PHYSICS I: EXPERIMENTS

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Foreword

This book is written with a dual purpose in mind. Firstly, it aims to guide the students in the experiments of the elementary physics courses. Secondly, it incorporates the worksheets that the students use during their 2-hour laboratory session.

There are six books to accompany the six elementary physics courses taught at Bogazici University. After renovating our laboratories, replacing most of the equipment, and finally removing the 110-V electrical distribution in the laboratories, it has become necessary to prepare these books. Each book starts with the basic methods for data taking and analysis. These methods include brief descriptions for some of the instruments used in the experiments and the graphical method for fitting the data to a straight line. In the second part of the book, the specific experiments performed in a specific course are explained in detail. The objective of the experiment, a brief theoretical background, apparatus and the procedure for the experiment are given in this part. The worksheets designed to guide the students during the data taking and analysis follows this material for each experiment. Students are expected to perform their experiment and data analysis during the allotted time and then hand in the completed worksheet to the instructor by tearing it out of the book.

We would like to thank the members of the department that made helpful suggestions and supported this project, especially Arsin Arşık and Işın Akyüz who taught these laboratory classes for years. Our teaching assistants and student assistants were very helpful in applying the procedures and developing the worksheets. Of course, the smooth operation of the laboratories and the continuous well being of the equipment would not be possible without the help of our technicians, Erdal Özdemir and Hüseyin Yamak, who took over the job from Okan Ertuna.

Erhan Gülmez & Zuhal Kaplan
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Part I. BASIC METHODS

Introduction

Physics is an experimental science. Physicists try to understand how nature works by making observations, proposing theoretical models and then testing these models through experiments. For example, when you drop an object from the top of a building, you observe that it starts with zero speed and hits the ground with some speed. From this simple observation you may deduce that the speed or the velocity of the object starts from zero and then increases, suggesting a nonzero acceleration.

Usually when we propose a new model we start with the simplest explanation. Assuming that the acceleration of the falling object is constant, we can derive a relationship between the time it takes to reach the ground and the height of the building. Then measuring these quantities many times we try to see whether the proposed relationship is valid. The next question would be to find an explanation for the cause of this motion, namely the Newton's Law. When Newton proposed his law, he derived it from his observations. Similarly, Kepler's laws are also derived from observations. By combining his laws of motion with Kepler's laws, Newton was able to propose the gravitational law of attraction. As you see, it all starts with measuring lengths, speeds, etc. You should understand your instruments very well and carry out the measurements properly. Measuring things correctly is absolutely essential for the success of your experiment.

Every time a new model or law is proposed, you can make some predictions about the outcome of new and untried experiments. You can test the proposed models by comparing the results of these actual experiments with the predictions. If the results disagree with the predictions, then the proposed model is discarded or modified. However, an agreement between the experimental results and the predictions is not sufficient for the acceptance of the specific model. Models are tested continuously to make sure that they are valid. Galilean relativity is modified and turned into the special relativity when we started measuring speeds in the order of the speed of light. Sometimes the modifications may occur before the tests are done. Of course, all physical laws are based on experimental studies. Experimental results always take precedence over theory. Obviously, experiments have to be done carefully and objectively without any bias. Uncertainties and any contributing systematic effects should be studied carefully.

This book is written for the laboratory part of the Introductory Physics courses taken by freshman and sophomore classes at Bogazici University. The first part of the book gives you basic information about statistics and data analysis. A brief theoretical background and a procedure for each experiment are given in the second part.

Experiments are designed to give students an understanding of experimental physics regardless of their major study areas, and also to complement the theoretical part of the course. They will introduce you to the experimental methods in physics. By doing these experiments, you will also be seeing the application of some of the physics laws you will be learning in the accompanying course.

You will learn how to use some basic instruments and interpret the results, to take and analyze data objectively, and to report their results. You will gain experience in data taking and improve your insight into the physics problems. You will be performing the experiments by following the procedures outlined for each experiment, which will help you gain confidence in experimental work. Even though the experiments are designed to be simple, you may have some errors due to systematical effects and so your results may be different from what you would expect theoretically. You will see that there is a difference between real-life physics and the models you are learning in class.

You are required to use the worksheets to report your results. You should include all your calculations and measurements to show that you have completed the experiment fully and carried out the required analysis yourself.

DATA TAKING AND ANALYSIS

Dimensions and Units

A physical quantity has one type of dimension but it may have many units. The dimension of a quantity defines its characteristic. For example, when we say that a quantity has the dimension of length (L), we immediately know that it is a distance between two points and measured in terms of units like meter, foot, etc. This may sound too obvious to talk about, but dimensional analysis will help you find out if there is a mistake in your derivations. Both sides of an equation must have the same dimension. If this is not the case, you may have made an error and you must go back and recheck your calculations. Another use of a dimensional analysis is to determine the form of the empirical equations. For example, if you are trying to determine the relationship between the distance traveled under constant acceleration and the time involved empirically, then you should write the equation as

$$d = kat^n$$

where k is a dimensionless quantity and a is the acceleration. Then, rewriting this expression in terms of the corresponding dimensions:

$$L = \left(LT^{-2}\right)T^n$$

will give us the exponent n as 2 right away. You will be asked to perform dimensional analysis in most of the experiments to help you familiarize with this important part of the experimental work.

Measurement and Instruments

To be a successful experimenter, one has to work in a highly disciplined way. The equipment used in the experiment should be treated properly, since the quality of the data you will obtain will depend on the condition of the equipment used. Also, the equipment has a certain cost and it may be used in the next experiment. Mistreating the equipment may have negative effects on the result of the experiment, too.

In addition to following the procedure for the experiment correctly and patiently, an experimenter should be aware of the dangers in the experiment and pay attention to the warnings. In some cases, eating and drinking in the laboratory may have harmful effects on you because food might be contaminated by the hazardous materials involved in the experiment, such as radioactive materials. Spilled food and drink may also cause malfunctions in the equipment or systematic effects in the measurements.

Measurement is a process in which one tries to determine the amount of a specific quantity in terms of a pre-calibrated unit amount. This comparison is made with the help of an instrument. In a measurement process only the interval where the real value exists can be determined. Smaller interval means better precision of the instrument. The smallest fraction of the pre-calibrated unit amount determines the precision of the instrument.

You should have a very good knowledge of the instruments you will be using in your measurements to achieve the best possible results from your work. Here we will explain how to use some of the basic instruments you will come across in this course.

Reading analog scales:

You will be using several different types of scales. Examples of these different types of scales are rulers, vernier calipers, micrometers, and instruments with pointers.

The simplest scale is the **meter stick** where you can measure lengths to a millimeter. The precision of a ruler is usually the smallest of its divisions.

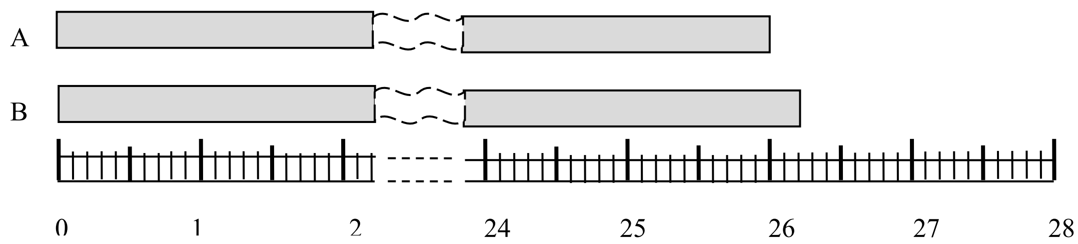


Figure 1. Length measurement by a ruler

In Figure 1, the lengths of object A and B are observed to be around 26 cm. Since we use a ruler with millimeter division the measurement result for the object A should be given as 26.0 cm and B as 26.2 cm. If you report a value more precise than a millimeter when you use a ruler with millimeter division, obviously you are guessing the additional decimal points.

Vernier Calipers (Figure 2) are instruments designed to extend the precision of a simple ruler by one decimal point. When you place an object between the jaws, you may obtain an accurate value by combining readings from the main ruler and the scale on the frame attached to the movable jaw. First, you record the value from the main ruler where the zero line on the frame points to. Then, you look for the lines on the frame and the main ruler that looks like the same line continuing in both scales. The number corresponding to this line on the frame gives you the next digit in the measurement. In Figure 2, the measurement is read as 1.23 cm. The precision of a vernier calipers is the smallest of its divisions, 0.1 mm in this case.

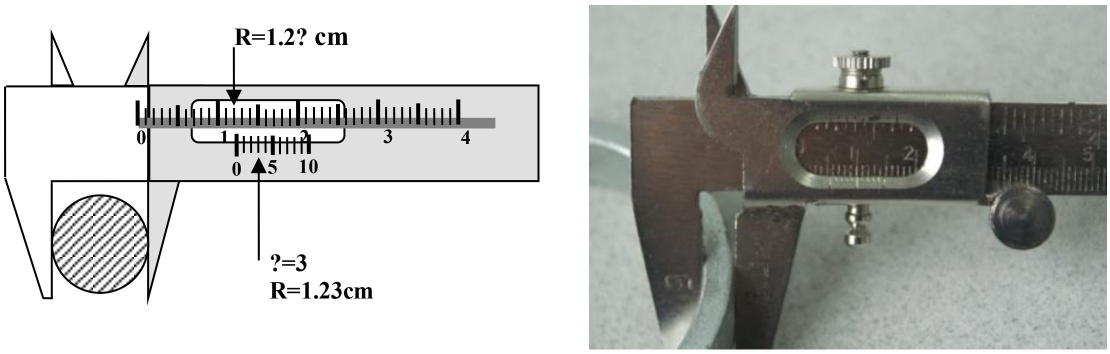


Figure 2. Vernier Calipers.

Micrometer (Figure 3) is similar to the vernier calipers, but it provides an even higher precision. Instead of a movable frame with the next decimal division, the micrometer has a cylindrical scale usually divided into a hundred divisions and moves along the main ruler like a screw by turning the handle. Again the coarse value is obtained from the main ruler and the more precise part of the measurement comes from the scale around the rim of the cylindrical part. Because of its higher precision, it is used mostly to measure the thickness of wires and similar things. In Figure 3, the measurement is read as 1.187 cm. The precision of a micrometer is the smallest of its divisions, 0.01 mm in this case.

Here is an example for the measurement of the radius of a disk where a ruler, a vernier calipers, and a micrometer are used, respectively:

Measurement	Precision	Instrument
$R = (23 \pm 1)mm$	1 mm	Ruler
$R = (23.1 \pm 0.1)mm$	0.1 mm	Vernier calipers
$R = (23.14 \pm 0.01)mm$	0.01 mm	Micrometer

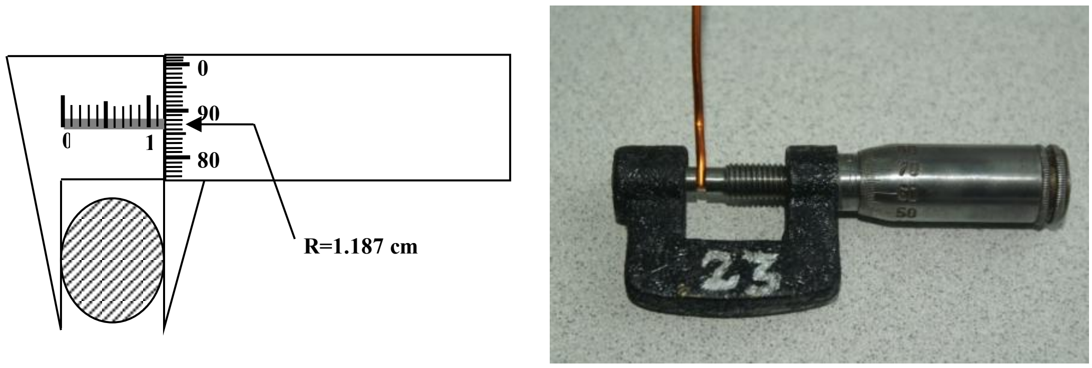


Figure 3. Micrometer.

