Boğaziçi University
Introductory Phys Labs

CONSERVATION OF ANGULAR MOMENTUM

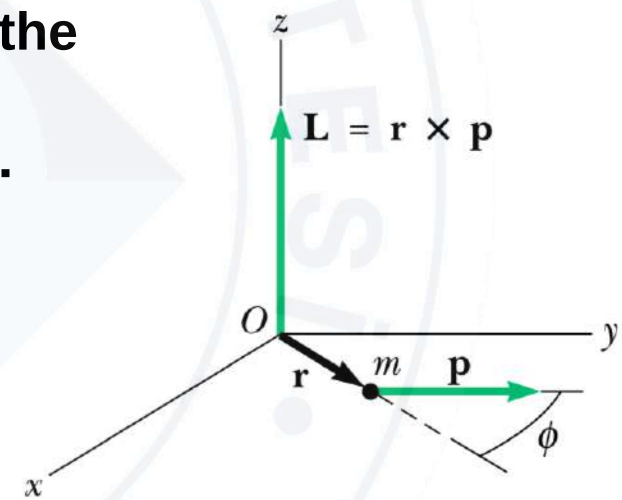
PHYL 101

Theory

TORQUE & ANGULAR MOMENTUM

Just as the idea of linear momentum helps us analyze translational motion, a rotational analog—*angular momentum*—helps us analyze the motion of objects undergoing rotational motion.

Consider a particle of mass *m* located at the vector position r and moving with linear momentum p as in the figure on the right.



In describing linear motion, we found that the net force on the particle equals the time rate of change of its linear momentum:

$$\Sigma \mathbf{F} = d\mathbf{p}/dt.$$

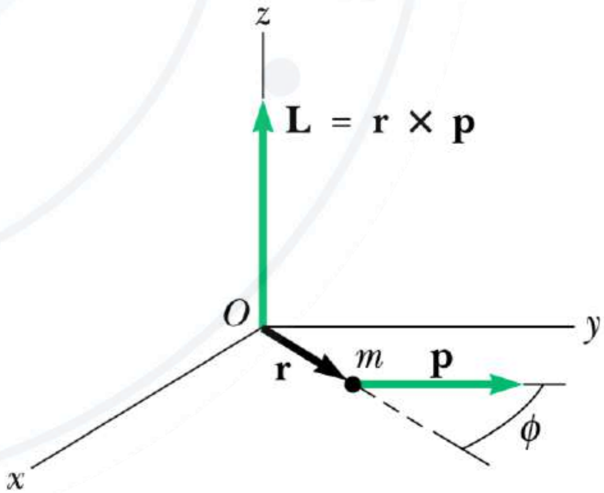
Let us take the cross product of each side of this equation with r, which gives us the net torque on the particle on the left side of the equation:

$$\mathbf{r} \times \Sigma \mathbf{F} = \Sigma \boldsymbol{\tau} = \mathbf{r} \times \frac{d\mathbf{p}}{dt}$$

Now let us add to the right-hand side the term $\frac{d\mathbf{r}}{dt} \times \mathbf{p}$,

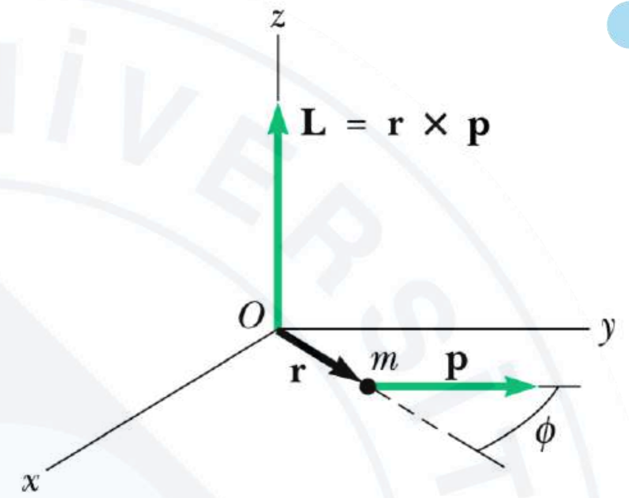
which is zero because *dr*/*dt* = ***v*** **and v and p are parallel. Thus,**

$$\Sigma \boldsymbol{\tau} = \mathbf{r} \times \frac{d\mathbf{p}}{dt} + \frac{d\mathbf{r}}{dt} \times \mathbf{p}$$



