALUATED"**

PART 1:

Description / Notation			**Value & Unit**

MASS - 1:

Mass on the holder m_1 = ...

Perpendicular Distance to the axis of rotation $d_{1\perp}$ = ...

Angle between the x-axis and the Force θ_1 = ...

Direction of the torque: *Clockwise* ☐ *Counterclockwise* ☐

MASS - 2:

Mass on the holder m_2 = ...

Perpendicular Distance to the axis of rotation $d_{2\perp}$ = ...

Angle between the x-axis and the Force θ_2 = ...

Direction of the torque: *Clockwise* ☐ *Counterclockwise* ☐

MASS - 3:

Mass on the holder m_3 = ...

Perpendicular Distance
to the axis of rotation $d_{3\perp}$ = ...

Angle between the
x-axis and the Force θ_3 = ...

Direction of
the torque: *Clockwise* ☐ *Counterclockwise* ☐

MASS - 4:

Mass on the holder m_4 = ...

Perpendicular Distance
to the axis of rotation $d_{4\perp}$ = ...

Angle between the
x-axis and the Force θ_4 = ...

Direction of
the torque: *Clockwise* ☐ *Counterclockwise* ☐

PART 2:

Description / Symbol	**Value & Unit**

UNKNOWN MASS - 1:

Mass on the holder m_1 = ...

Perpendicular Distance
to the axis of rotation $d_{1\perp}$ = ...

Angle between the
x-axis and the Force θ_1 = ...

Direction of
the torque: *Clockwise* ☐ *Counterclockwise* ☐

UNKNOWN MASS - 2:

Mass on the holder m_2 = ...

Perpendicular Distance
to the axis of rotation $d_{2\perp}$ = ...

Angle between the
x-axis and the Force θ_2 = ...

Direction of the torque: *Clockwise* ☐ *Counterclockwise* ☐

MASS - 3:

Mass on the holder m_3 = ...

Perpendicular Distance
to the axis of rotation $d_{3\perp}$ = ...

Angle between the
x-axis and the Force θ_3 = ...

Direction of the torque: *Clockwise* ☐ *Counterclockwise* ☐

MASS - 4:

Mass on the holder m_4 = ...

Perpendicular Distance
to the axis of rotation $d_{4\perp}$ = ...

Angle between the
x-axis and the Force θ_4 = ...

Direction of the torque: *Clockwise* ☐ *Counterclockwise* ☐

CALCULATIONS :

For PART 1:

ΣF_x : ....

....

ΣF_y : ....

....

$\Sigma \tau_z$: ....

....

For PART 2:

ΣF_x : ....

....

ΣF_y : ....

....

$\Sigma \tau_z$: ....

....

R E S U L T S :

Use F_x or F_y and τ_Z to solve for $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$:

m_1 = ...

. . . .

m_2 = ...

. . . .

2. *EMPIRICAL EQUATIONS*

OBJECTIVE **:** To study a nonlinear phenomenon and determine the parameters related to the motion through a linear representation.

THEORY **:** Physics laws are based on experiments. We may obtain some relationships starting from the first principles or established physics laws through physical and mathematical reasoning. These relationships are accepted as valid laws if they are shown to be valid by all sorts of experiments. However, in some cases we may not know the underlying physical principle. We may have only our observation of the phenomenon. From the observation we may try to develop a relationship between the quantities that are being measured. Of course, if there are more than two quantities involved, we should set all the quantities to a constant value except two of them, and then measure one of these two by varying the value of the other quantity.

For example, in the periodic motion of metal rings placed on a knife edge fixed on the wall, there are several quantities; the radius, thickness of the rings, and the period of the oscillations are some of the quantities that we can think of. If we want to determine the relationship between the radius and the period of the oscillations, we should have rings made of the same material and thickness. Then we should let the rings oscillate and measure the period as a function of the radius, making sure that the initial amplitudes are the same. Once we obtain the data we can try different relationships between the period and the radius; linear, quadratic, cubic, etc. However, this would be a time consuming process. Instead we assume that the relationship is in the form of $T = ar^n$, which is not linear. By taking the logarithm (base-10) of both sides, we get $\log T = \log a + n \log r$. This is a linear expression whose slope and y-intercept can be easily obtained through graphical analysis. We can either plot the data on a log-log graph paper or the logarithm of the values on a regular graph paper. Then we can determine the exponent ***n*** from the slope of the straight line.

Establishing physics laws in this way produces expressions that are already validated by the experiment. Of course, we should still try to derive the same expression through logical reasoning and starting from the known and well established physics laws.

APPARATUS **:** A set of five metal rings, vernier calipers, stop watch, meter stick,

PROCEDURE **:** Each one of the five metal rings is suspended successively from a knife edge. The rings are made to oscillate from side to side. The period of oscillations is determined by taking average over at least 10 oscillations. The mean diameter of each ring is also determined. After obtaining the data, you should plot them on a log-log graph paper and the logarithm of the values on a regular graph paper. Determine the slope and intercept from both plots and compare them. Report the average of both values.

EMPIRICAL EQUATIONS

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description	**Symbol (unit)**	**RING NUMBER**				
		- 1 -	**- 2 -**	**- 3 -**	**- 4 -**	**- 5 -**
Inner Diameter *(first measurement)*	D_{i1} ( )					
Inner Diameter *(second measurement)*	D_{i2} ( )					
Average Inner Diameter	$\mathbf{D_{iave}}$ ( )					
Outer Diameter *(first measurement)*	D_{o1} ( )					
Outer Diameter *(second measurement)*	D_{o2} ( )					
Average Outer Diameter	$\mathbf{D_{oave}}$( )					
# of Periods:	t ( )					

CALCULATIONS:

Description	**Symbol (unit)**	**RING NUMBER**				
		- 1 -	**- 2 -**	**- 3 -**	**- 4 -**	**- 5 -**
Average Diameter	D_{ave} ( )					
One Period	T ( )					
Logarithm of D_{ave}	Log D_{ave}					
Logarithm of T	Log T					

1) Use **Log D** & **Log T** data set:

A) From the graph, choose two SLOPE POINTS other than data points,

SP_1 : (;)

SP_2 : (;)

B) Calculate "n" using SP_1 and SP_2,

n_1 = ...

. . . .

C) By reading the *y*-intercept of the line from the graph, determine *A*,

Intercept_1 = ...

. . . .