Spherometer (Figure 4) is an instrument to determine very small thicknesses and the radius of curvature of a surface. First you should place the spherometer on a level surface to get a calibration reading (CR). You turn the knob at the top until all four legs touch the surface. When the middle leg also touches the surface, the knob will first seem to be free and then tight. The reading at this position will be the calibration reading (CR). Then you should place the spherometer on the curved surface and turn the knob until all four legs again touch the surface. The reading at this position will be the measurement reading (MR). You will read the value from the vertical scale first and then the value on the dial will give you the fraction of a millimeter. Then you can calculate the radius of curvature of the surface as:

$$R = \frac{D}{2} + \frac{A^2}{6D}$$

where D = |CR-MR| and A is the distance between the outside legs.

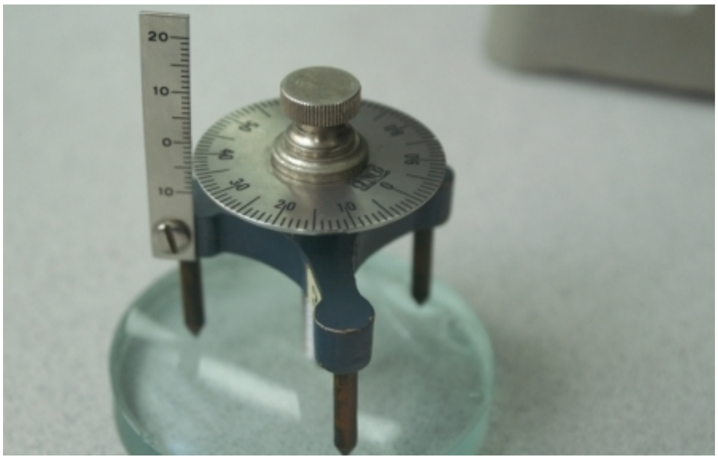


Figure 4. Spherometer.

Instruments with pointers usually have a scale along the path that the pointer moves. Mostly the scales are curved since the pointers move in a circular arc. To avoid the systematic errors introduced by the viewing angle, one should always read the value from the scale where the pointer is projected perpendicularly. You should not read the value by looking at the pointer and the scale sideways or at different angles. You should always look at the scale and the pointer perpendicularly. Usually in most instruments there is a mirror attached to the scale to make sure the readings are done similarly every time when you take a measurement (Figure 5). When you bring the scale and its image on the mirror on top of each other, you will be looking at the pointer and the scale perpendicularly. Then you can record the value that the pointer shows on the scale. Whenever you measure something by such an instrument, you should follow the same procedure.

Figure 5. A voltmeter with a mirror scale.

Data Logger

In some experiments we will be using sensors to measure some quantities like position, angle, angular velocity, temperature, etc. The output of these sensors will be converted into numbers with the help of a data acquisition instrument called DATA LOGGER (Figure 6).

Data Logger is a versatile instrument that takes data using changeable sensors. When you plug a sensor to its receptacle at the top, it recognizes the type of the sensor. When you turn the data logger on with a sensor attached, it will start displaying the default mode for that sensor. Data taking with the data logger is very simple. You can start data taking by pressing the Start/Stop button (7). You may change the display mode by pressing the button on the right with three rectangles (6). To change the default measurement mode, you should press the plus or minus buttons (3 or 4). If there is more than one type of quantity because of the specific sensor you are using, you may select the type by pressing the button with a check mark (5) to turn on the editing mode and then selecting the desired type by using the plus and minus buttons (3 or 4). You will exit from the editing mode by pressing the button with the check mark (5) again. You may edit any of the default settings by using the editing and plus-minus buttons. For a more detailed operation of the instrument you should consult your instructor.

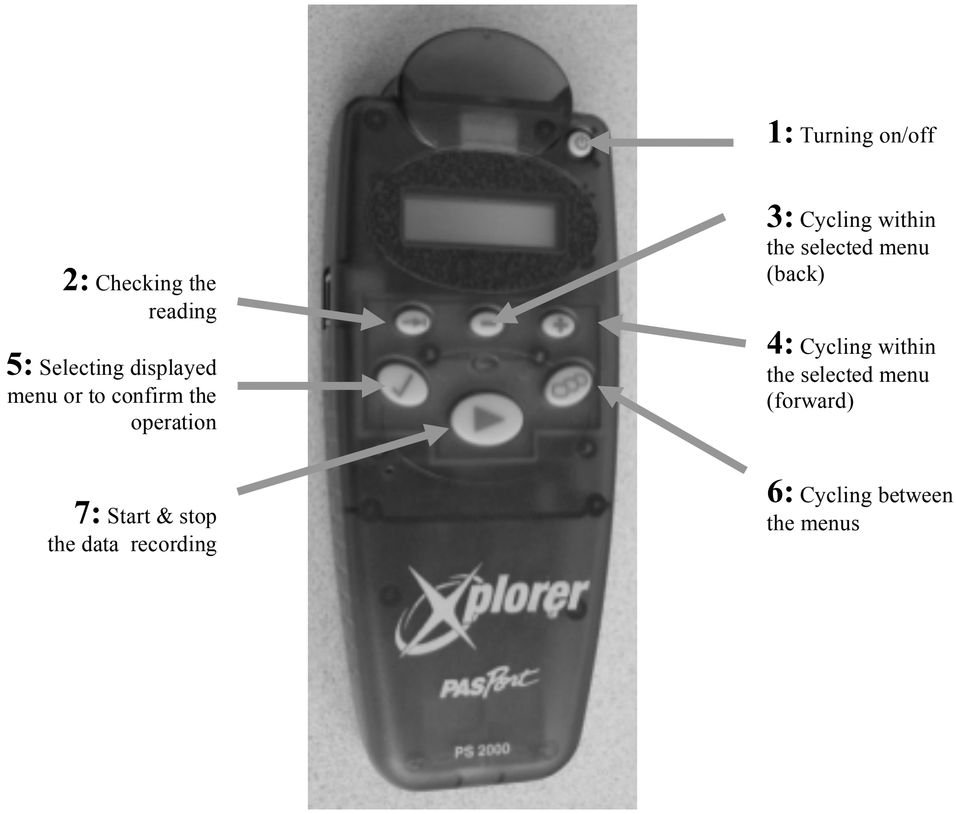


Figure 6. Data Logger.

Basics of Statistics and Data Analysis

Here, you will have an introduction to statistical methods, such as distributions and averages.

All the measurements are done for the purpose of obtaining the value for a specific quantity. However, the value by itself is not enough. Determining the value is half the experiment. The other half is determining the uncertainty. Sometimes, the whole purpose of an experiment may be to determine the uncertainty in the results.
Error and uncertainty are synonymous in experimental physics even though they are two different concepts. Error is the deviation from the true value. Uncertainty, on the other hand, defines an interval where the true value is. Since we do not know the true value, when we say error we actually mean uncertainty. Sometimes the accepted value for a quantity after many experiments is assumed to be the true value.

Sample and parent population

When you carry out an experiment, usually you take data in a finite number of trials. This is our sample population. Imagine that you have infinite amount of time, money, and effort available for the experiment. You repeat the measurement infinite times and obtain a data set that has all possible outcomes of the experiment. This special sample population is called parent population since all possible sample populations can be derived from this infinite set. In principle, experiments are carried out to obtain a very good representation of the parent population, since the parameters that we are trying to measure are those that belong to the parent population. However, since we can only get an approximation for the parent population, values determined from the sample populations are the best estimates.

Mean and Standard deviation

Measuring a quantity usually involves statistical fluctuations around some value. Multiple measurements included in a sample population may have different values. Usually, taking an average cancels the statistical fluctuations to first degree. Hence, the average value or the mean value of a quantity in a sample population is a good estimate for that quantity.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Even though the average value obtained from the sample population is the best estimate, it is still an estimate for the true value. We should have another parameter that tells us how close we are to the true value. The variance of the sample:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

gives an idea about how scattered the data are around the mean value. Variance is in fact a measure of the average deviation from the mean value. Since there might be negative and positive deviations, squares of the deviations are averaged to avoid a null result. Because the variance is the average of the squares, square root of variance is a better quantity that shows the scatter around the mean value. The square root of the variance is called standard deviation:

$$\sigma_s = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

However, the standard deviation calculated this way is just the standard deviation of the sample population. What we need is the standard deviation of the parent population. The best estimate for the standard deviation of the parent population can be shown to be:

$$\sigma_p = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

As the number of measurements, N, becomes large or as the sample population approaches parent population, standard deviation of the sample is almost equal to the standard deviation of the parent population.

Distributions

The probability of obtaining a specific value can be determined by dividing the number of measurements with that value to the total number of measurements in a sample population. Obviously, the probabilities obtained from the parent population are the best estimates. Total probability should be equal to 1 and probabilities should be larger as one gets closer to the mean value. The set of probability values associated with a population is called the probability distribution for that measurement. Probability distributions can be experimental distributions obtained from a measurement or mathematical functions. In physics, the most frequently used mathematical distributions are Binomial, Poisson, Gaussian, and Lorentzian. Gaussian and Poisson distributions are in fact special cases of Binomial distribution. However, in most cases, Gaussian distribution is a good approximation. In fact, all distributions approach Gaussian distribution at the limit (Central Limit Theorem).

Errors

The result of an experiment done for the first time almost always turns out to be wrong because you are not familiar with the setup and may have systematic effects. However, as you continue to take data, you will gain experience in the experiment and learn how to reduce the systematical effects. In addition to that, increasing number of measurements will result in a better estimate for the mean value of the parent population.

Errors in measurements: Statistical and Systematical errors

As mentioned above, error is the deviation between the measured value and the true value. Since we do not know the true value, we cannot determine the error in this sense. On the other hand, uncertainty in our measurement can tell us how close we are to the true value. Assuming that the probability distribution for our measurement is a Gaussian distribution, 68% of all possible measurements can be found within one standard deviation of the mean value. Since most physical distributions can be approximated by a Gaussian, defining the standard deviation as our uncertainty for that measurement will be a reasonable estimate. In some cases, two-standard deviation or two-sigma interval is taken as the uncertainty. However, for our purposes using the standard deviation as the uncertainty would be more than enough. Also, from now on, whenever we use error, we will actually mean uncertainty.

Errors or uncertainties can be classified into two major groups; statistical and systematical.

Statistical Errors

Statistical errors or random errors are caused by statistical fluctuations in the measurements. Even though some unknown phenomenon might be causing these fluctuations, they are mostly random in nature. If the size of the sample population is large enough, then there is equal number of measurements on each side of the mean at about similar distances. Therefore, averaging over such a large number of measurements will smooth the data and cancel the effect of these fluctuations. In fact, as the number of measurements increases, the effect of the random fluctuations on the average will diminish. Taking as much data as possible improves statistical uncertainty.

Systematical Errors

On the other hand, systematical errors are not caused by random fluctuations. One could not reduce systematical errors by taking more data. Systematical errors are caused by various reasons, such as, the miscalibration of the instruments, the incorrect application of the procedure, additional unknown physical effects, or anything that affects the quantity we are measuring. Systematic errors caused by the problems in the measuring instruments are also called instrumental errors. Systematical errors are reduced or avoided by finding and removing the cause.

Example 1: You are trying to measure the length of a pipe. The meter stick you are going to use for this purpose is constructed in such a way that it is missing a millimeter from the beginning. Since both ends of the meter stick are covered by a piece of metal, you do not see that your meter stick is 1 mm short at the beginning. Every time you use this meter stick, your measurement is actually 1 mm longer than it should be. This will be the case if you repeat the measurement a few times or a few million times. This is a systematical error and, since it is caused by a problem in the instrument used, it is considered an instrumental error. Once you know the cause, that is, the shortness of your meter stick, you can either repeat your measurement with a proper meter stick or add 1 mm to every single measurement you have done with that particular meter stick.

Example 2: You might be measuring electrical current with an ammeter that shows a nonzero value even when it is not connected to the circuit. In a moving coil instrument this is possible if the zero adjustment of the pointer is not done well and the pointer always shows a specific value when there is no current. The error caused by this is also an instrumental error.

Example 3: At CERN, the European Research Center for Nuclear and Particle Physics, there is a 28 km long circular tunnel underground. This tunnel was dug about 100 m below the surface. It was very important to point the direction of the digging underground with very high precision. If there were an error, instead of getting a complete circle, one would get a tunnel that is not coming back to the starting point exactly. One of the inputs for the topographical measurements was the direction towards the center of the earth. This could be determined in principle with a plumb bob (or a piece of metal hung on a string) pointing downwards under the influence of gravity. However, when there is a mountain range on one side and a flat terrain on the other side (like the location of the CERN accelerator ring), the direction given by the plumb bob will be slightly off towards the mountainous side. This is a systematic effect in the measurement and since its existence is known, the result can be corrected for this effect.