Therefore,

$$\Sigma \tau = \frac{d(\mathbf{r} \times \mathbf{p})}{dt}$$



This looks very similar in form to the equation $\Sigma F = \frac{dp}{dt}$. This suggests that the combination r x p should play the same role in rotational motion that p plays in translational motion. We call this combination the *angular momentum* of the particle:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

Thus, we can write:

$$\Sigma \tau = \frac{d\mathbf{L}}{dt}$$

Similar to $\sum \mathbf{F}_{\text{ext}} = \frac{d\mathbf{p}_{\text{tot}}}{dt}$ **, we can show that**

$$\sum \boldsymbol{\tau}_{\text{ext}} = \frac{d\mathbf{L}_{\text{tot}}}{dt}$$

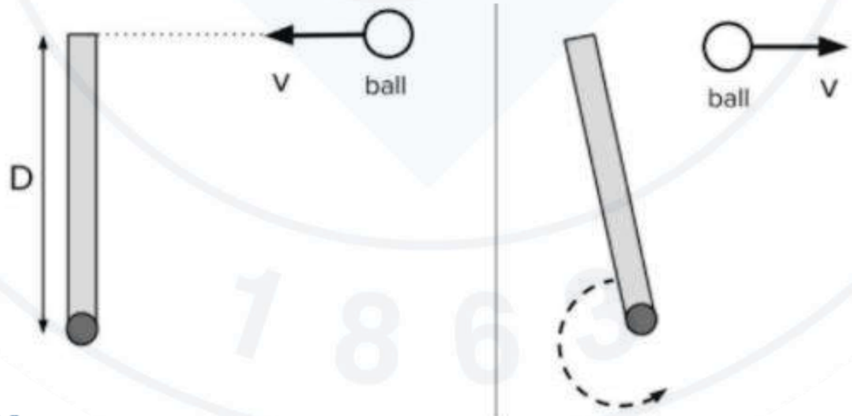
CONSERVATION OF ANGULAR MOMENTUM

The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero.

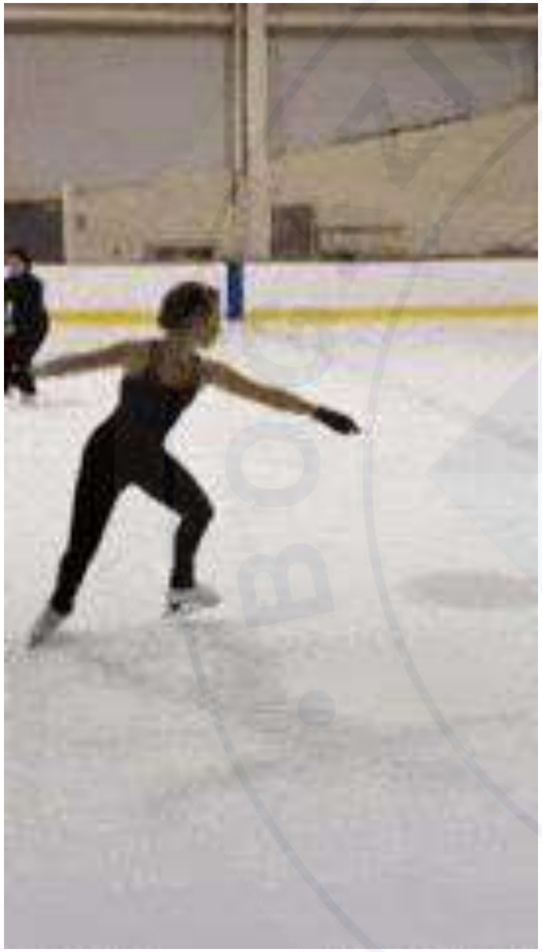
$$\sum \tau_{external} = \frac{dL_{total}}{dt} = 0$$