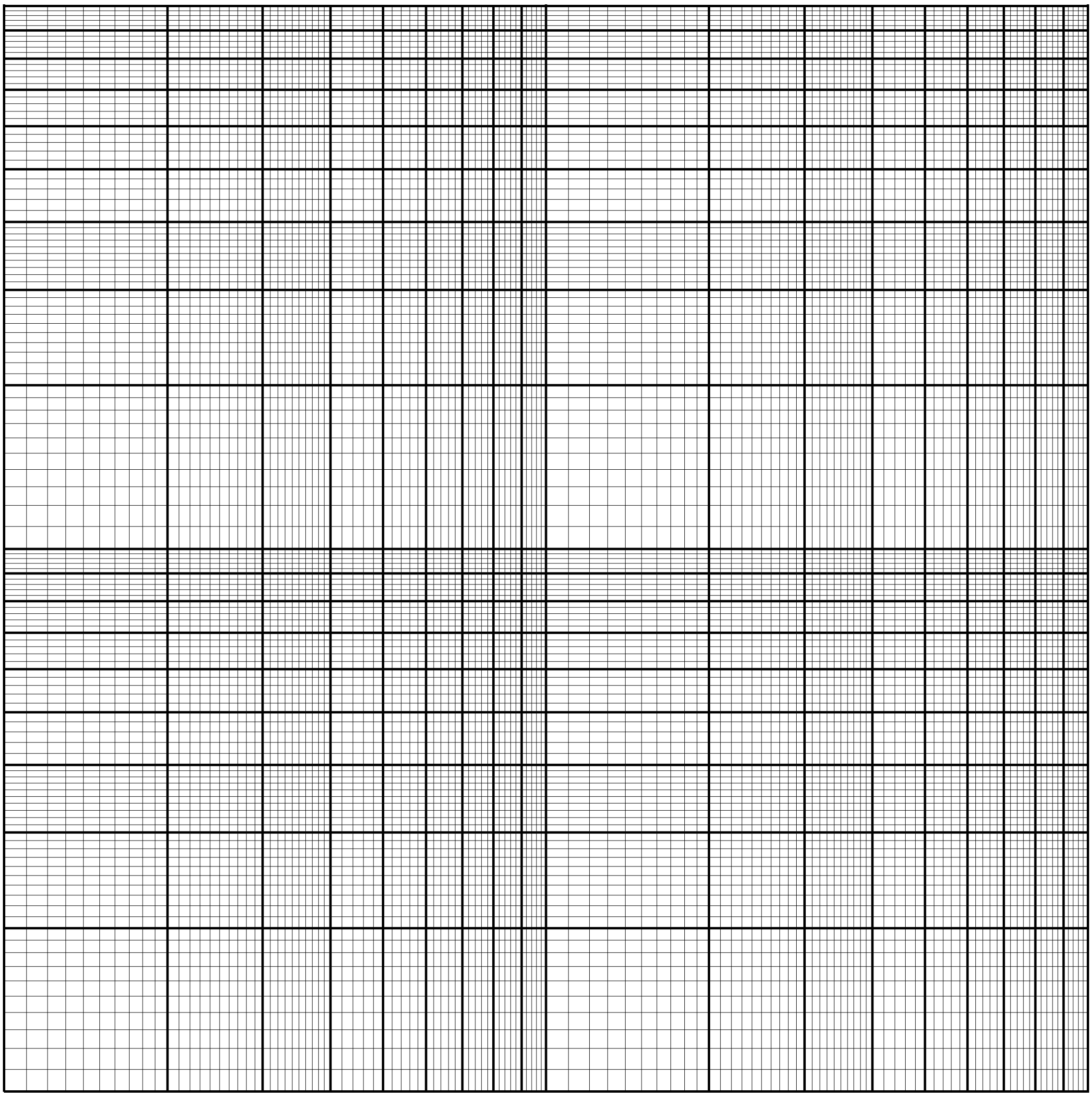
A_1 = ...

. . . .

$D_{\text{(for T=1 sec)}}$ = ...

. . . .

2) Use **D** & **T** data set:



A) From the graph, choose two SLOPE POINTS other than data points,

SP_1 : (;   )

SP_2 : (;   )

B) Calculate "*n*" using SP_1 and SP_2
(*Show your calculations clearly)*

n_2 = ...

. . . .

C) By reading the *y*-intercept of the line from the graph, determine *A*,
(*Show your calculations clearly)*

Intercept_2 = ...

. . . .

A_2 = ...

. . . .

***D* (for T=1 sec)** = ...

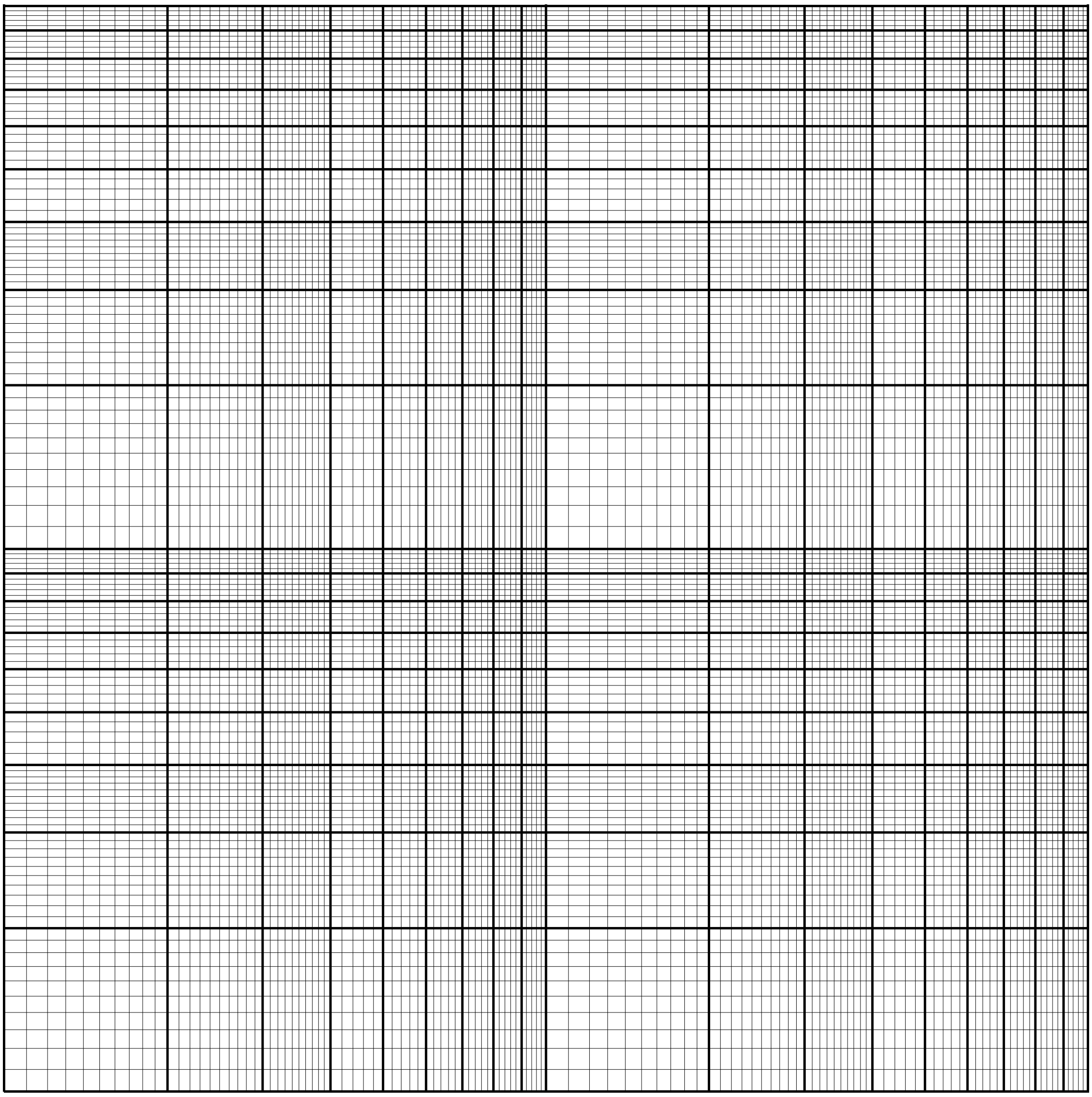
. . . .

RESULTS:

Symbol		Calculations	Result	Dimension
n_{ave}	=	....	....	... .
A_{ave}	=	....	....	... .

QUESTION :

1. Can we use this set of rings to determine the gravitational acceleration? Explain.



3. *THE PHYSICAL PENDULUM*

OBJECTIVE **:** To study the properties of the physical pendulum and to use the physical pendulum to determine the acceleration due to gravity.