Once the existence and the cause of a systematic effect are known, it is possible to either change the procedure to avoid it or correct it. However, we may not always be fortunate enough to know if there is a systematic effect in our measurements. Sometimes, there might be unknown factors that affect our experiment. The repetition of the measurement under different conditions, at different locations, and with totally different procedures is the only way to remove the unknown systematic effects. In fact, this is one of the fundamentals of the scientific method.

We should also mention the accuracy and precision of a measurement. The meaning of the word "accuracy" is closeness to the true value. As for "precision," it means a measurement with higher resolution (more significant figures or digits). An instrument may be accurate but not precise or vice versa. For example, a meter stick with millimeter divisions may show the correct value. On the other hand, a meter stick with 0.1 mm division may not show the correct value if it is missing a one-millimeter piece from the beginning of the scale. However, if an instrument is precise, it is usually an expensive and well designed instrument and we expect it to be accurate.

Reporting Errors: Significant figures and error values

As mentioned above, determining the error in an experiment requires almost the same amount of work as determining the value. Sometimes, almost all the effort goes into determining the uncertainty in a measurement.

Using significant figures is a crude but an effective way of reporting the errors. A simple definition for significant figures is the number of digits that one can get from a measuring instrument (but not a calculator!). For example, a digital voltmeter with a four-digit display can only provide voltage values with four digits. All these four digits are significant unless otherwise noted. On the other hand, reporting a six digit value when using an analog voltmeter whose smallest division corresponds to a four-digit reading would be wrong. One could try to estimate the reading to the fraction of the smallest division, but then this estimate would have a large uncertainty.

Significant figures are defined as following:

- Leftmost nonzero digit is the most significant figure.

  Examples: 0.0000**6**520 m

  1234 m

  41.02 m

  126.1 m

  4120 m

  12000 m

- Rightmost nonzero digit is the least significant figure if there is no decimal point.

  Examples: 123**4** m

  41**2**0 m

  1**2**000 m

- If there is a decimal point, rightmost digit is the least significant figure even if it is zero.

  Examples: 0.0000652**0** m

  41.0**2** m

  126.**1** m

Then, the number of significant figures is the number of digits between the most and the least significant figures including them.

Examples:	0.0000**6**52**0** m	4 significant figures
	123**4** m	4 sf
	41.0**2** m	4 sf
	126.**1** m	4 sf
	41**2**0 m	3 sf
	12000 m	2 sf
	1.200**0** x 10^4 m	5 sf

Significant figures of the results of simple operations usually depend on the significant figures of the numbers entering into the arithmetic operations. Multiplication or division of two numbers with different numbers of significant figures should result in a value with a number of significant figures similar to the one with the smallest number of significant figure. For example, if you multiply a three-significant-figure number with a two-significant-figure number, the result should be a two-significant-figure number. On the other hand, when adding or subtracting two numbers, the outcome should have the same number of significant figures as the smallest of the numbers entering into the calculation. If the numbers have decimal points, then the result should have the number of significant figures equal to the smallest number of digits after the decimal point. For example, if three values, two with two significant figures and one with four significant figures after the decimal point, are added or subtracted, the result should have two significant figures after the decimal point.

Example: Two different rulers are used to measure the length of a table. First, a ruler with 1-m length is used. The smallest division in this ruler is one millimeter. Hence, the result from this ruler would be 1.000 m. However, the table is slightly longer than one meter. A second ruler is placed after the first one. The second ruler can measure with a precision of one tenth of a millimeter. Let's assume that it gives a reading of 0.2498 m. To find the total length of the table we should add these two values. The result of the addition will be 1.2498, but it will not have the correct number of significant figures since one has three and the other has four significant figures after the decimal point. The result should have three significant figures after the decimal point. We can get the correct value by rounding off the number to three significant figures after the decimal point and report it as 1.250 m.

More Examples for Addition and Subtraction:

$$\begin{array}{rl} & 4.122 \\ & 3.\mathbf{74} \\ + & \underline{0.011} \\ & 7.873 = 7.\mathbf{87} \end{array}$$

(2 digits after the decimal point)

Examples for Multiplication and Division:

4.782 x **3.05** = 14.5851 = **14.6** (3 significant figures)

3.728 / 1.6781 = 2.22156 = **2.222** (4 significant figures)

Rounding off

Sometimes you may have more numbers than the correct number of significant figures. This might happen when you divide two numbers and your calculator may give you as many digits as it has in its display. Then you should reduce the number of digits to the correct number of significant figures by rounding it off. One common mistake is by starting from the rightmost digit and repeatedly rounding off until you reach the correct number of significant figures. However, all the extra digits above and beyond the number of correct significant figures have no significance. Usually you should keep one extra digit in your calculations and then round this extra digit at the end. You should just discard the extra digits other than the one next to the least significant figure. The reasoning behind the rounding off process is to bring the value to the correct number of significant figures without adding or subtracting an amount in a statistical sense. To achieve this you should follow the procedure outlined below:

- If the number on the right is less than 5, discard it.
- If it is more than 5, increase the number on its left by one.
- If the number is exactly five, then you should look at the number on its left.
  - If the number on its left is even then again discard it.
  - If the number on the left of 5 is odd, then you should increase it by one.

This special treatment in the case of 5 is because there are four possibilities below and above five and adding five to any of them will introduce a bias towards that side. Hence, grouping the number on the left into even and odd numbers makes sure that this ninth case is divided into exactly two subsets; five even and five odd numbers. We count zero in this case since it is in the significant part. We do not count zero on the right because it is not significant.

Example: Rounding off 2.4456789 to three significant figures by starting from all the way to the right, namely starting from the number 9, and repeatedly rounding off until three significant figures are left would result in 2.45 but this would be wrong. The correct way of doing this is first dropping all the non-significant figures except one and then rounding it off, that is, after truncation 2.445 is rounded off to 2.44.

More Examples: Round off the given numbers to 3 significant figures:

43.3**7**468	=	43.37	=	43.**4**
43.3**4**468	=	43.34	=	43.**3**
43.*3***5**468	=	43.35	=	43.**4**
43.*4***5**568	=	43.45	=	43.**4**

If we determine the standard deviation for a specific value, then we can use that as the uncertainty since it gives us a better estimate. In this case, we should still pay attention to the number of significant figures since reporting extra digits is meaningless. For example if you have the average and the standard deviation as 2.567 and 0.1, respectively, then it would be appropriate to report your result as 2.6±0.1.

Weighted Averages

Sometimes we may measure the same quantity in different sessions. As a result we will have different sets of values and uncertainties. By combining all these sets we may achieve a better result with a smaller uncertainty. To calculate the overall average and standard deviation, we can assign weight to each value with the corresponding variance and then calculate the weighted average.

$$\mu = \frac{\sum_{i=1}^{m} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{m} \frac{1}{\sigma_i^2}}$$

Similarly we can also calculate the overall standard deviation.

$$\sigma_{\mu} = \sqrt{\frac{1}{\sum_{i=1}^{m} \frac{1}{\sigma_i^2}}}$$

Error Propagation

If you are measuring a single quantity in an experiment, you can determine the final value by calculating the average and the standard deviation. However, this may not be the case in some experiments. You may be measuring more than one quantity and combining all these quantities to get another quantity. For example, you may be measuring *x* and *y* and by combining these to obtain a third quantity *z*:

$$z = ax + by \qquad \text{or} \qquad z = f(x, y)$$

You could calculate *z* for every single measurement and find its average and standard deviation. However, a better and more efficient way of doing it is to use the average values of *x* and *y* to calculate the average value of *z*. In order to determine the variance of *z,* we have to use the square of the differential of *z*:

$$(dz)^2 = \left(\sum (\frac{\partial f}{\partial x})dx + (\frac{\partial f}{\partial y})dy \right)^2$$

Variance would be simply the sum of the squares of both sides over the whole sample set divided by the number of data points *N* (or *N*-1 for the parent population). Then, the general expression for determining the variance of the calculated quantity as a function of the measured quantities would be:

$$\sigma_z^2 = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial x_j} \right)^2 \sigma_j^2 \qquad \text{for } k \text{ number of measured quantities.}$$

Applying this expression to specific cases would give us the corresponding error propagation rule. Some special cases are listed below:

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 \qquad \text{for} \qquad z = ax \pm by$$

$$\frac{\sigma_z^2}{z^2} = \frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2} \qquad \text{for} \qquad z = axy \text{ or } \frac{ax}{y} \text{ or } \frac{ay}{x}$$

$$\frac{\sigma_z^2}{z^2} = b^2 \frac{\sigma_x^2}{x^2} \qquad \text{for} \qquad z = ax^b$$

$$\frac{\sigma_z^2}{z^2} = (b \ln a)^2 \sigma_x^2 \qquad \text{for} \qquad z = a^{bx}$$

$$\frac{\sigma_z^2}{z^2} = b^2 \sigma_x^2 \qquad \text{for} \qquad z = ae^{bx}$$

$$\sigma_z^2 = a^2 \frac{\sigma_x^2}{x^2} \qquad \text{for} \qquad z = a \ln(bx)$$

Multivariable measurements: Fitting procedures

When you are measuring a single quantity or several quantities and then calculating the final quantity using the measured values, all the measurements involve unrelated quantities. There are no relationships between them other than the calculated and measured quantities. However, in some cases you may have to set one or more quantities and measure another quantity determined by the independent variables. This is the case when you have a function relating some quantities to each other. For example, the simplest function would be the linear relationship:

$$y = ax + b$$

where a is called the slope and b the y-intercept. Since we are setting the value of the independent variable x, we assume its uncertainty to be negligible compared to the dependent variable y. Of course, we should be able to determine the uncertainty in y. From such an experiment, usually we have to determine the parameters that define the function; a and b. This can be done by fitting the data to a straight line.

The least squares (or maximum likelihood, or chi-square minimization) method would provide us with the best possible estimates. However, this method involves lengthy calculations and we will not be using it in this course.

We will be using a graphical method that will give us the parameters that we are looking for. It is not as precise as the least squares method and does not give us the uncertainties in the parameters, but it provides answers in a short time that is available to you.

Graphical method is only good for linear cases. However, there are some exceptions to this either by transforming the functions to make them linear or plotting the data on a semi-log or log-log or polar graph paper (Figure 7). $1/r$, $1/r^2$, $y = ax^5$, $y = ae^{-bx}$, are some examples for nonlinear functions that can be transformed to linear expressions. $1/r^n$ type expressions can be linearized by substituting $1/r^n$ with a simple *x*: $y = A + B/r^n \rightarrow y = A + Bx$ where $x = 1/r^n$. Power functions can be linearized by taking the logarithm of the function: $y = ax^n$ becomes $\log y = \log a + n \log x$ and then through $y' = \log y$, $a' = \log a$, and $x' = \log x$ transformation it becomes $y' = a' + nx'$. Exponential functions can be transformed similar to the power functions by taking the natural logarithm: $y = ae^{-bx}$ becomes $\ln y = \ln a - bx$ and through $y' = \ln y$ and $a' = \ln a$ transformation it becomes $y' = a' - bx$.

Before attempting to obtain the parameters that we are looking for, we have to plot the data on a graph paper. As long as we have linearly dependent quantities or transformed quantities as explained above, we can use regular graph paper.

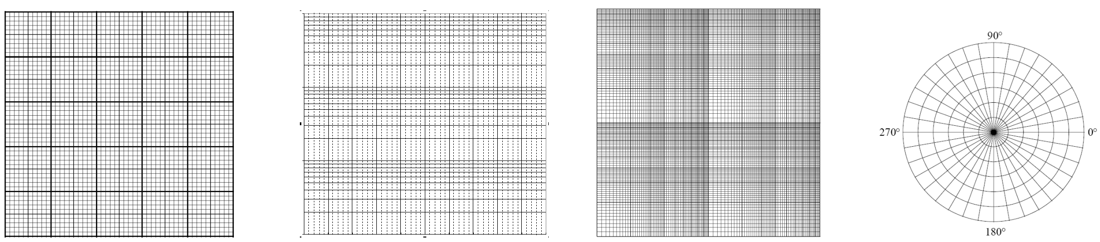


Figure 7: Different types of graph papers: linear, semi-log, log-log, and polar.