$$L_{total} = constant$$

$$L_i = L_f$$



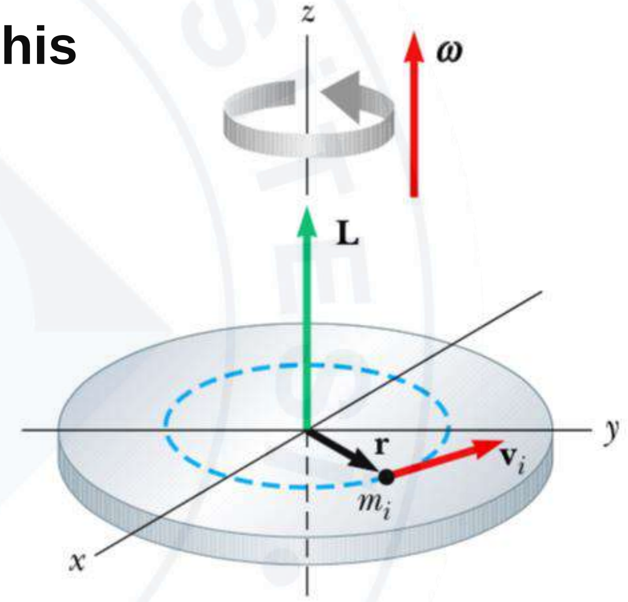
CONSERVATION OF ANGULAR MOMENTUM



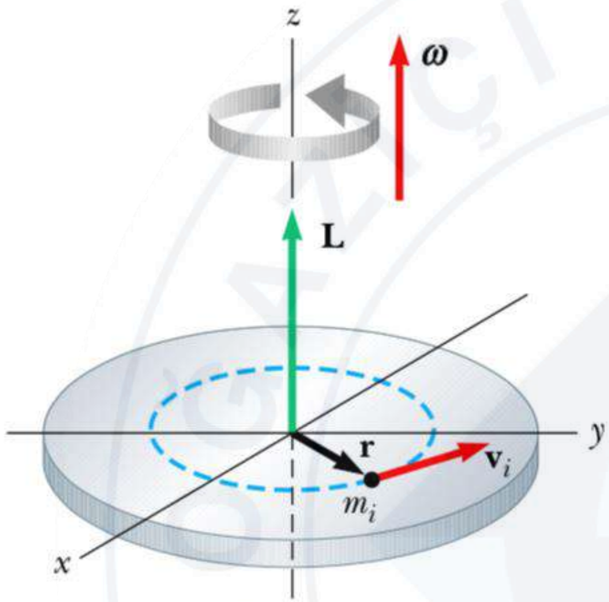
ANGULAR MOMENTUM OF A RIGID OBJECT

Because $v_i = r_i \omega_i$, we can express the magnitude of the angular momentum of this particle *i* as

$$L_i = m_i r_i^2 \omega$$



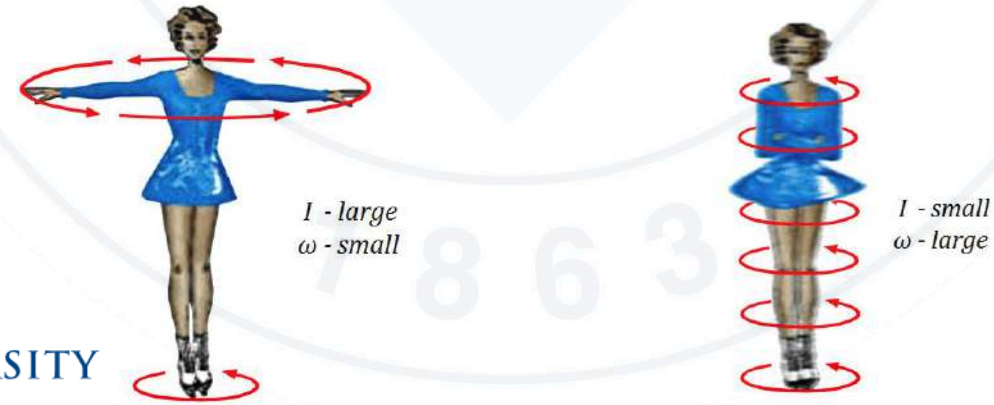
The vector L_i is directed along the *z* axis, as is the vector ω.