THEORY **:**

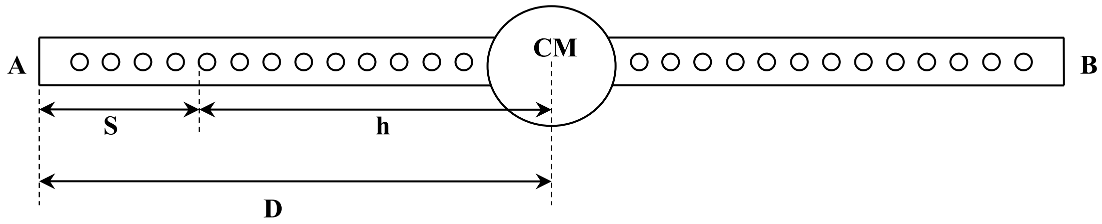


Figure 1. Physical pendulum.

In simple pendulum we determined the expression for the period by solving the force equation with the assumption that the mass hanging at the end of the string is a point mass. Since we used a small ball our assumption was acceptable. When we have an object that is much larger and can not be treated as a point particle, we can still determine the period of oscillations if we hang this object from any point and let it oscillate. In this case we should write the torque equation and solve it. Of course we should know the moment of inertia of the object with respect to the point that the object is hung. Then the period of oscillations will be

$$T = 2\pi\sqrt{\frac{I}{Mgh}} \qquad (1)$$

where I is the moment of inertia about the axis of rotation or the point that the object is hung and h is the distance between this point and its center of mass. The moment of inertia about any given point can be expressed in terms of the moment of inertia about the center of mass using the parallel axis theorem:

$$I = I_{CM} + Mh^2 \qquad (2)$$

and I_{CM} can be written in terms of the radius of gyration k:

$$I_{CM} = Mk^2 \qquad (3)$$

Then combining these equations we can express the period as

$$\left(T = 2\pi\left[\frac{h^2 + k^2}{gh}\right]^{1/2} \right) \qquad (4)$$

This is equivalent to a simple pendulum with a length:

$$L = \left(h^2 + k^2\right)/h \qquad (5)$$

This simple pendulum is called "the equivalent simple pendulum" to the physical pendulum.

From the figure above we see that

$$h = D - S \qquad (6)$$

and plugging this into the expression for the period results in

$$T = 2\pi\left(\frac{k^2 + (D-S)^2}{g(D-S)}\right)^{1/2}. \qquad (7)$$

Plotting the period as a function of *S* will give us the graph in Figure 2. As you can see from the graph, there are four possible points for a specific period value that we can hang the pendulum. These four points collapse down to two for the minimum period. Radius of gyration is the distance at which the physical pendulum is hung to get the minimum period. We can determine the radius of gyration by measuring the period while varying the distance between the center of mass and the point that the pendulum is hung. Then we can simply read the distance corresponding to the minimum period from the graph. Radius of gyration is the distance between this point and the center of mass.

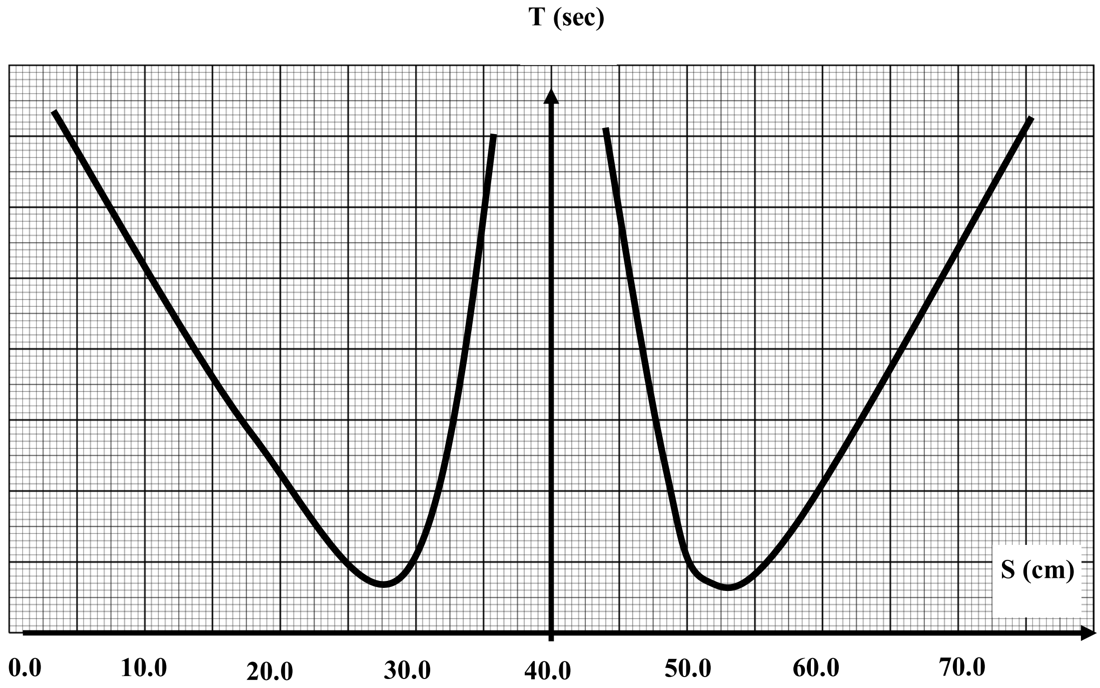


Figure 2. Plot of the period as a function of *S* (Equation (7)).

From the plot we can also see that the period of oscillations become infinite if we hang the object from its center of mass.

Because of the symmetry around the center of mass we can limit ourselves to one side of the center of mass. Equating the expressions for the two points that result in the same period:

$$2\pi\sqrt{\frac{(h_1^2 + k^2)}{gh_1}} = 2\pi\sqrt{\frac{(h_2^2 + k^2)}{gh_2}}, \qquad (8)$$

after simplifying we get:

$$\frac{h_1 + k^2}{h_1} = \frac{h_2 + k^2}{h_2}, \qquad (9)$$

and solving for k

$$k^2 = \frac{\left(h_1^2 h_2 - h_2^2 h_1\right)}{\left(h_1 - h_2\right)} = h_1 h_2 \qquad (10)$$