You should use as much area of the graph paper as possible when you plot your data. Your graph should not be squeezed to a corner with lots of empty space. To do this, first you should determine the minimum and maximum values for each variable, *x* and *y*, then choose a proper scale value. For example, if you have values ranging from 3 to 110 and your graph paper is 23 centimeters long, then you should choose a scale factor of 1 cm to 5 units of your variable and label your axis from 0 to 115 and marking each big square (usually linear graph papers prepared in cm and millimeter divisions) at increasing multiples of 5. You should choose the other axis in a similar way. When you select a scale factor you should select a factor that is easy to divide by, like 1, 2, 4, 5, 10, etc. Usually scale factors like 3, 4.5, 7.9 etc., are bad choices. Both axis may have different scale factors and may start from a nonzero value. You should clearly label each axis and write down the scale factors. Then you should mark the position corresponding to each data pair with a cross or similar symbols. Usually you should also include the uncertainties as vertical bars above and below the data point whose lengths are determined according to the scale factor. Once you finish marking all your data pairs, then you should try to pass a straight line through all the data points. Usually, this may not be possible since the data points may not fall into a straight line. However, since you know that the relationship is linear there should be a straight line that passes through the data points even though not all of them fall on a line. You should make sure that the straight line passes *through* the data points in a balanced way. An equal number of data points should be below and above the straight line. Then, by picking two points on the line as far apart from each other as possible, you should draw parallel lines to the axes, forming a triangle (Figure 8). The slope is the slope of the straight line. You can calculate the slope as:

$$Slope = \frac{\Delta y}{\Delta x}$$

and read the y-intercept from the graph by finding the point where the straight line crosses the y-axis. You can estimate the uncertainties of the slope and intercept by finding different straight lines that still pass through all the data points in an acceptable manner. The minimum and maximum values obtained from these different trials would give us an idea about the uncertainties. However, obtaining the parameters will be sufficient in this course.

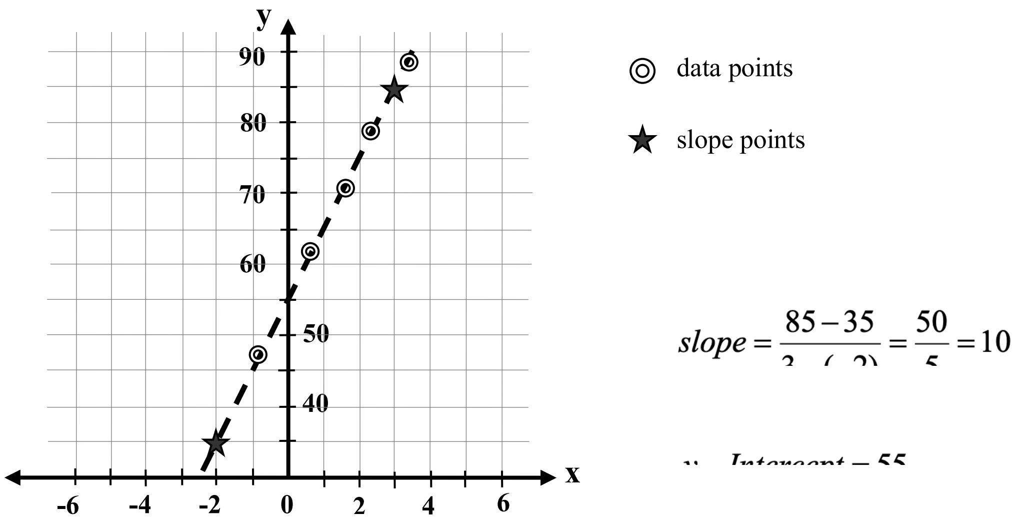


Figure 8: Determining the slope and y-intercept.

Special graph papers, like semi-log and log-log graph papers, are used when you have relationships that can be transformed into linear relationships by taking the base-10 logarithm of both sides. Semi-log graph papers are used if one side of the expression contains powers of ten or single exponential function resulting in a linear variable when you take the base-10 logarithm of both sides.

Logarithmic graph papers are used when you prefer to use the measured values directly without taking the logarithms and still obtaining a linear graph. Each logarithmic axis is divided in such a way that when you use the divisions marked on the paper it will have the same effect as if you first took the logarithm and then plotted on a regular graph paper. Logarithmic graph papers are divided linearly into decades and in each decade is divided logarithmically. There is no zero value in a logarithmic axis. You should plot your data by choosing appropriate scale factors for each axis and then mark the data points directly without taking the logarithms. You should again draw a straight line that will pass through all the data points in a balanced way. The slope of the line would give us the exponent in the relationship. For example, a relationship like $y = ax^n$ would be linearized as $\log y = \log a + n \log x$. If you plot this on a regular graph paper, the slope will be given by $n = (\log y_2 - \log y_1)/(\log x_2 - \log x_1)$ where you will read the logarithms directly from the graph. On the other hand, when you plot your data on a log-log paper, you will be using the measured values directly. When you picked the two points from the straight line that fits the data points best, the slope should be calculated by

$n = (\log y_2 - \log y_1)/(\log x_2 - \log x_1)$ where you will calculate the logarithms using the values read from the graph. y-intercept would be directly the value where the straight line crosses the vertical axis at $x = 1$.

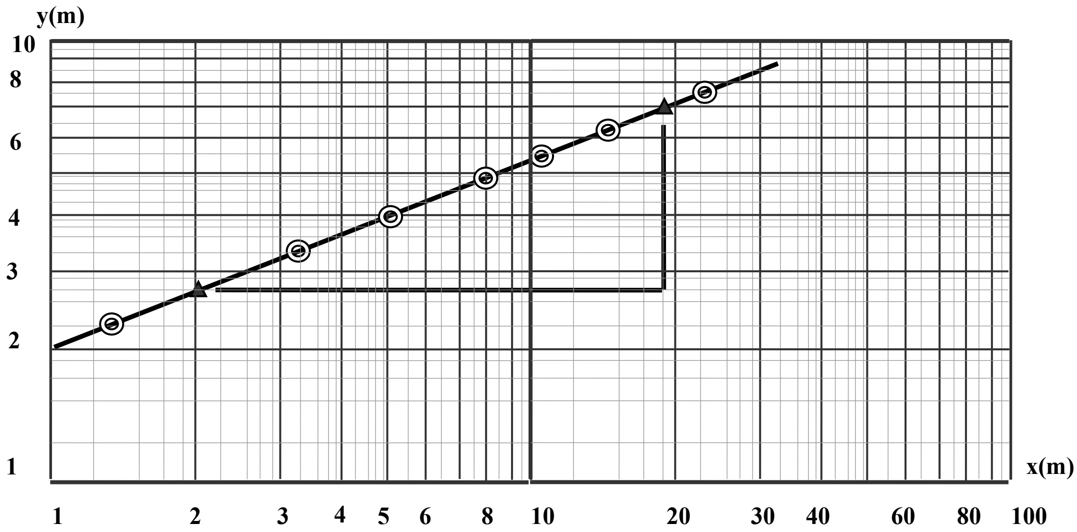


Figure 9: Determining the slope and y-intercept.

slope point 1: (2.0 ; 2.6) and slope point 2 : (18.0 ; 7.0)

$$\text{slope} = \frac{\log(7.0) - \log(2.6)}{\log(18.0) - \log(2.0)} = \frac{0.4301}{0.9542} = 0.4507 \qquad \text{and} \qquad y\text{-intercept} = 2.0.$$

Reports

Obviously, doing an experiment and getting some results are not enough. The results of the experiment should be published so that others working on the same problem will know your results and use them in their calculations or compare with their results. The reports should have all the details so that another experimenter could repeat your measurements and get the same results. However, in an introductory teaching lab there is no need for such extensive reports since the experiments you will be doing are well established and time is limited. You have to include enough details to convince your lab instructor that you have performed the experiment appropriately and analyzed it correctly. The results of your analysis, including the uncertainties in the measurements, should be clearly expressed. The comparisons with the accepted values may also be included if possible.

Part II: EXPERIMENTS

1. *THE SIMPLE PENDULUM*

OBJECTIVE **:** To study the motion of a simple pendulum and to determine the acceleration due to gravity using a simple pendulum.

THEORY **:** For small angular displacements less than about ten degrees, it can be shown that the motion of a point mass attached to the end of a string of length L is a periodic motion with the period:

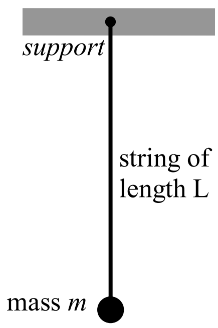
$$T = 2\pi\sqrt{\frac{L}{g}}.$$

We can calculate the gravitational acceleration, *g*, if we measure the length of the string and the period of oscillations:

$$g = 4\pi^2 \frac{L}{T^2}.$$

APPARATUS **:** A string of length L, a stopwatch, a metal ball and a meter stick.

PROCEDURE **:**



Choose an initial length for the pendulum which should not be less than 120 cm. Set the pendulum into oscillation making sure that the maximum amplitude is less than ten degrees. Measure the time, ***t*,** for 10 complete oscillations and determine the period, ***T***, corresponding to the chosen length. Repeat this for 4 more length values. Calculate ***g*** for each measurement.

THE SIMPLE PENDULUM

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description	**Symbol**		**Value & Unit**
Acceleration due to gravity	g_{TV}	=	. . . .
Number of Oscillations	N	=	. . . .

Length of Pendulum ***L* ( )**	***10 periods*** ***t* ( )**	***One Period*** ***T* ( )**
# of Significant Figures :	*# of Significant Figures :*	*# of Significant Figures :*

CALCULATIONS and RESULT:

Symbol		Calculations (show each step)	Result & Unit
g_1	=	. . . .	. . . .
g_2	=	. . . .	. . . .
g_3	=	. . . .	. . . .
g_4	=	. . . .	. . . .
g_5	=	. . . .	. . . .
$g_{average}$	=	. . . .	. . . .

% Deviation for g: . . . .

. . . .

Show the dimensional analysis for g: . . . .

. . . .

QUESTIONS :

1) What are the possible sources of systematic errors in this experiment?

2) At what point of its swing, does the ball have its maximum velocity? Maximum acceleration?

3) Assume that your pendulum passes through its equilibrium point every second,

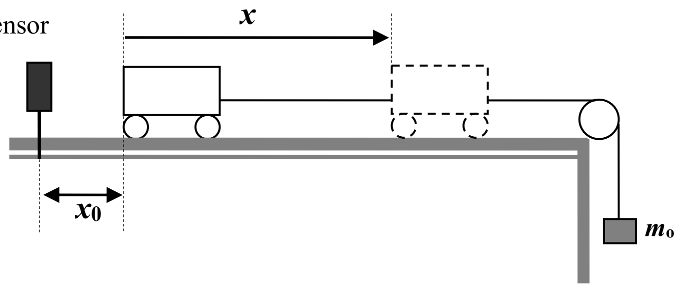
a. What is the period of this pendulum?

b. What must the length of this pendulum be?

2. *FORCE AND ACCELERATION*

OBJECTIVE : To measure the effect of force acting on a mass.

THEORY :



In this experiment, the motion of the car on a special track is studied. Masses are placed on the mass holder that is attached to the car. When the masses are released, they fall to the floor while applying a force on the cars due to the gravitation. Acceleration of the car can be calculated from the Newton's Law:

$$M_{total} a = m_o g$$

$$a = \frac{m_o}{M_{total}} g$$

$$M_{total} = m_c + m_0$$

To measure the acceleration we have to record the motion of the mass+car system as a function of time. As you know, the velocity is the derivative of the position and the acceleration is the derivative of the velocity with respect to the time. So, if we know the position as a function of time, we can determine the velocity and the acceleration. However, it is difficult to record the position on a continuous base experimentally. We can only determine the position at specific times. Even though the velocity and the acceleration may not be constant, we can still determine the average velocity for a specific interval.

$$v_{average} = \Delta x / \Delta t$$

From the plot of the average velocity versus the time we can determine the acceleration by taking the derivative of the function defined by this graph.