We can now find the angular momentum of the whole object by taking the sum of L_i over all particles:

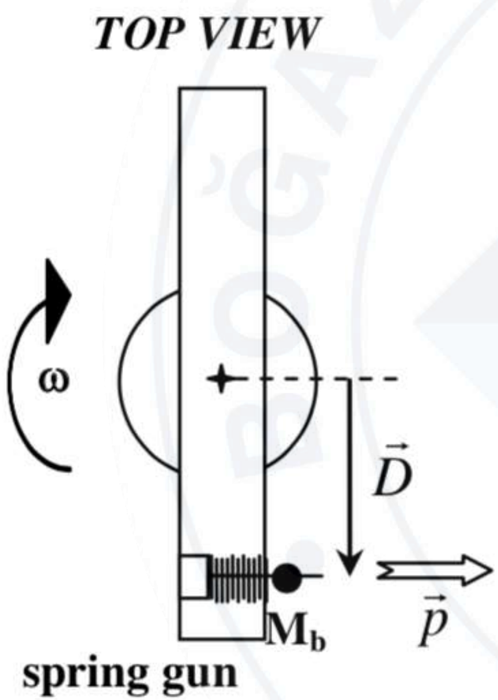
$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega = \left(\sum_i m_i r_i^2\right)\omega$$

$$L_z = I\omega$$



EXPERIMENT

In the experiment we are going to study the conservation of angular momentum.



We have spring gun attached to a turntable, which is free to rotate.

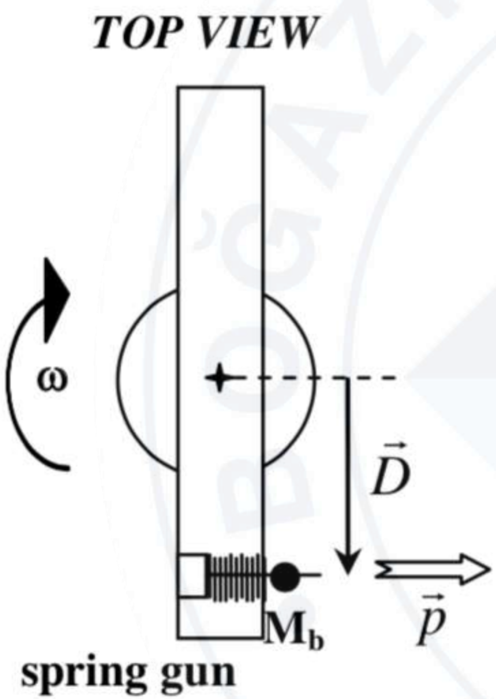
When we shoot a ball from the spring gun, this is like an inverse collision, the angular momentum of the system is conserved:

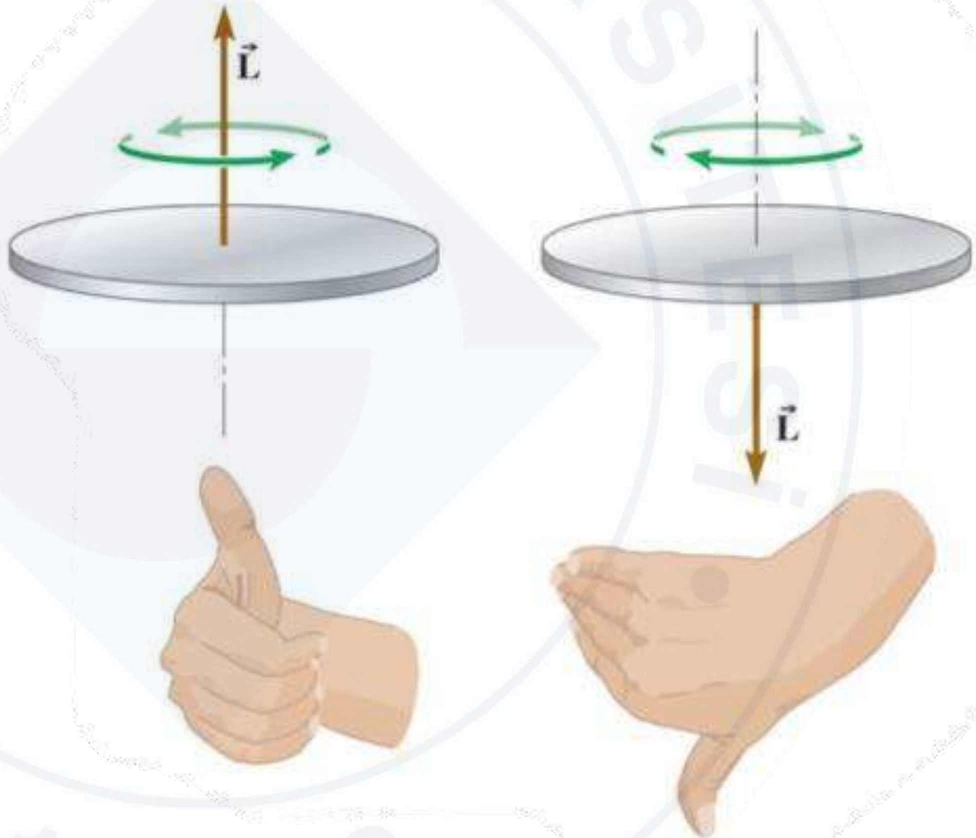
$$L_i = L_f$$
$$0 = \mathbf{L}_{ball} + \mathbf{L}_{turntable}$$

In terms of magnitudes:

$$L_{ball} = L_{turntable}$$

Angular momentum of the ball and turntable is on the opposite directions.