Hence, the period expression given in Equation (7) becomes

$$T = 2\pi \sqrt{\frac{\left(h_1 + h_2\right)}{g}}$$

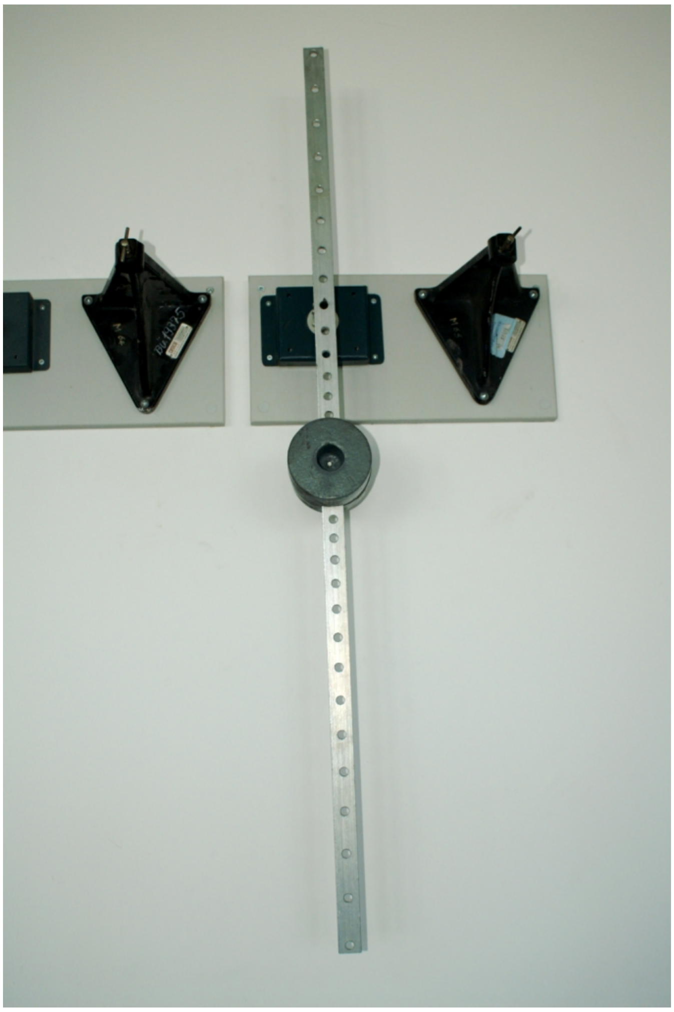
and similarly the length of the equivalent simple pendulum (Equation (5)) becomes

$$L = h_1 + h_2$$

APPARATUS : Physical pendulum, meter stick, stopwatch

PROCEDURE :

- Support the pendulum on the knife edge at the hole nearest to one end of the bar. Observe the time for 10 full oscillations and determine the period. In the same way determine the period about an axis through each and every hole in the bar.
- Remove the pendulum from its support and measure the distance of the various points of suspension from one end of the bar.
- Record these values of ***S*** as a function of the corresponding values of period ***T***.
- Plot the values of ***S*** versus period ***T*** and draw a horizontal line corresponding to a period ***T***. Determine the radius of gyration, *k*, from the graph.
- Determine the length of the equivalent simple pendulum and calculate the gravitational acceleration using this value. Compare your result with the known value of *g*.



THE PHYSICAL PENDULUM

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			**Value & Unit**
Distance from one end to the center of the pendulum	$\boldsymbol{D}$	=	. . . .
Mass of of the pendulum	$\boldsymbol{M}$	=	. . . .
Acceleration due to gravity	$\boldsymbol{g}_{\mathbf{TV}}$	=	. . . .

Distance from one end of the pendulum to the suspension point ***S*** **( )**	**Time for 10 Period** ***t*** **( )**	**One Period** ***T*** **( )**

Plot *S* versus *T*:

Read from the Graph:

Description / Symbol		**Value & Unit**
Period (any chosen) T	=	. . . .
Minimum Period T_o	=	. . . .
Distance from the center to the first suspension point for T, h_1	=	. . . .
Distance from the center to the second suspension point for *T*, h_2	=	. . . .
For minimum Period: h_o	=	. . . .
Radius of Gyration $k = h_o$	=	. . . .

CALCULATIONS and RESULT:

Description	**Symbol**		**Calculations (show each step)**	**Result**
Radius of Gyration	$k = \sqrt{h_1 h_2}$	=	. . . .	. . . .
Length of the Equivalent Simple Pendulum	L	=	. . . .	. . . .

.

Description/Symbol	Calculations (show each step)	Result
Moment of Inertia about the CM $I_o = I_{CM}$ =	. . . .	. . . .
	. . . .	
Moment of Inertia Corresponding to h_1 $I_{(\text{for T})}$ =	. . . .	. . . .
	. . . .	
Moment of Inertia Corresponding to h_2 $I_{(\text{for T})}$ =	. . . .	. . . .
	. . . .	
Acceleration due to Gravity g_{EV} =	. . . .	. . . .
	. . . .	
% Error for *g* =	. . . .	

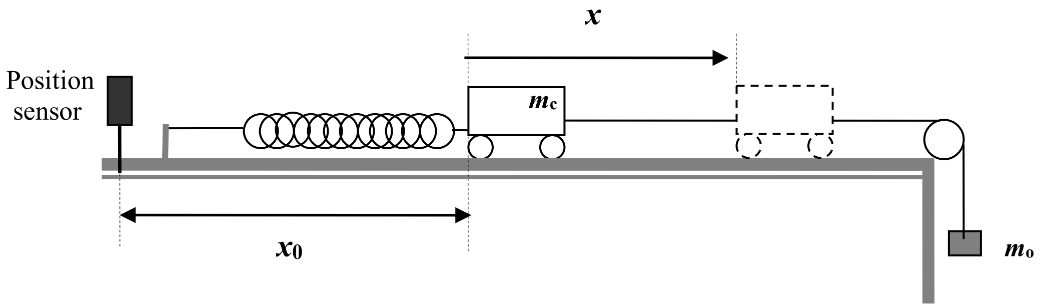
Dimensional analysis for the Radius of Gyration, *k* . . . .

Dimensional analysis for the moment of Inertia, I: . . . .

4. *SIMPLE HARMONIC MOTION*

OBJECTIVE **:** To investigate the resultant of two forces, one constant, the other depending on displacement from equilibrium (restoring force).

THEORY **:**



The system shown in the figure above will be exhibiting a periodic motion due to the variable restoring force in the spring. If we write the equation of motion:

$$\frac{d^2x}{dt^2} + \omega^2 x = \frac{m_o}{m_{total}} g$$

$$m_{total} = m_c + m_o$$

then, the solution of this equation will be:

$$x(t) = \frac{m_o g}{k} - ACos(\omega t + \delta)$$

whose period of oscillation is given by

$$\omega^2 = \frac{k}{m_{total}}.$$

Derivative of the position with respect to time will yield the velocity as a function of time and the second derivative will give us the acceleration:

$$v(t) = A\omega Sin(\omega t + \delta)$$

$$a(t) = A\omega^2 Cos(\omega t + \delta)$$

Notice that when the magnitude of the velocity reaches its maximum the acceleration becomes zero and vice versa.

APPARATUS : Car and track system, position sensor, data logger, spring, hanger and mass set.

PROCEDURE :

- Disconnect car from the spring and compensate for friction.
- Fix the spring to the car; locate the point where no force is acting on the car, keeping the car stationary, and place mass $\boldsymbol{m}$ on the holder.
- Place the position sensor at least 30 cm away from the car. Start the data logger at the desired rate (suggested value is 10 per second) and let the car go. The car first accelerates ($\boldsymbol{mg} > \boldsymbol{kx}$), attains its maximum velocity where $\boldsymbol{mg} = \boldsymbol{kx}$, then decelerates ($\boldsymbol{mg} < \boldsymbol{kx}$) and finally stops to come back.
- Using the data in the data logger's memory, calculate the average velocity for each interval.
- Plot the average velocity versus time and the total displacement versus time curves.
- From the velocity versus time graph, determine the maximum velocity which corresponds to zero acceleration and the corresponding time t and the period.
- From the displacement versus time graph, determine the maximum displacement x_{eq}
- Calculate other system parameters.

Simple Harmonic Motion

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol		**Value & Unit**
Mass on the holder m	=	. . . .
Initial distance of the Car x_0	=	. . . .
Number of the Cylinders in the Car	=	. . . .
Data Taking Rate	=	. . . .