APPARATUS : Car and track, masses with hanger, position sensor, data logger, balance

PROCEDURE :

- You will be determining the positions with the help of a position sensor. The sensor works by sending ultrasound pulses forward and listening for the echoes. From the known speed of sound in the air and the time between the transmission and reception of the ultrasound signals, the data logger determines the distance to the sensor.

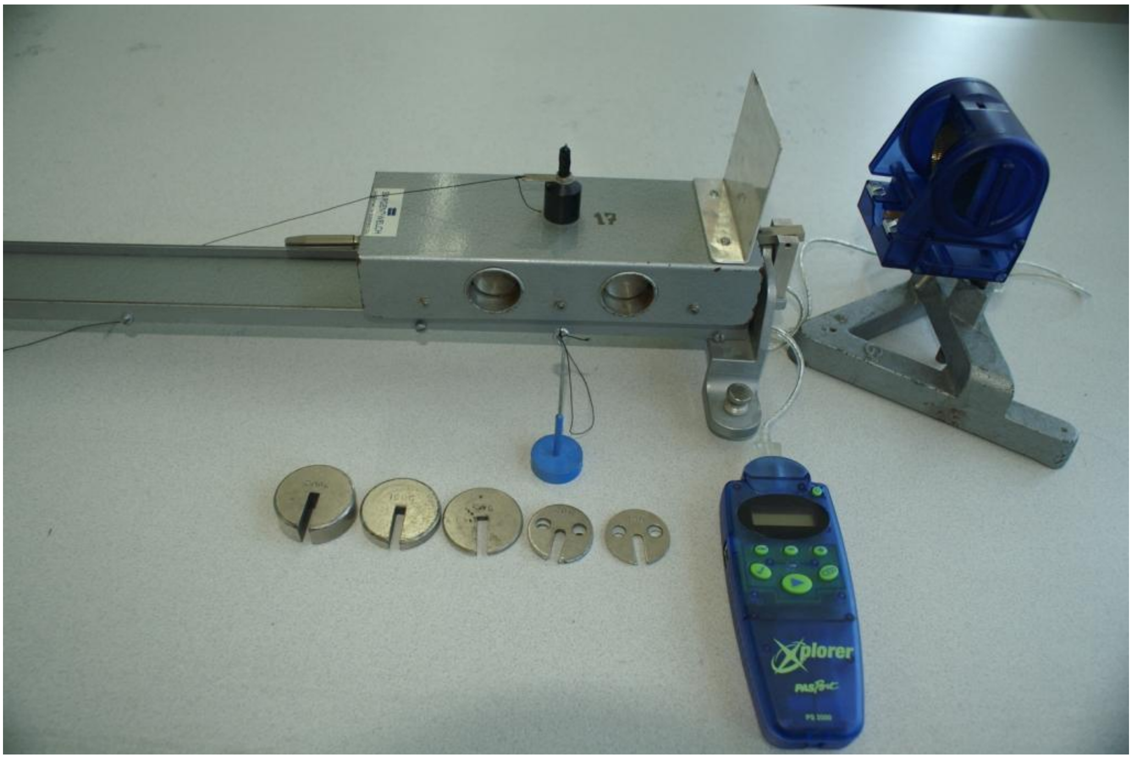
- Set the position sensor approximately 20 cm away from the car before releasing it.

- Adjust the data logger to an appropriate rate (suggested value is 10 per second) and compensate for the friction force.

- Place the given mass on the holder. Start the data logger and release the car. Stop the data logger when the mass holder hits the ground.

- Using the up and down buttons on the data logger, read the position information in its memory. Measure the length of each interval and calculate the average velocity for each interval.

- On a graph paper, plot the average velocity versus time and determine the acceleration.



FORCE AND ACCELERATION

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			**Value & Unit**
Mass on the holder	m_o	=	....
Initial distance of the Car	x_0	=	....
Number of Cylinders in the Car		=	....
Data Taking Rate		=	....

Number of Intervals	t ( )	x ( )	Δx ( )	$V_{ave} = \Delta x / \Delta t$ ( )

CALCULATIONS and RESULT:

From the graph, choose two SLOPE POINTS other than data points,

SP_1 : (;)

SP_2 : (;)

By using SP_1 and SP_2, calculate:

Description / Symbol		Calculations (show each step)	Result
SLOPE	=	. . . .	. . . .
		. . . .	
Acceleration a	=	. . . .	. . . .
Total Mass M_{total}	=	. . . .	. . . .
		. . . .	
Mass of the Car m_c	=	. . . .	. . . .
		. . . .	

QUESTIONS :

1) Should the masses of the washers, which are placed on the hanger to overcome the friction, be added to the total mass? Why?

2) If the velocity versus time graph does not pass through the origin, what is the meaning of this nonzero *y*-intercept value physically?

3. *BALLISTIC PENDULUM - PROJECTILE MOTION*

OBJECTIVE **:** To study the fundamentals of projectile motion.

THEORY **:** When the ball is shot with an initial speed v in the horizontal direction, its range will be

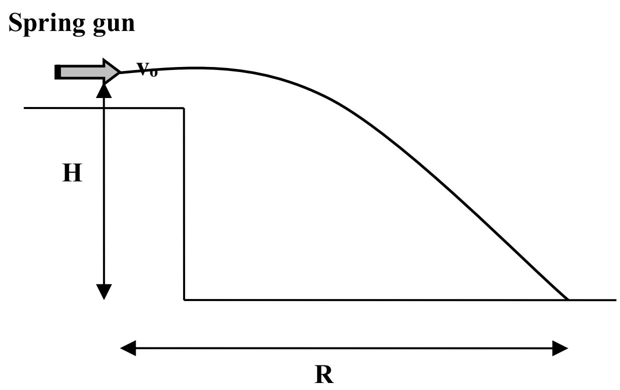
$$R = vt$$

where ***t*** is the time of flight and it will be free falling. The height it falls down will determine the flight time:

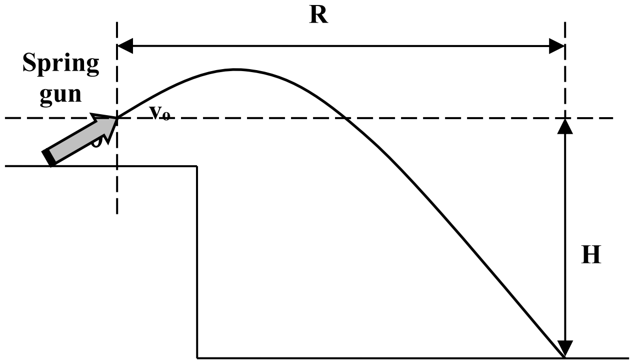
$$H = \frac{1}{2} g t^2$$

By combining these two equations, we can determine the initial speed in terms of the range and the height:

$$v_o = R\sqrt{\frac{g}{2H}}$$



On the other hand, when the ball is shot at an angle θ, it will follow a parabolic trajectory:



It can be shown that the trajectory equation is

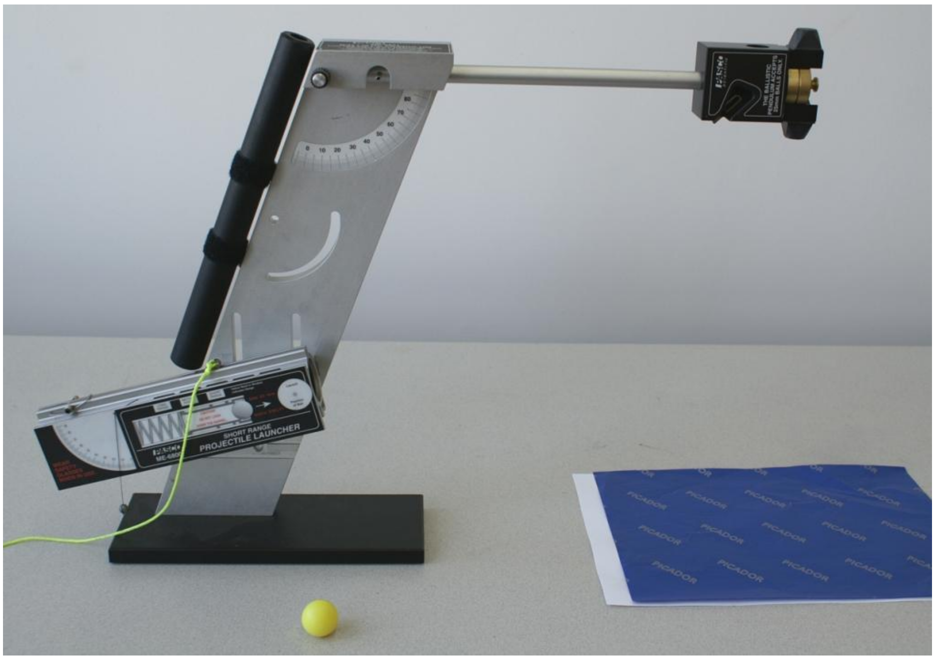
$$\frac{gR^2}{2v_o^2}\tan^2\theta - R\tan\theta + \left(\frac{gR^2}{2v_o^2} - H\right) = 0 \,.$$

APPARATUS : Ballistic pendulum with plastic ball, meter stick, balance, carbon paper.

PROCEDURE :

Part 1: The spring gun is leveled on the table and the plastic ball is projected horizontally. The initial velocity of the ball can be determined by measuring the range, R, and the initial height, H, of the ball.

Part 2: The spring gun is inclined at an angle θ with the horizontal and the ball is shot freely. Range, height and the initial velocity of the ball are used to calculate θ.



BALLISTIC PENDULUM-PROJECTILE MOTION

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

PART 1 – HORIZONTAL MOTION

Description / Symbol			**Value & Unit**
Range of Spring Gun		=	....
Height	H_h	=	....
Range (1st trial)	R_{h_1}	=	....
Range (2nd trial)	R_{h_2}	=	....
Range (3rd trial)	R_{h_3}	=	....
Average Range	R_{h_ave}	=	....

PART 2 – PROJECTILE MOTION

Description and Symbol			**Value & Unit**
Height	H_p	=	. . . .
Range (1st trial)	R_{p_1}	=	. . . .
Range (2nd trial)	R_{p_2}	=	. . . .
Range (3rd trial)	R_{p_3}	=	. . . .
Average Range	$R_{\mathrm{p\_ave}}$	=	. . . .
Measured Angle	θ_{MV}	=	. . . .

CALCULATIONS and RESULT:

Show the analytic solution of the equation - NO NUMERICAL SOLUTION

Equation for θ : $\frac{gR^2}{2v_o^2}\tan^2\theta - R\tan\theta + \left(\frac{gR^2}{2v_o^2} - H\right) = 0$ → $\mathbf{ax^2 + bx + c = 0}$

Solve for tanθ, and express your result in terms of a, b, and c:

Description		Calculations (show each step)	Result & Unit
Initial velocity of the ball v_o	=	. . . .	. . . .
		. . . .	
Coefficient a	=	. . . .	. . . .
Coefficient b	=	. . . .	. . . .
Coefficient c	=	. . . .	. . . .
$\tan\theta_{EV1}$	=	. . . .	. . . .
θ_{EV1}	=	. . . .	. . . .
$\tan\theta_{EV2}$	=	. . . .	. . . .
θ_{EV2}	=	. . . .	. . . .

% Difference in θ values: . . . .

QUESTIONS :

1) What would the uncertainty Δv in the velocity be if the uncertainty in the height and range values were about a millimeter?

2) What are the possible sources of error in this experiment?

