

## THE RATIO OF HEAT CAPACITIES <br> OF AIR, $\gamma=\mathrm{C}_{\mathrm{p}} / \mathrm{C}_{\mathrm{v}}$

PHYL102

## THEORY

## THE RATIO OF HEAT CAPACITIES OF AIR, $\gamma=\mathbf{C}_{\mathbf{p}} / \mathbf{C}_{v}$

## HEAT CAPACITY

The heat capacity of a gas is the amount of energy needed to increase its temperature by $1^{\circ} \mathrm{C}$.

$$
\mathrm{Q}=\mathrm{C} \Delta \mathrm{~T}
$$

- $\mathrm{C}_{\mathrm{p}}$ is the heat capacity at constant pressure (isobaric process).
- $\mathrm{C}_{\mathrm{V}}$ is the heat capacity at constant volume (isovolumetric process).
- The ratio of these heat capacities is represented by the Greek letter $\gamma$ (gamma) and defined as

$$
\gamma=\mathbf{C}_{\mathbf{P}} / \mathbf{C}_{\mathbf{V}}
$$

The ratio of heat capacities of air is a thermodynamic quantity but it can be determined by observing a totally mechanical process: oscillation of a steel ball.


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## OSCILLATION OF THE BALL



If a ball with mass $M$ is in equilibrium inside a tube with cross-sectional area A, the air pressure inside the tube is equal to atmospheric pressure + pressure exerted by the ball.

$$
P_{e}=P_{0}+\frac{M g}{A}
$$

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$$
\mathrm{P}_{\mathrm{e}}=\mathrm{P}_{0}+\frac{\mathrm{Mg}}{\mathrm{~A}}
$$

- Atmospheric Pressure $\mathrm{P}_{0}$

$$
\mathrm{P}_{0}=\rho \mathrm{gh}
$$

$\star P_{0}$ is measured with barometer
( $\rho$ : density of Mercury)
( h : height of Mercury in the barometer)

- Cross-Sectional Area A

$$
\mathbf{A}=\pi \mathbf{R}^{2}
$$

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A small deviation $d x$ of the ball from the equilibrium position makes the air inside the tube expand and compress.
The expansion/compression causes a change $\Delta P$ in the pressure.

The pressure change applies a net force on the ball and makes it accelerate.
$\mathrm{A} \Delta \mathrm{P}=\mathrm{M} \frac{\mathrm{d}^{2} \mathrm{x}}{\mathrm{dt}^{2}}$

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The expansion/compression of the air is an adiabatic process since there is no heat exchange.

In an adiabatic process, the pressure and volume obeys the following relation

## $\mathrm{P} \mathrm{V}^{\gamma}=$ constant

where

$$
\gamma=\mathbf{C}_{\mathbf{P}} / \mathrm{C}_{\mathbf{V}}
$$


$\rightarrow$ V
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Taking the variation of this expression,

$$
\triangle \mathbf{P} \mathbf{V}^{\gamma}+\mathbf{P} \gamma \mathbf{V}^{\gamma-1} \triangle \mathbf{V}=0
$$

we can calculate the pressure change $\Delta P$,

$$
\Delta \mathbf{P}=-\frac{\mathbf{P} \gamma \Delta \mathbf{V}}{\mathbf{V}}
$$

and substitute it into the acceleration equation:

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Now, the equation of motion of the ball is

$$
\frac{\mathrm{d}^{2} \mathrm{x}}{\mathrm{dt}^{2}}+\frac{\gamma \mathrm{PA} \triangle \mathrm{~V}}{\mathrm{MV}}=0
$$


$\Delta V$ is the volume traced by the ball when it moves up and down inside the tube.

$$
\Delta \mathrm{V}=\mathrm{Ax}
$$

Finally, the acceleration equation of the ball becomes

$$
\frac{\mathrm{d}^{2} \mathrm{x}}{\mathrm{dt}^{2}}+\frac{\gamma \mathbf{P A}^{2}}{\mathbf{M V}} \mathbf{x}=0
$$

Since the equation of simple harmonic motion is the period of the oscillation is given by

$$
\frac{d^{2} x}{d t^{2}}+\omega^{2} x=0
$$

$$
T=\frac{2 \pi}{\omega}
$$

$$
\mathrm{T}=2 \pi \sqrt{\frac{\mathrm{MV}}{\gamma \mathrm{PA}^{2}}}
$$

$$
\text { where } P=P_{e}
$$

## APPARATUS

THE RATIO OF HEAT CAPACITIES OF AIR, $\gamma=\mathbf{C}_{\mathrm{p}} / \mathbf{C}_{\mathrm{v}}$
Apparatus: Glass Flask (Vessel), Glass Tube, Steel Ball, Stopwatch, Vernier Calipers


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## Glass flask in details



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The glass tube is attached to the top of the glass vessel thanks to a cork which prevents the air inside the vessel from leaking when the ball released through the tube.

* The steel ball fits to the diameter of the tube but can move up and down easily.
* Cross-sectional areas of ball and tube are equal.



## EXPERIMENT

## THE RATIO OF HEAT CAPACITIES OF AIR, $\gamma=\mathbf{C}_{\mathbf{p}} / \mathbf{C}_{v}$

What to measure : Height of the barometer (h), mass and diameter of the ball (M,D), time for $\mathbf{n}$ oscillations ( $\mathbf{t}$ )

What to calculate : Period of oscillation
(T), cross-sectional area (A),
atmospheric pressure $\left(P_{0}\right)$, equilibrium pressure ( $\mathrm{P}_{\mathrm{e}}$ )

Experimental findings : Ratio of heat capacities of air ( $\gamma_{\text {air }}$ )


Volume V of the flask can be read from the list according to the flask number. The list and your flask number are in Classroom.

Be aware that volume is in unit of liter but you work in CGS units.

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## PROCEDURE:

The ball is dropped into the tube. The oscillation of the ball is observed.
$\square$ The time for n oscillations (at least 5) is measured using a stopwatch.
$\square$ This procedure is repeated for 5 trials.

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(fill in the Table)

| \# of | \# of <br> Oscillations <br> Trials | Time for n <br> Oscillations <br> $t($ | Time for One <br> Oscillation (Period) |
| :---: | :---: | :---: | :---: |
| 1 |  |  | $T(\quad)$ |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

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## By using the data taken, calculate:

Radius of the ball $R=$

Cross sectional Area
of the precision tube $A=$

Atmospheric Pressure $P_{\mathrm{o}}=\rho g h=$
(fill in the page)

Pressure inside the bottle
at Equilibrium Position
of the Ball $P_{e}=P_{o}+\frac{m g}{A}=$

## THE RATIO OF HEAT CAPACITIES OF AIR, $\gamma=\mathbf{C}_{\mathbf{p}} / \mathbf{C}_{v}$

## (fill in the page)

Average Period $T_{\text {average }} \quad=\ldots \ldots \ldots \ldots$.

Ratio of Heat
Capacities

$$
\gamma=C_{\mathrm{p}} / C_{\mathrm{v}}=
$$

Show the Dimensional Analysis of $\boldsymbol{\gamma}$ clearly

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