Number of Intervals	***t*** **( )**	***x*** **( )**	**Δx** **( )**	**$v_{ave} = \Delta x / \Delta t$** **( )**

CALCULATIONS and RESULT:

Read from the Graphs:

Description	Symbol	Value & Unit
Maximum velocity	v_{max} =	. . . .
Time Corresponding to to the max. Velocity	t =	. . . .
Equilibrium Displacement	$x_{eq.}$ =	. . . .

Calculate:

Description / Symbol	Calculations (show each step)	Result	Dimension
Spring Constant k =	. . . .	. . . .	. . . .
	. . . .		
Period of Oscillation T =	. . . .	. . . .	. . . .
	. . . .		
Frequency of Oscillation ω =	. . . .	. . . .	. . . .
	. . . .		

Description / Symbol			**Calculations (show each step)**	**Result**	**Dimension**
System Parameter	A	=	. . . .	. . . .	. . . .
			. . . .		
Maximum Displacement	x_{max}	=	. . . .	. . . .	. . . .
			. . . .		
Maximum Acceleration	a_{max}	=	. . . .	. . . .	. . . .
			. . . .		
Total Mass	m_{total}	=	. . . .	. . . .	. . . .
			. . . .		
Mass of the Car	m_{car}	=	. . . .	. . . .	. . . .
			. . . .		

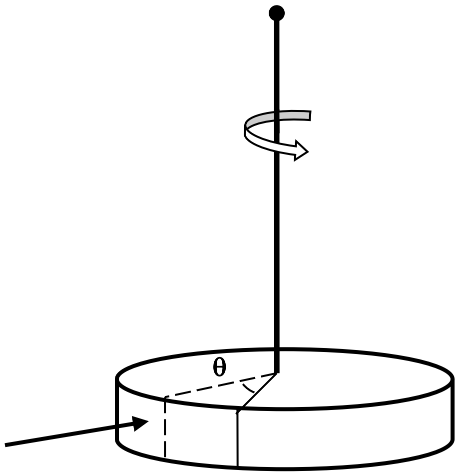
QUESTIONS :

1) What would the amplitude of the simple harmonic motion be if the mass on the hanger were doubled?
2) Calculate the acceleration of the car at time $t=T/4$.
3) If there were 10% error in determining the distance between successive intervals, what would the error in the tenth average velocity be?

5. *ANGULAR HARMONIC MOTION*

OBJECTIVE **:** To study angular oscillatory motion and the dependence of the period of oscillation on the moment of inertia of the system.

THEORY **:**



We can study the angular harmonic motion in a torsional system where an object is attached to a straight rod and rotated to some angle initially. This initial rotation causes some torsion in the wire thereby producing a restoring torque. Resulting torque equation is similar to the force equations that we obtained for the simple harmonic oscillation; hence it has the same type of solution for the period:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

Below are the formulas to calculate the moment of inertia of uniform disk and ring masses:

$$I_{disk} = \frac{MR^2}{2}$$

$$I_{ring} = \frac{1}{2}M\left[\left(\overline{R}_{inner}\right)^2 + \left(\overline{R}_{outer}\right)^2\right]$$

APPARATUS **:** Torsion pendulum, disk and ring masses, meter stick, stopwatch.

PROCEDURE :

Part 1: Measure the time, *t*, for the disc to complete 50 oscillations, and determine the mean period of oscillation *T*.

Measure the diameter of the disk at two different positions, compute the mean diameter and mean radius *R* for the disc. By using this radius, compute the moment of inertia of the disc and the torsion constant of the rod.

Part 2: Place the ring whose moment of inertia is unknown, on the disc, measure the time to complete 50 oscillations, and determine the mean period of oscillation *T*.

Compute the sum of the moment of inertias of the disc and the ring. Evaluate the moment of inertia of the ring. Compute the theoretical value of moment of inertia and determine the percentage error.

ANGULAR HARMONIC MOTION

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Part 1: Moment of Inertia of the Disk

Description / Symbol			**Value & Unit**
Time for 50 oscillations	t	=	. . . .
Time for one oscillation	T	=	. . . .
Diameter of the disc *1st measurement*	D_{disc1}	=	. . . .
Diameter of the disc *2nd measurement*	D_{disc2}	=	. . . .
Average diameter of the disc	D_{disc}	=	. . . .
Radius of the disc	Rdisc	=	. . . .
Mass of the disc	M_{disc}	=	. . . .

Part 2: Moment of Inertia of the Ring & the Disc

Description / Symbol			Value & Unit
Time for 50 oscillations	t^*	=	. . . .
Time for one oscillation	T^*	=	. . . .
Outer Diameter of the ring *1st measurement*	D_{out-1}	=	. . . .
Outer Diameter of the ring *2nd measurement*	D_{out-2}	=	. . . .
Average outer diameter of the ring	D_{outer}	=	. . . .
Outer Radius of the ring	R_{outer}	=	. . . .
Inner Diameter of the ring *1st measurement*	$D_{inner-1}$	=	. . . .
Inner Diameter of the ring *2nd measurement*	$D_{inner-2}$	=	. . . .
Average inner diameter of the ring	D_{inner}	=	. . . .
Inner Radius of the ring	R_{inner}	=	. . . .
Mass of the ring	M_{ring}	=	. . . .

CALCULATIONS:

Description			Calculations (show each step)	Result
Moment of Inertia of the system	I_{disc}	=	. . . .	. . . .
Torsion constant of the rod	κ	=	. . . .	. . . .
Total Moment of Inertia of the disk and the ring	I_{total}	=	. . . .	. . . .
Moment of Inertia of the ring	$I_{ring\text{-}EV}$	=	. . . .	. . . .
Theoretical value of the Moment of Inertia of the ring	$I_{ring\text{-}TV}$	=	. . . .	. . . .

% Error for the Moment of Inertia of the ring: ....

Dimensional Analysis for the Torsion Constant, κ: ....

Dimensional Analysis for the Moment of Inertia, I: ....

QUESTIONS :

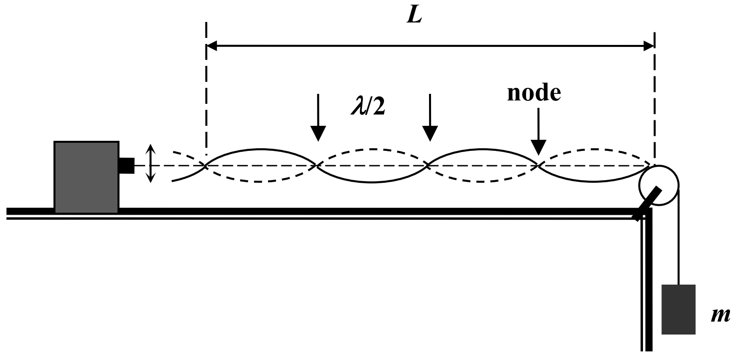
1) What would the uncertainty in determining the torsion constant κ be, if the period and the radius of the ring are determined with 1% uncertainties?

2) How would the period of the oscillations be affected if you place another object on the disc while it is oscillating?

6. *STANDING WAVES IN A STRING*

OBJECTIVE **:** To study the standing waves in a cord, and to verify the equation for the velocity of a wave on a string.