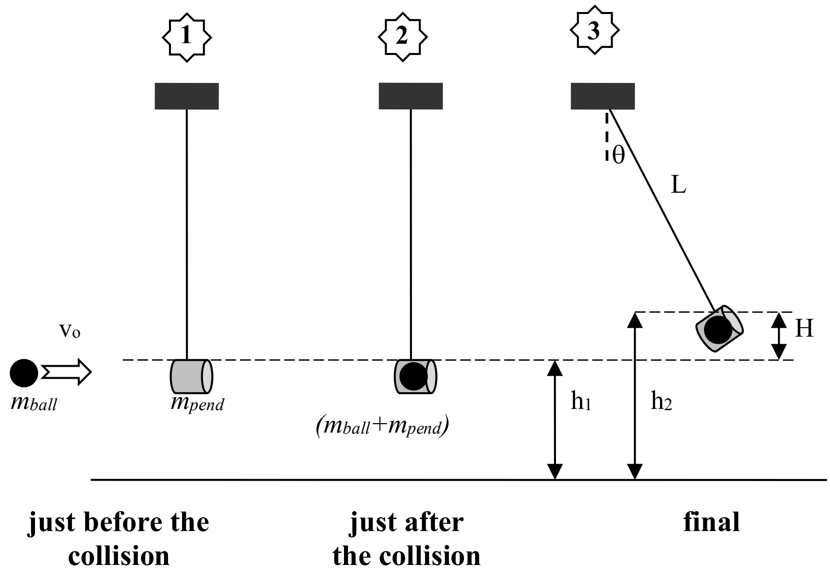
3) In this experiment we are ignoring the effect of air friction. Assuming that the experiment is done in a very viscous liquid, discuss the effect of the friction due to the liquid on the motion of the ball.

4) Assume that the ballistic pendulum is moving upward with a speed of v_b in the first part. Derive the equations for the range and the final velocity with which the ball strikes the floor.

4. *BALLISTIC PENDULUM - CONSERVATION OF MOMENTUM*

OBJECTIVE **:** To study the principle of conservation of momentum, and by applying this principle to measure the initial velocity of a ball.

THEORY **:**



In this experiment we will study the conservation of momentum using the ballistic pendulum. When the steel ball is shot towards the pendulum attachment of the apparatus, it will hit and stay inside the pendulum attachment. This is an example of a completely inelastic collision. We can express the conservation of momentum during the collision as:

$$m_{ball} v_o = (m_{ball} + m_{pend}) v_{final} .$$

Since the pendulum attachment is free to swing up, it will do so until all its kinetic energy turns into the potential energy:

$$\frac{1}{2}(m_{ball} + m_{pend}) v^2{}_{final} = (m_{ball} + m_{pend}) gH$$

The pendulum attachment pushes a pointer as it swings up until it reaches the maximum. Using this maximum angle information and the length of the pendulum attachment, we can determine *H*:

$$H = L(1-\cos\theta)\,.$$

Then using this value and working backwards from the equations above, we can determine the initial velocity of the ball:

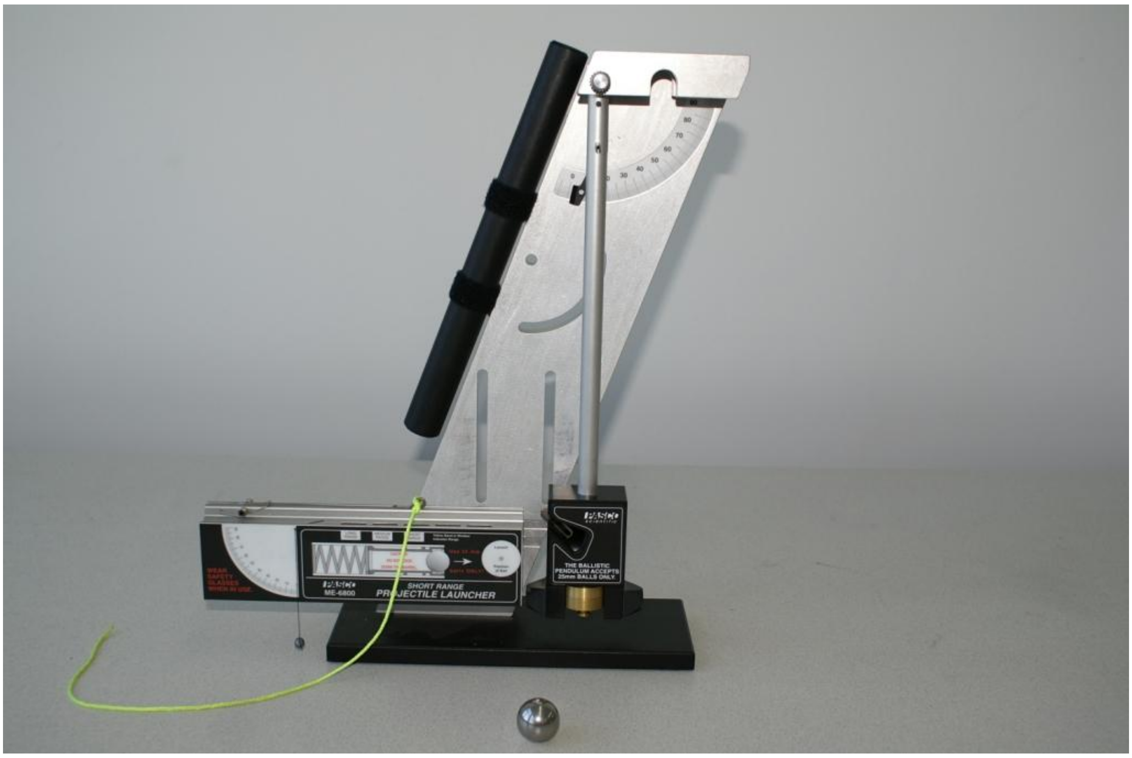
$$v_0 = \frac{m_{ball} + m_{pend}}{m_{ball}}\sqrt{2gH} = \frac{m_{ball} + m_{pend}}{m_{ball}}\sqrt{2gL(1-\cos\theta)}\,.$$

APPARATUS **:** Ballistic pendulum with the pendulum attachment, meter stick, balance, steel ball

PROCEDURE **:**

- By equating the momentum before the collision to that after the collision, and equating the kinetic energy of the system just after the collision to the increase in potential energy at the height h_2, the initial velocity of the ball can be calculated.

- Fire the ball into the pendulum three times for each compression level of the spring gun and determine the mean increase in height *H*. Do not forget to reset the angle pointer just before shooting the ball to the pendulum attachment. Calculate the initial velocity of the ball v_o for corresponding compression level.



BALLISTIC PENDULUM – CONSERVATION OF MOMENTUM

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description/Symbol			**Value & Unit**
Mass of the ball	m_{ball}	=	....
Mass of the pendulum	m_{pend}	=	....
Length of the pendulum	L	=	....
Acceleration due to gravity	g	=	....

Level of Compression	# (i)	θ	$H = L(1\text{-}Cos\,\theta)$ ( )
		# of Significant Figure:	***# of Significant Figure:***
Short Range Compression	1		
	2		
	3		
Average of $H^{SR} = \frac{1}{3}\sum_{i=1}^{3} H_i^{SR} =$			
Medium Range Compression	1		
	2		
	3		
Average of $H^{MR} = \frac{1}{3}\sum_{i=1}^{3} H_i^{MR} =$			
Long Range Compression	1		
	2		
	3		
Average of $H^{LR} = \frac{1}{3}\sum_{i=1}^{3} H_i^{LR} =$			

CALCULATIONS:

Description / Symbol		Calculations (show each step)	Result	Dimension
Velocity of the ball for SR $v_{\text{SR-ave}}$	=	....	....	....
		....		
Velocity of the ball for MR $v_{\text{MR-ave}}$	=	....	....	....
		....		
Velocity of the ball for LR $v_{\text{LR-ave}}$	=	....	....	....
		....		

QUESTIONS :

1) Calculate the energy loss during the calculation. Where does this energy go?

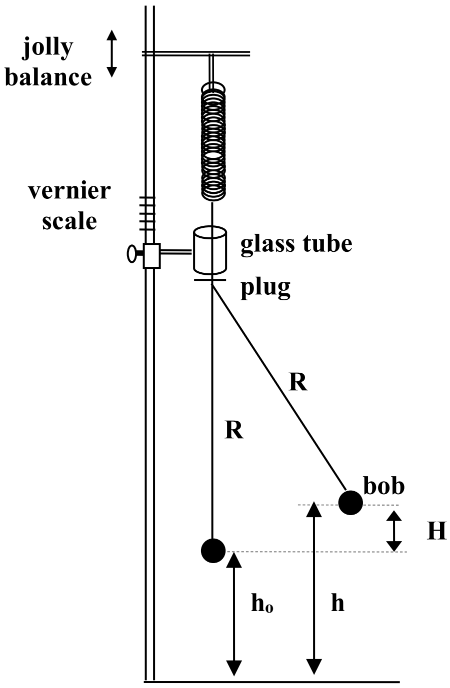
2) What are the possible sources of error in this experiment?

3) At what height is half the kinetic energy converted into potential energy? Give your answer with respect to the initial height.

5. *CENTRIPETAL FORCE*

OBJECTIVE **:** To study the motion of a body moving in a circle and verify the centripetal force equation.

THEORY **:**



A mass swinging at the end of a string does a circular motion. The mass undergoing a uniform circular motion has the acceleration given by

$$a_{cent} = \frac{v^2}{R} \qquad (1)$$

where *R* is the length of the string. Since the velocity of the mass in a simple pendulum is not constant, the acceleration will be changing.

We will start the motion of the mass by releasing it from a height that we measure. When the ball passes through its lowest position, all the potential energy difference between the initial height and the lowest position will be converted into kinetic energy.

$$mgH = \frac{mv^2}{2} \qquad (2)$$

Using the speed calculated from this expression, we can determine the acceleration at this position. The centripetal acceleration is usually caused by the tension in the string if the pendulum is just a string hanging from the ceiling.

$$F_{cent} = ma_{cent} \qquad (3)$$

But we will hang the string from a spring to be able to measure the centripetal force. In this case the restoring force in the spring will be the centripetal force.

$$F_{rest} = kx = F_{cent} = \frac{mv^2}{R} \qquad (4)$$

By combining Equations (2), (4), and the initial extension of the spring due to the mass of the bob:

$$mg = kD \qquad (5)$$

we can get

$$\frac{mg}{D}x = \frac{2mgH}{R} \qquad (6)$$

and

$$H = \frac{Rx}{2D}. \qquad (7)$$

This is a straight line with a slope of $R/2D$. Hence, recording the height from which we release the bob and the corresponding extension of the spring, we can determine the slope by plotting the data. Then, we can calculate the length of the pendulum R and compare it with the measured value.

APPARATUS **:** Centripetal force apparatus, meter stick.

PROCEDURE **:**

- Place the bob on the table.
- To read r_1, adjust the jolly balance until the glass tube barely touches the shoulder of the plug.
- Let the bob hang freely, pulling the spring down and adjust the jolly balance again until the glass tube barely touches the shoulder of the plug. Read r_2.
- For the first measurement, extend the spring by a distance of 1.90 cm.
- Find the height ***h***, so that it will pull the plug out of the tube by a distance of Δx cm when the bob swings through its equilibrium position. Since the elongation due to the centripetal force is $\Delta x \sim 0.10$ cm, total spring extension, ***x*** will be 2.00 cm.

- Increase the spring extension at certain increments (e.g. 2.00 cm), measure the corresponding ***h*** as a function of the spring extension.

- Plot your data and determine the slope of the straight line that fits the data best.

CENTRIPETAL FORCE

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			**Value & Unit**
Length of the pendulum	R_{TV}	=	. . . .
Height from the floor to the center of the bob	h_o	=	. . . .
Reading in vernier scale without the bob	r_1	=	. . . .
Reading in vernier scale with the bob	r_2	=	. . . .
Extension in the spring = $r_2 – r_1$ = due to bob	D	=	. . . .

increment in vernier	**Reading in Vernier** r ( )	**Additional Extension in the spring** x ( )	**Height of the bob from floor** h ( )	$H = h - h_o$ ( )

CALCULATIONS and RESULTS:

From the graph, choose two SLOPE POINTS other than data points,

SP$_1$: (;)

SP$_2$: (;)

Calculate:

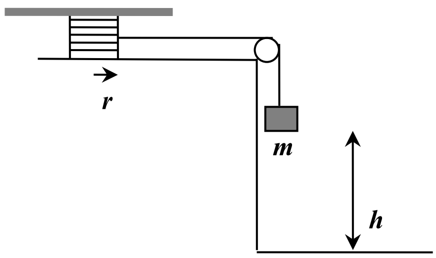
Symbol		Calculation (show each step)	Result	Dimension
Slope	=	. . . .	. . . .	. . . .
		. . . .		
$\boldsymbol{R_{EV}}$ **/ 2*D***	=	. . . .	. . . .	. . . .
$\boldsymbol{R}_{\mathrm{EV}}$	=	. . . .	. . . .	. . . .
		. . . .		