THEORY **:**



When a string fixed on both ends and under tension is excited on one end, there will be waves traveling along the string. If we continue to excite the string, the waves reflected from the other end will interfere with the waves traveling in the forward direction. If the length of the string is exactly equal to the integer multiples of the half wavelengths, there will be standing waves along the string. The points where the string is motionless are called *nodes* and the distance between successive nodes will be equal to the half wavelength. Speed and the wavelength of the waves traveling along the string depend on the tension and the mass per unit length of the string:

$$v = \left(\frac{T}{\mu}\right)^{\frac{1}{2}} = f\lambda$$

$$T = \mu\lambda^2 f^2$$

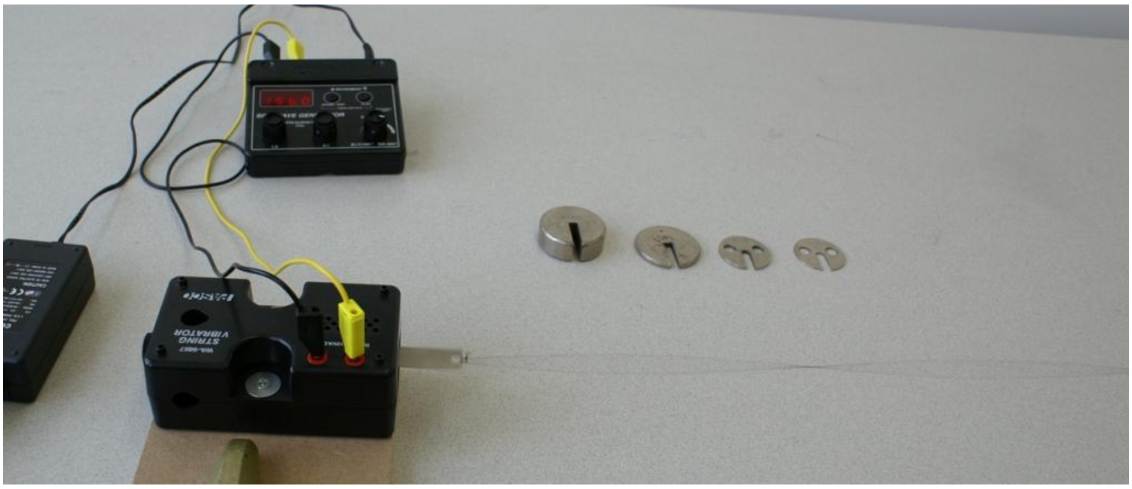
A Plot of the tension versus the square of the frequency data pairs that produce standing waves should follow a straight line whose slope is equal to the mass per unit length times the square of the wavelength. Tension on the string is provided by the masses placed on the hanger on the other end.

APPARATUS **:** String vibrator and its variable frequency power supply, hanger and mass set, string.

PROCEDURE **:**

- Length of the cord between the vibration generator and the pulley is kept constant. Place a mass on the mass holder and set the vibration generator in motion. Arrange the frequency of the vibration generator until standing waves are clearly observed.
- Determine the number of nodes and the wavelengths. Record the frequency value along with the corresponding mass on the mass holder.
- By keeping the **wavelength constant**, change the mass and read the corresponding frequency for clearly observed standing waves for 4 more times.

- Plot tension, T, versus f^2 and determine the slope. Calculate the mass per unit length for the cord.



STANDING WAVES IN A STRING

Name & Surname :

Experiment #:

Section :

Date :

DATA:

Description / Symbol			**Value & Unit**
Mass per unit length of the Cord	μ_{TV}	=	. . . .
Length of the Cord	L	=	. . . .
Acceleration due to gravity	g	=	. . . .

Mass, ***m*** **( )**	**# of λ / 2** ***(keep constant)***	**λ ( )** ***(keep constant)***	**Frequency,** ***f*** **( )**	**f^2** **( )**	**Tension** **$T = m.g$** **( )**

CALCULATIONS:

From the graph, choose two SLOPE POINTS other than data points,

SP_1 : (; **)**

SP_2 : (; **)**

Calculate:

SLOPE = ...

RESULTS:

Description / Symbol	Calculations (show each step)	Result
Mass per unit length of the Cord μ_{EV} =	. . . .	. . . .
	. . . .	
% Error for μ =	. . . .	. . . .

Dimensional analysis for μ : . . . .

7. *SPECIFIC HEAT OF METALS AND HEAT OF FUSION OF ICE*

OBJECTIVE **:** To measure the specific heat of a metal and to determine the unknown mass of an ice block, through the method of mixtures.

THEORY **:**

It is experimentally shown that the heat absorbed by an object is directly proportional to the change in the temperature and the mass of the object. Proportionality constant is the specific heat of the material that the object is made of:

$$Q = mc\Delta T$$

If you use the SI unit system, the specific heat is defined as the amount of heat absorbed to increase the temperature by one centigrade for a 1 kg object. Unit for the heat is the same as Joule but in these calculations mostly calories are used (1 cal = 4.187 Joules).

When you place two objects at different temperatures in close contact, they will exchange heat until the temperatures are equal. The heat gained by one object is equal to the heat given by the other object since the energy is conserved. For example, if you have a calorimeter with a known mass $\boldsymbol{m_c}$ and specific heat $\boldsymbol{c_c}$ filled with water with mass $\boldsymbol{m_w}$ at a known temperature $\boldsymbol{T_1}$. When you place a specimen into the water inside the calorimeter at a higher temperature $\boldsymbol{T_2}$, assuming that there is no heat lost to the surroundings, we can write the following heat exchange equation:

$$m_s c_s (T_2 - T_3) = m_c c_c (T_3 - T_1) + m_w c_w (T_3 - T_1)$$

where T_3 is the final temperature of the mixture when it comes into equilibrium.

These expressions are valid unless there is no phase change. If there is a phase change involved, then the corresponding heat necessary for the phase change should be added into the appropriate side of the equation. For example, if we add a block of ice with a mass $\boldsymbol{m_i}$ at a temperature of $\boldsymbol{T_i}$ into the calorimeter mentioned in the previous paragraph, we should write the heat exchange as follows:

$$m_i c_i (0 - T_i) + m_i L_f + m_i c_w (T_4 - 0) = m_c c_c (T_3 - T_4) + m_w c_w (T_3 - T_4) + m_s c_s (T_3 - T_4)$$

since the heat of fusion, L_f, depends only on the mass. Temperature is constant during the phase change and the final temperature is T_4.

APPARATUS **:** Calorimeter, stirrer, thermometer, heater, water, specimen, ice, temperature sensor, data logger.

PROCEDURE:

Calorimeter = Inner vessel of calorimeter + stirrer

Part 1: Determine the mass of the calorimeter (*inner vessel of the calorimeter and the stirrer*), and the mass of the specimen and its container.

Put your sample in its container into a water boiler one-third full and heat it until the temperature is 95°C.

Add 80g of water at room temperature to the calorimeter. Measure the initial temperature of calorimeter and water, $T_{i\text{-}cal}$ and $T_{i\text{-}w}$. Quickly pour the hot sample into the calorimeter and observe the temperature rise of the water and calorimeter combination. Note the highest temperature as equilibrium temperature, T_{1e}. Before calculating the specific heat of given metal, continue with Part 2.

Part 2: Get an ice block and drop it into the calorimeter immediately and keep the system closed and well mixed. The temperature will first drop, then it will stay stationary as the ice melts, and finally, it will decrease to the equilibrium temperature T_{2e}. Record this temperature.

Calculate the specific heat of metal and the mass of ice block.

Specific Heat of Metals and Heat of Fusion of Ice

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			**Value & Unit**
Specific Heat of Water	c_{w}	=	....
Specific Heat of the Calorimeter	c_{cal}	=	....
Specific Heat of Ice	c_{ice}	=	....
Heat of Fusion of Ice	L_{f}	=	....

P A R T 1 – S P E C I F I C H E A T O F M E T A L S

Description / Symbol			**Value & Unit**
Mass of the Calorimeter	m_{cal}	=	. . . .
Mass of Water	m_w	=	. . . .
Mass of the Specimen + container	m_{s+con}	=	. . . .
Mass of the Specimen	m_s	=	. . . .
Initial Temperature of the Calorimeter	$T_{i\text{-}cal}$	=	. . . .
Initial Temperature of Water	$T_{i\text{-}w}$	=	. . . .
Initial Temperature of the Specimen	$T_{i\text{-}s}$	=	. . . .
Equilibrium Temperature	T_{1e}	=	. . . .

PART 2 – HEAT OF FUSION OF ICE

Description / Symbol			**Value & Unit**
Initial Temperature of Ice	$T_{i\text{-}ice}$	=	. . . .
Initial Temperature of the Calorimeter, Water and the Specimen	$T_{i\text{-}cal\ +\ contents}$	=	. . . .
Equilibrium Temperature	T_{2e}	=	. . . .

CALCULATIONS :

For PART-1: ****** NO NUMERICAL EVALUATION ******

Heat Lost: . . . .

. . . .

Heat Gained: . . . .

. . . .

Specific Heat of the Specimen: c_s = ...

NO NUMERICAL EVALUATION

For PART-2: **(NO NUMERICAL EVALUATION)**

Heat Lost: . . . .

. . . .

Heat Gained: . . . .

. . . .

Mass of Ice: $m_{\text{ice-EV}} =$ . . . .

NO NUMERICAL EVALUATION

RESULTS:

Description		**Calculations (show each step)**	**Result**
Specific Heat of the Specimen c_s	=	. . . .	. . . .
		. . . .	
Total Mass m_{total}	=	. . . .	. . . .
Experimental Value of the Mass of Ice, $m_{ice\text{-}EV}$	=	. . . .	. . . .
		. . . .	
Measured Value of the Mass of Ice, $m_{ice\text{-}MV}$	=	. . . .	. . . .
		. . . .	

% Error for the Mass of Ice:. . . . .

Dimensional analysis for the Specific Heat:. . . . .

QUESTIONS :

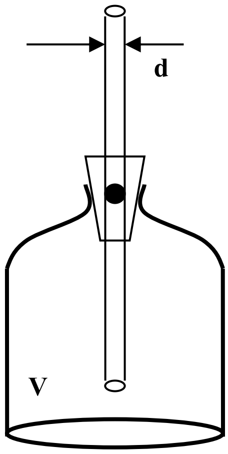
1) The calorimeter used in this experiment does not have perfect insulation and some heat is lost to the surroundings. Estimate the effect of 10% heat loss (10% of the total heat exchanged) on the specific heat value you determine for the specimen.

2) In the heat exchange equations, the thermal effect of the thermometer is neglected; discuss the thermal effect of the thermometer or the thermal sensor in the results.

8. *THE RATIO OF HEAT CAPACITIES OF AIR, $\gamma = C_P/C_V$*

OBJECTIVE : To determine the ratio of the specific heats of air, C_p / C_v.

THEORY : We can determine the ratio of the heat capacities air in a glass vessel by observing the oscillation of a steel ball inside the glass tube attached to the top of the glass bottle.



The pressure inside the bottle is given by:

$$P = P_o + \frac{Mg}{a}$$

where P_0 is the atmospheric pressure and M is the mass of the steel ball. This is the case when the ball is in equilibrium and it closes the opening completely but can move up and down easily. When the ball is disturbed away from the equilibrium position by an infinitesimal amount, dx, there will be a change in the pressure ΔP. This change in the pressure applies a net force causing the ball to accelerate:

$$A\Delta P = M\frac{d^2x}{dt^2}$$

where A is the cross section of the glass tube. Changes in the pressure can be considered adiabatic, so that

$$PV^\gamma = cons.$$

where γ is the ratio of the specific heats. Through differentiation of this expression and using the expression for the volume change as Ax, we can show that the equation of motion can be expressed as:

$$\frac{d^2x}{dt^2} + \frac{\gamma PA^2}{MV}x = 0$$

where V is the volume of the bottle. Then the period of oscillations can be given as

$$T = 2\pi\sqrt{\frac{MV}{\gamma PA}}$$

and

$$\gamma = 4\pi^2 \frac{MV}{A^2 PT^2}$$

By measuring all the quantities on the right you can determine the ratio of the specific heat of air.

APPARATUS **:** C_p/C_V apparatus and a stopwatch.

PROCEDURE **:** After cleaning the inside of the tube and the steel ball, drop the ball into the tube. Start the time when the ball is at its lowest position and determine the total time for as many oscillations as possible as long as the amplitude of the oscillation is greater than 2-3 cm.

The Ratio of Heat Capacities of Air, $\gamma = C_P/C_V$

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			**Value & Unit**
cm Hg	h	=	....
Density of Mercury	ρ	=	13.6 g / cm^3
Acceleration due to gravity	g	=	....
Flask Number	N	=	....
Volume of the Flask	V	=	....
Diameter of the ball	D	=	....
Mass of the ball	m	=	....

# of Trials	# of Oscillations *(n)*	Time for n Oscillations *t* ( )	Time for One Oscillation (Period) *T* ( )
1			
2			
3			
4			
5			

CALCULATIONS:

Description	Symbol	Calculations (show each step)	Result
Radius of the ball	$R =$	. . . .	. . . .
Cross sectional Area of the precision tube	$A =$	. . . .	. . . .
Atmospheric Pressure	$P_o = \rho g h \quad =$	. . . .	. . . .
		. . . .	
Pressure inside the bottle at Equilibrium Position of the Ball	$P_e = P_o + \dfrac{mg}{A} =$	. . . .	. . . .
		. . . .	

Description / Symbol		**Calculations (show each step)**	**Result**
Average Period, T_{average}	=	. . . .	. . . .
		. . . .	
Ratio of Heat Capacities, $\gamma = C_p / C_v$	=	. . . .	. . . .
		. . . .	

Show the Dimensional Analysis of γ clearly

APPENDICES

A. Physical Constants:

Planck's constant	h	$6.626x10^{-34}$ J or $4.136x10^{-21}$Mev. sec
	$\hbar$	$1.05x10^{-34}$J.sec or $6.58x10^{-22}$ Mev.sec
Universal Gas Constant	R	8.314 J/^{o}K mole
Avagadro's Number	N_A	$6.022x10^{23}$
Boltzman Constant	k	$1.381x10^{-23}$ J/^{o}K or $8.617x10^{-5}$ ev/^{o}K
Electron charge	e	$1.602x10^{-19}$ C
Speed of light in vacuum	c	$2.998x10^{8}$ m/sec
Stefan-Boltzman Constant	σ	$5.67x10^{-8}$ W/m^2.^{o}K^4
Gravitational Constant	G	$6.672x10^{-11}$ N.m^2/kg^2
Gravitational acceleration	g	9.81 m/sec^2
Permeability of Vacuum	μ_o	$1.257x10^{-6}$ H/m
Permitivity of Vacuum	ε_o	or $8.854x10^{-12}$ C^2/J.m
Rydberg Constant	R_∞	$1.097x10^{7}$ m^{-1}
Fine structure constant	$\alpha = e^2/(2\varepsilon_o hc)$	$7.297x10^{-3}$
First Bohr radius	α_o	$5.29x10^{-11}$ m
Charge to mass ratio of the electron	e/m	$1.759x10^{11}$ C/kg
Bohr Magneton	μ_B	$9.27x10^{-24}$ A.m^2
Atomic mass unit (amu)	u	$1.66x10^{-27}$ kg or 931.5 Mev
Electron rest mass	m_e	$9.11x10^{-31}$ kg or 511 kev
Proton rest mass	M_p	$1.672x10^{-27}$ kg or 938.2 Mev
Neutron rest mass	M_n	$1.675x10^{-27}$ kg or 939.6 Mev
Compton wavelength of electron	λ_C	$2.43x10_{-12}$ m
$\hbar c$		197 Mev. Fermi
Standard volume of ideal gas		$2.24x10^{-2}$ m^3/mole
1 eV		$1.602x10^{-19}$ J
1 amu		931.14 Mev
1 g		$5.610x10^{26}$ Mev
1 electron mass		0.51098 Mev
Ice point	T_o	273.16 ^{o}K