% Error for the Length of the Pendulum, *R*: . . . .

6. *ROTATIONAL INERTIA*

OBJECTIVE **:** To determine experimentally the rotational inertia of a body.

THEORY **:**



A mass connected to a rotating drum is free to descend down to the floor. For this mass the loss in potential energy is equal to the gain in the translational and rotational kinetic energy:

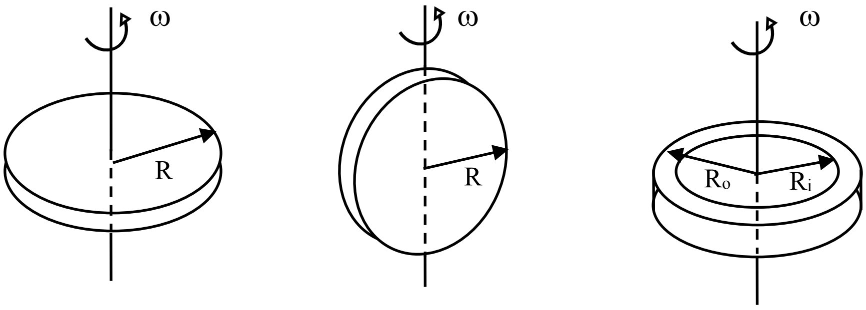
$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

The velocity of mass where it touches the floor and the corresponding angular velocity are:

$$v = \frac{2h}{t} \qquad \text{and} \qquad \omega = v / r$$

As a result, rotational inertia of the drum is given as:

$$I = mr^2\left[\frac{gt^2}{2h} - 1\right]$$



Disk about its CM   Disk about its diameter   Ring

As a special case, the rotational inertia of a uniform disk about an axis passing through its center of mass (CM) and perpendicular to the disk is given by

$$I_z = I_{CM} = \frac{1}{2}MR^2 ,$$

or about its diameter:

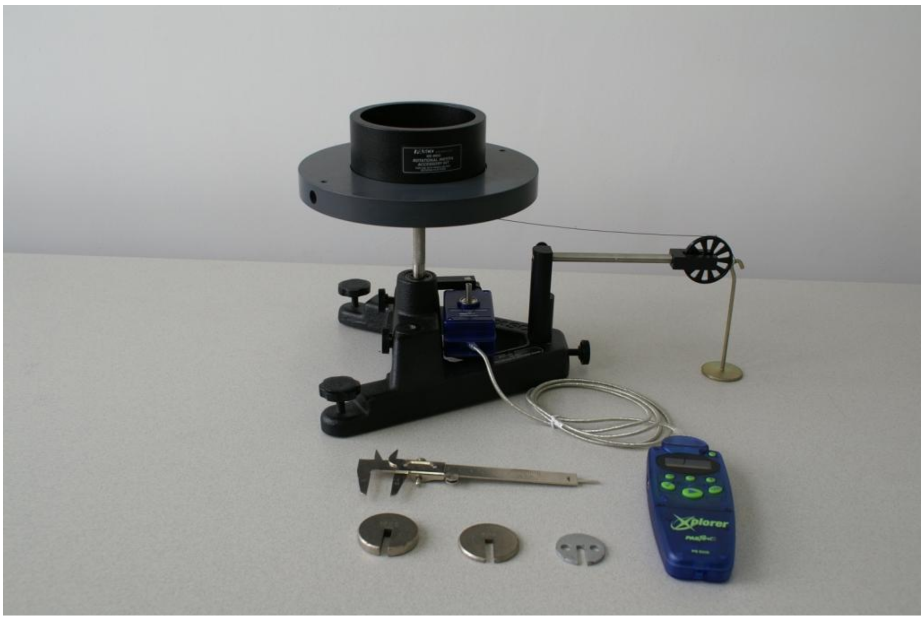
$$I_r = I_{diameter} = \frac{1}{4}MR^2 .$$

Moment of inertia of a ring about an axis passing through its center perpendicularly:

$$I_{ring} = \frac{1}{2}M\left(R_i^2 + R_o^2\right)$$

APPARATUS : Rotational inertia apparatus, disk and ring masses, mass and hanger set, string.

PROCEDURE :



- Wind the cord onto the drum and hang a mass *m* at the end of the cord. Place the disk mass on the drum. After determining the height of the mass above the floor, release the mass and determine the time for descent. Repeat twice and find the average *t*. Calculate the rotational inertia of the disk + drum assembly.

- Repeat the same procedure with the disk mounted on its side. The difference of the moment of inertias should be equal to the moment of inertia of the disk mounted on its side or half the moment of inertia when it is mounted horizontally.

- Calculate the moment of inertia of the disk and the drum part separately for both cases.

- Repeat the measurements for the ring to determine the moment of inertia of the ring and the drum assembly. Calculate the moment of inertia of the ring using the value for the drum part from the previous step.

- Calculate the moment of inertias theoretically from the geometry of the disk and the ring masses and compare your results to the values you determined in the previous step.

ROTATIONAL INERTIA

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol		**Value & Unit**
Diameter of the drum	d =	....
Radius of the drum	r =	....
Height of mass holder from the floor	h =	....
Mass on the mass holder	m^* =	....

Part 1: Rotational Inertia of Disk

		ABOUT CM		**ABOUT DIAMETER**
Time for descent	t_1 =	....	$t`_1$ =	....
Time for descent	t_2 =	....	$t`_2$ =	....
Average time for descent	t_{ave} =	....	$t`_{ave}$ =	....

Part 2: Rotational Inertia of the Ring

Time for
descent -1 $t``_1$ = ...

Time for
descent -2 $t``_2$ = ...

Average time
for descent $t``_{ave}$ = ...

Description / Symbol	**Value & Unit**
Mass of the Disk M_{disk} =	. . . .
Diameter of the Disk D_{disk} =	. . . .
Radius of the Disk R_{disk} =	. . . .
Mass of the Ring M_{ring} =	. . . .
Inner Diameter of the Ring $D_{i\text{-}ring}$ =	. . . .
Outer Diameter of the Ring $D_{o\text{-}ring}$ =	. . . .
Inner Radius of the Ring $R_{i\text{-}ring}$ =	. . . .
Outer Radius of the Ring $R_{o\text{-}ring}$ =	. . . .

CALCULATIONS and RESULTS:

Description / Symbol	Calculations (show each step)	Result
I_1 - Rotational Inertia of the drum + Disk about its CM $I^{CM}_{drum+disk}$ =	. . . .	. . . .
	. . . .	
I_2 - Rotational Inertia of the drum + Disk about its DIAMETER $I^{DIAMETER}_{drum+disk}$ =	. . . .	. . . .
	. . . .	

$2 I_2 > I_1$, If not, measure time for descent again

I_3 - Rotational Inertia of the drum + the Ring + disk $I_{drum+ring+disk}$ =	. . . .	. . . .
	. . . .	

SHOW YOUR CALCULATIONS CLEARLY (You may use back of the page)

Rotational Inertia of the DISK about its CM I^{CM}_{DISK} =	. . . .	. . . .
Rotational Inertia of the DISK about its DIAMETER $I^{diameter}_{DISK}$ =	. . . .	. . . .
Rotational Inertia of the RING I_{RING} =	. . . .	. . . .
Rotational Inertia of the DRUM I_{DRUM} =	. . . .	. . . .

Theoretical Values for *I*:

I_{DISK}^{CM} = ...

$I_{DISK}^{diameter}$ = ...

I_{RING} = ...

% Error for Rotational Inertia:

ΔI_{DISK}^{CM} :

$\Delta I_{DISK}^{diameter}$:

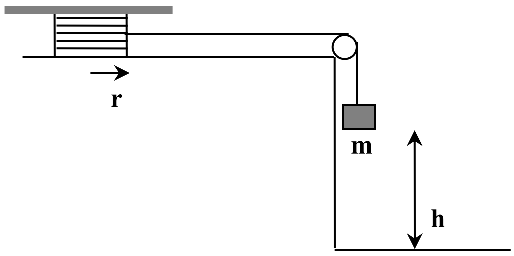
ΔI_{RING} :

Show the Dimensional Analysis of Rotational Inertia, *I*:

7. TORQUE AND ANGULAR ACCELERATION

OBJECTIVE : To measure the effect of torque acting on a rotating mass.

THEORY :



A rotating object also obeys the Newton's Laws of motion. When we apply a torque on an object, we can express the Newton's Law in terms of the torque and the angular acceleration:

$$\tau = I\alpha$$

where torque is applied through a string wrapped around the drum with a radius r attached to a free falling object of mass *m*. T is tension in the string:

$$\tau = \vec{F} x \vec{r} = mgr = T\,r$$

Then the torque and angular acceleration equation becomes

$$I\alpha = T\,r$$

Using the force equation

$$T - mg = -ma \quad (a=\alpha r)$$

we can determine the moment of inertia by measuring the angular acceleration.

$$I = \frac{mgr}{\alpha} - mr^2$$

We can also determine the moment of inertia from the free fall time:

$$I = mr^2 \left[\frac{gT^2}{2h} - 1 \right]$$

APPARATUS : Rotational inertia apparatus with rotational sensor, data logger, mass and hanger set.

PROCEDURE :

- Using a small mass (50 g) on the mass holder, observe the rotational motion of the disk on the rotational inertia apparatus. You should set the data logger to **2 samples/s**. When the free fall is completed, retrieve the rotation angles as a function of time from the data logger.

- Calculate the average angular velocity for successive time intervals and plot the result as a function time. ($\omega_{\text{average}} = \Delta\theta / \Delta t$)

- From your graph, obtain the angular acceleration of the disk assembly by determining the slope of the straight line fit to your data.

- Determine the moment of inertia of the disk assembly using the angular acceleration.

- Determine the free fall time and the height of the mass holder from the floor and calculate the moment of inertia using the equation given above.

- Compare both results for the moment of inertia and calculate the percentage difference between them:

$$\%\text{diff} = \frac{|I_1 - I_2|}{I_1 + I_2} \times 100$$

TORQUE AND ANGULAR ACCELERATION

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			**Value & Unit**
Diameter of the drum	d	=	. . . .
Radius of the drum	r	=	. . . .
Mass on the mass holder	m	=	. . . .
Height of the mass holder from the floor	h	=	. . . .
Time for descent	T	=	. . . .

Number of Intervals	Δt ( )	t ( )	Θ ( )	$\Delta\Theta$ ( )	$\omega_{average} = \Delta\Theta / \Delta t$ ( )

CALCULATIONS and RESULT:

From the graph, choose two SLOPE POINTS other than data points,

SP_1 : (;)

SP_2 : (;)

By using SP_1 and SP_2, calculate:

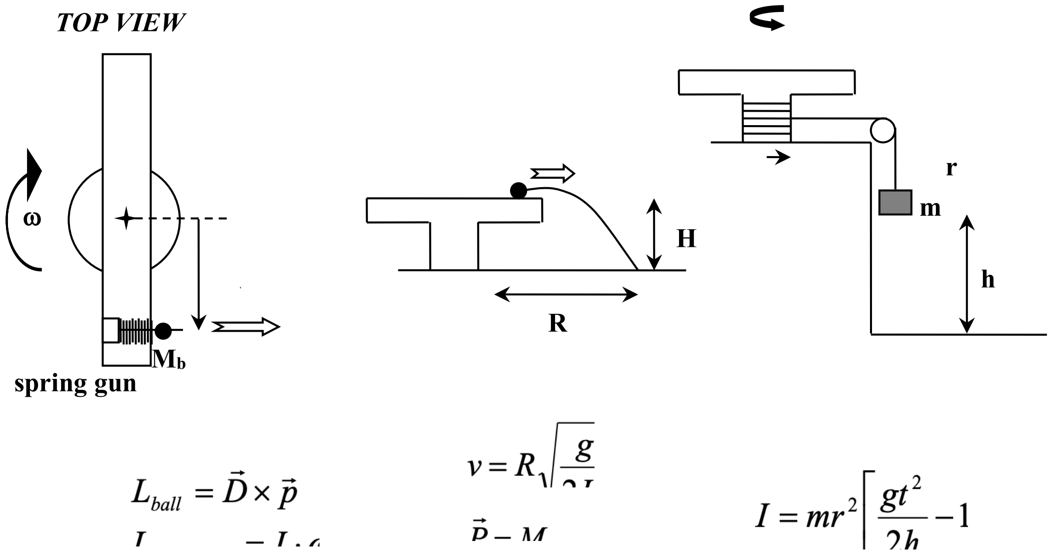
Description / Symbol		**Calculations (show each step)**	**Result**	**Dimension**
SLOPE	=	....	....	....
		....		
Angular Acceleration α	=	....	....	....
Moment of Inertia $I = \frac{mgr}{\alpha} - mr^2$	=	....	....	....
$I = mr^2\left[\frac{gT^2}{2h} - 1\right]$	=	....	....	....
		....		