B. Conversion Tables:

LENGTH

	cm	meter	km	A^0	inch	foot	mile
cm	1	10^{-2}	10^{-5}	10^8	0.3937	$3.281x10^{-2}$	$6.214x10^{-6}$
meter	100	1	10^{-3}	10^{10}	39.37	3.281	$6.214x10^{-4}$
km	10^5	1000	1	10^{13}	$3.937x10^4$	3281	0.6214
A^0	10^8	10^{10}	10^{13}	1	$3.937x10^{-9}$	$3.281x10^{-10}$	$4.214x10^{-14}$
inch	28.540	0.0254	$2.540x10^{-5}$	$2.540x10^8$	1	0.0833	$1.578x10^{-5}$
foot	30.48	0.3048	$3.048x10^{-4}$	$3.048x10^9$	12	1	$1.894x10^{-4}$
mile	$1.609x10^5$	1609	1.609	$1.609x10^{13}$	$6.336x10^4$	5280	1

AREA

	m^2	cm^2	ft^2	$in.^2$	circ mile
m^2	1	10^4	10.76	1550	$1.974x10^9$
cm^2	10^{-4}	1	$1.076x10^{-3}$	0.1550	$1.974x10^5$
ft^2	$9.290x10^{-2}$	929.0	1	144	$1.833x10^8$
$in.^2$	$6.452x10^{-4}$	6.452	$6.944x10^{-3}$	1	$1.273x10^6$
circular mill	$5.067x10^{-10}$	$5.065x10^{-6}$	$5.454x10^{-9}$	$7.854x10^{-7}$	1

VOLUME

	m^3	cm^3	liter	ft^3	$in.^3$
m^3	1	10^6	1000	35.31	$6.102x10^4$
cm^3	10^{-6}	1	$1.000x10^{-3}$	$3.531x10^{-5}$	$6.102x10^{-2}$
liter	$1.000x10^{-3}$	1000	1	$3.531x10^{-2}$	61.02
ft^3	$2.832x10^{-2}$	$2.832x10^4$	28.32	1	1728
$in.^3$	$1.639x10^{-5}$	16.39	$1.639x10^{-2}$	$5.787x10^{-4}$	1

MASS

	kg	gram	ounce	pound	amu	m slug	ton
kg	1	10^3	35.27	2.205	$6.024x10^{26}$	$1.021x10^{-1}$	10^{-3}
gram	10^{-3}	1	$3.527x10^{-2}$	$2.205x10^{-3}$	$6.024x10^{23}$	$1.021x10^{-4}$	10^{-6}
ounce	$2.835x10^{-2}$	28.35	1	$6.250x10^{-2}$	$1.708x10^{25}$	$2.895x10^{-3}$	$2.835x10^{-5}$
pound	$4.536x10^{-1}$	$4.536x10^2$	16	1	$2.372x10^{25}$	$4.630x10^{-2}$	$4.536x10^{-4}$
amu	$1.66x10^{-27}$	$1.66x10^{-24}$	$5.854x10^{-26}$	$3.66x10^{-27}$	1	$1.695x10^{-28}$	$1.660x10^{-30}$
m slug	9.806	$9.806x10^3$	$3.454x10^2$	21.62	$5.9x10^{27}$	1	$9.806x10^{-3}$
ton	10^3	10^6	$3.527x10^4$	$2.205x10^{-3}$	$6.024x10^{29}$	$1.021x10^2$	1

TIME

	second	minute	hour	year
second	1	1.667×10^{-2}	2.778×10^{-4}	3.165×10^{-8}
minute	60	1	1.667×10^{-2}	1.901×10^{-6}
hour	3600	60	1	1.140×10^{-4}
year	3.156×10^7	5.259×10^5	8.765×10^3	1

FORCE

	Nt	Dyne	Kg F
Nt	1	10^5	0.1020
Dyne	10^{-5}	1	$1.020x10^{-6}$
Kg F	9.807	$9.807x10^5$	1

PRESSURE

	pa	mm Hg	mbar	kgf/m^2	$dyne/cm^2$	atmosphere
Pascal	1	$7.501x10^{-3}$	10^{-2}	0.1020	10	$9.869x10^{-6}$
torr	$1.333x10^2$	1	1.333	13.6	$1.333x10^3$	$1.316x10^{-3}$
mbar	10^2	0.7501	1	10.20	10^3	$9.869x10^{-4}$
$dyne/cm^2$	0.1	$7.501x10^{-4}$	10^{-3}	$10.20x10^{-3}$	1	$9.869x10^{-7}$
kgf/m^2	9.807	$9.807x10^{-2}$	$9.807x10^{-2}$	1	98.07	$9.679x10^{-5}$
atm	$1.013x10^5$	$7.601x10^2$	$1.013x10^{-3}$	$1.033x10^4$	$1.013x10^6$	1

ENERGY

	Joule	kilowatt-hour	Btu	erg	Calorie	electron volt
Joule	1	$2.778x10^{-7}$	$9.480x10^{-4}$	10^7	0.2389	$6.242x10^{18}$
kilowatt-hour	$3.6x10^6$	1	$3.412x10^3$	$3.6x10^{13}$	$8.6x10^5$	$2.247x10^{25}$
Btu	$1.055x10^3$	$2.930x10^{-4}$	1	$1.055x10^{10}$	$2.468x10^2$	$6.585x10^{21}$
erg	10^{-7}	$2.778x10^{-14}$	$9.480x10^{-11}$	1	$2.389x10^{-8}$	$6.242x10^{11}$
calorie	4.187	$1.163x10^{-6}$	$4.053x10^{-3}$	$4.187x10^7$	1	$2.613x10^{19}$
electron volt	$1.602x10^{-19}$	$4.450x10^{-26}$	$1.519x10^{-22}$	$1.602x10^{-12}$	$3.827x10^{-20}$	1

POWER

	watt	erg/sec	calorie/sec	kgfm/sec	Btu/sec	HP
watt	1	10^7	0.2388	0.1020	3.413	$1.360x10^{-3}$
erg/sec	10^{-7}	1	$2.388x10^{-8}$	$1.020x10^{-8}$	$3.413\ x10^{-7}$	$1.360x10^{-10}$
calorie/sec	4.187	$4.187x10^7$	1	0.4268	14.29	$5.694x10^{-3}$
kgfm/sec	9.807	$9.807x10^7$	2.343	1	33.47	133.3
Btu/sec	0.2931	$2.931x10^6$	$6.999x10^{-2}$	$2.987x10^{-2}$	1	$3.982x10^{-4}$
HP	735.5	$7.355x10^9$	175.7	75	$2.511x10^3$	1

MAGNETIC FIELD

	gauss	TESLA	milligauss
gauss	1	10^{-4}	1000
TESLA	10^4	1	10^7
milligauss	0.001	10^{-7}	1

REFERENCES