%difference for *I*:

8. *CONSERVATION OF ANGULAR MOMENTUM*

OBJECTIVE : To study the conservation of angular momentum of a system about a fixed axis.

THEORY :



In a system shown on the left of the figure above we can study the conservation of angular momentum. When the spring gun is released and shoots the ball, the ball has also an angular momentum defined by its linear momentum since the spring gun is fixed on the turntable. The turntable is free to rotate around its axis. Since this is like an inverse collision, the momentum and the angular momentum are conserved:

$$L_{ball} = L_{turn-table}$$

or

$$M_b vD = I\omega$$

Determining the moment of inertia of the spring gun assembly will be done similar to the previous experiment, **Rotational Inertia**. The important points are summarized next to the figure above.

PROCEDURE :

- Fire the ball at 5 different positions (every 4 cm) on the aluminum rotating platform by releasing the compressed spring. The initial velocity of the ball can be determined by measuring the range and the initial height of the ball.

- Read the angular velocity of the turntable from the data logger.

- For the rotational inertia of the turntable when the spring gun is placed at the center, wind the cord onto the drum and hang a mass m at the end of the cord. After determining the height of the mass above the floor, release the mass and determine the time for descent. Calculate the Rotational Inertia of the assembly when the spring gun is at the center ($I_{SPRINGGUN}^{CM}$).

- Calculate *the rotational Inertia, I* at 5 different positions (*D)* on the aluminum rotating platform by using parallel axes theorem ($I_D = I_{SPRINGGUN}^{CM} + M_{gun} D^2$)

- Calculate M_b for different D values.

CONSERVATION OF ANGULAR MOMENTUM

Name & Surname : **Experiment #:**

Section : **Date :**

DATA:

Description / Symbol			Value & Unit
Mass of the ball	M_b	=	....
Mass of the spring gun	M_{gun}	=	....
Initial height of the ball	H	=	....
Diameter of the drum	d	=	....
Radius of the drum	r	=	....
Mass on the mass holder	m	=	....
Height of the mass holder from the floor	h	=	....
Time for descent	t	=	....

CALCULATIONS :

D ( )	***R*** ( )	***Velocity of the ball*** ***v*** ( )

$I_{SPRINGGUN}^{CM}$ = ...

D ( )	$\boldsymbol{\omega}$ ( )	$\boldsymbol{I_D}$ ( )	$\boldsymbol{M_b}$ ( )
	$\sum_{i=1}^{5} M_b^i =$		

RESULTS:

Average mass of the ball M_b = ...

% Error for M_b = ...

APPENDICES

A. Physical Constants:

Planck's constant	h	6.626×10^{-34} Jsec or 4.136×10^{-21} Mev sec
	$\hbar$	1.05×10^{-34} J.sec or 6.58×10^{-22} Mev.sec
Universal Gas Constant	R	8.314 J/°K mole
Avagadro's Number	N_A	6.022×10^{23}
Boltzman Constant	k	1.381×10^{-23} J/°K or 8.617×10^{-5} ev/°K
Electron charge	e	1.602×10^{-19} C
Speed of light in vacuum	c	2.998×10^{8} m/sec
Stefan-Boltzman Constant	σ	5.67×10^{-8} W/m^2.°K^4
Gravitational Constant	G	6.672×10^{-11} N.m^2/kg^2
Gravitational acceleration	g	9.81 m/sec^2
Permeability of Vacuum	μ_o	1.257×10^{-6} H/m or
Permitivity of Vacuum	ε_o	8.854×10^{-12} C^2/J.m
Rydberg Constant	R_∞	1.097×10^{7} m^{-1}
Fine structure constant	$\alpha = e^2/(2\varepsilon_o hc)$	7.297×10^{-3}
First Bohr radius	α_o	5.29×10^{-11} m
Charge to mass ratio of the electron	e/m	1.759×10^{11} C/kg
Bohr Magneton	μ_B	9.27×10^{-24} A.m^2
Atomic mass unit (amu)	u	1.66×10^{-27} kg or 931.5 Mev
Electron rest mass	m_e	9.11×10^{-31} kg or 511 kev
Proton rest mass	M_p	1.672×10^{-27} kg or 938.2 Mev
Neutron rest mass	M_n	1.675×10^{-27} kg or 939.6 Mev
Compton wavelength of electron	λ_C	$2.43 \times 10_{-12}$ m
$\hbar c$		197 Mev. Fermi
Standard volume of ideal gas		2.24×10^{-2} m^3/mole
1 eV		1.602×10^{-19} J
1 amu		931.14 Mev
1 g		5.610×10^{26} Mev
1 electron mass		0.51098 Mev
Ice point	T_o	273.16 °K

B. Conversion Tables:

LENGTH

	cm	meter	km	A^0	inch	foot	mile
cm	1	10^{-2}	10^{-5}	10^{8}	0.3937	$3.281x10^{-2}$	$6.214x10^{-6}$
meter	100	1	10^{-3}	10^{10}	39.37	3.281	$6.214x10^{-4}$
km	10^{5}	1000	1	10^{13}	$3.937x10^{4}$	3281	0.6214
A^0	10^{8}	10^{10}	10^{13}	1	$3.937x10^{-9}$	$3.281x10^{-10}$	$4.214x10^{-14}$
inch	28.540	0.0254	$2.540x10^{-5}$	$2.540x10^{8}$	1	0.0833	$1.578x10^{-5}$
foot	30.48	0.3048	$3.048x10^{-4}$	$3.048x10^{9}$	12	1	$1.894x10^{-4}$
mile	$1.609x10^{5}$	1609	1.609	$1.609x10^{13}$	$6.336x10^{4}$	5280	1

AREA

	m^2	cm^2	ft^2	$in.^2$	circ mile
m^2	1	10^{4}	10.76	1550	$1.974x10^{9}$
cm^2	10^{-4}	1	$1.076x10^{-3}$	0.1550	$1.974x10^{5}$
ft^2	$9.290x10^{-2}$	929.0	1	144	$1.833x10^{8}$
$in.^2$	$6.452x10^{-4}$	6.452	$6.944x10^{-3}$	1	$1.273x10^{6}$
circular mill	$5.067x10^{-10}$	$5.065x10^{-6}$	$5.454x10^{-9}$	$7.854x10^{-7}$	1

VOLUME

	m^3	cm^3	liter	ft^3	$in.^3$
m^3	1	10^{6}	1000	35.31	$6.102x10^{4}$
cm^3	10^{-6}	1	$1.000x10^{-3}$	$3.531x10^{-5}$	$6.102x10^{-2}$
liter	$1.000x10^{-3}$	1000	1	$3.531x10^{-2}$	61.02
ft^3	$2.832x10^{-2}$	$2.832x10^{4}$	28.32	1	1728
$in.^3$	$1.639x10^{-5}$	16.39	$1.639x10^{-2}$	$5.787x10^{-4}$	1

MASS

	kg	gram	ounce	pound	amu	m slug	ton
kg	1	10^{3}	35.27	2.205	$6.024x10^{26}$	$1.021x10^{-1}$	10^{-3}
gram	10^{-3}	1	$3.527x10^{-2}$	$2.205x10^{-3}$	$6.024x10^{23}$	$1.021x10^{-4}$	10^{-6}
ounce	$2.835x10^{-2}$	28.35	1	$6.250x10^{-2}$	$1.708x10^{25}$	$2.895x10^{-3}$	$2.835x10^{-5}$
pound	$4.536x10^{-1}$	$4.536x10^{2}$	16	1	$2.372x10^{25}$	$4.630x10^{-2}$	$4.536x10^{-4}$
amu	$1.66x10^{-27}$	$1.66x10^{-24}$	$5.854x10^{-26}$	$3.66x10^{-27}$	1	$1.695x10^{-28}$	$1.660x10^{-30}$
m slug	9.806	$9.806x10^{3}$	$3.454x10^{2}$	21.62	$5.9x10^{27}$	1	$9.806x10^{-3}$
ton	10^{3}	10^{6}	$3.527x10^{4}$	$2.205x10^{-3}$	$6.024x10^{29}$	$1.021x10^{2}$	1

TIME

	second	minute	hour	year
second	1	1.667×10^{-2}	2.778×10^{-4}	3.165×10^{-8}
minute	60	1	1.667×10^{-2}	1.901×10^{-6}
hour	3600	60	1	1.140×10^{-4}
year	3.156×10^{7}	5.259×10^{5}	8.765×10^{3}	1

FORCE

	Nt	Dyne	Kg F
Nt	1	10^5	0.1020
Dyne	10^{-5}	1	$1.020x10^{-6}$
Kg F	9.807	$9.807x10^5$	1

PRESSURE

	pa	mm Hg	mbar	kgf/m^2	dyne/cm^2	atmosphere
Pascal	1	$7.501x10^{-3}$	10^{-2}	0.1020	10	$9.869x10^{-6}$
torr	$1.333x10^2$	1	1.333	13.6	$1.333x10^3$	$1.316x10^{-3}$
mbar	10^2	0.7501	1	10.20	10^3	$9.869x10^{-4}$
dyne/cm^2	0.1	$7.501x10^{-4}$	10^{-3}	$10.20x10^{-3}$	1	$9.869x10^{-7}$
kgf/m^2	9.807	$9.807x10^{-2}$	$9.807x10^{-2}$	1	98.07	$9.679x10^{-5}$
atm	$1.013x10^5$	$7.601x10^2$	$1.013x10^{-3}$	$1.033x10^4$	$1.013x10^6$	1

ENERGY

	Joule	kilowatt-hour	Btu	erg	Calorie	electron volt
Joule	1	$2.778x10^{-7}$	$9.480x10^{-4}$	10^7	0.2389	$6.242x10^{18}$
kilowatt-hour	$3.6x10^6$	1	$3.412x10^3$	$3.6x10^{13}$	$8.6x10^5$	$2.247x10^{25}$
Btu	$1.055x10^3$	$2.930x10^{-4}$	1	$1.055x10^{10}$	$2.468x10^2$	$6.585x10^{21}$
erg	10^{-7}	$2.778x10^{-14}$	$9.480x10^{-11}$	1	$2.389x10^{-8}$	$6.242x10^{11}$
calorie	4.187	$1.163x10^{-6}$	$4.053x10^{-3}$	$4.187x10^7$	1	$2.613x10^{19}$
electron volt	$1.602x10^{-19}$	$4.450x10^{-26}$	$1.519x10^{-22}$	$1.602x10^{-12}$	$3.827x10^{-20}$	1

POWER

	watt	erg/sec	calorie/sec	kgfm/sec	Btu/sec	HP
watt	1	10^7	0.2388	0.1020	3.413	$1.360x10^{-3}$
erg/sec	10^{-7}	1	$2.388x10^{-8}$	$1.020x10^{-8}$	$3.413\ x10^{-7}$	$1.360x10^{-10}$
calorie/sec	4.187	$4.187x10^7$	1	0.4268	14.29	$5.694x10^{-3}$
kgfm/sec	9.807	$9.807x10^7$	2.343	1	33.47	133.3
Btu/sec	0.2931	$2.931x10^6$	$6.999x10^{-2}$	$2.987x10^{-2}$	1	$3.982x10^{-4}$
HP	735.5	$7.355x10^9$	175.7	75	$2.511x10^3$	1

MAGNETIC FIELD

	gauss	TESLA	milligauss
gauss	1	10^{-4}	1000
TESLA	10^4	1	10^7
milligauss	0.001	10^{-7}	1

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