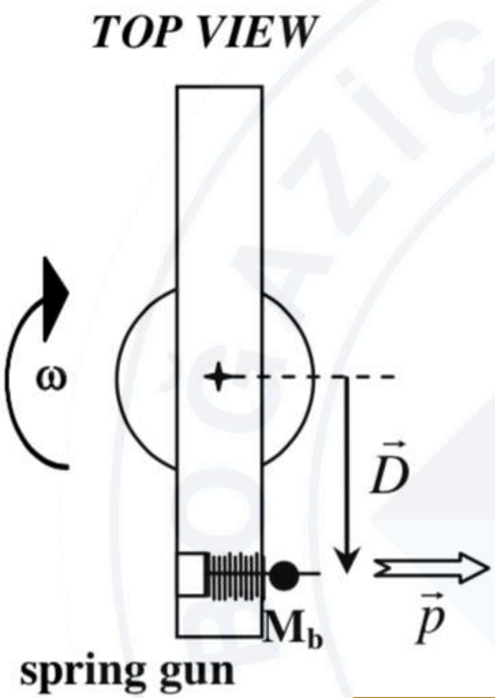




$I.\vec{\omega}$ ⊗

$\vec{D} \times \vec{p}$ ⊙

Right hand rule



$$L_{ball} = L_{turntable}$$

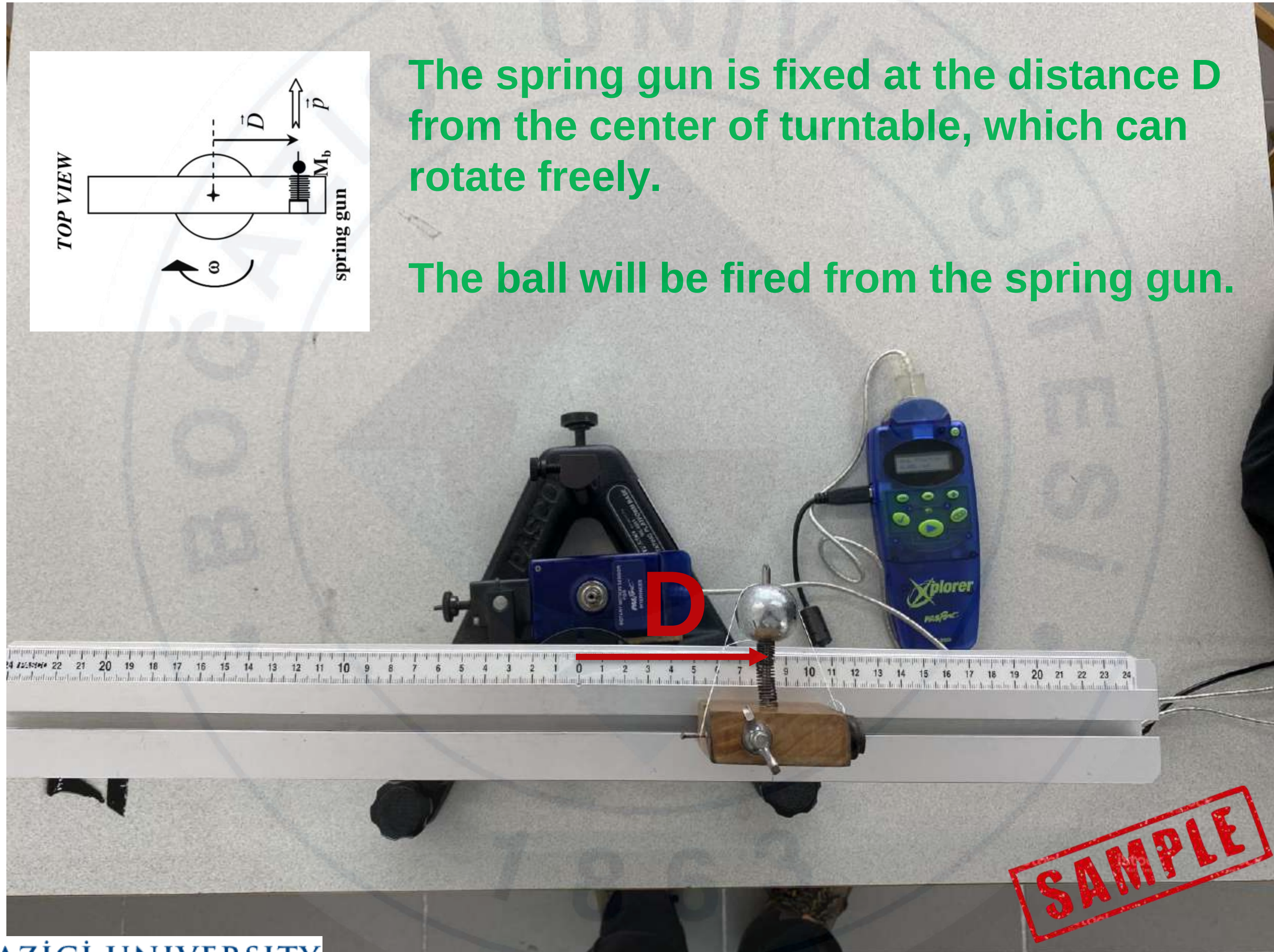
$$L_{ball} = \vec{D} \times \vec{p}$$

$$L_{turntable} = I.\omega$$

Therefore, we find:

$$M_b \nu D = I_D \omega$$

Our aim is finding the mass of the ball using the equation above.



The spring gun is fixed at the distance D from the center of turntable, which can rotate freely.

The ball will be fired from the spring gun.

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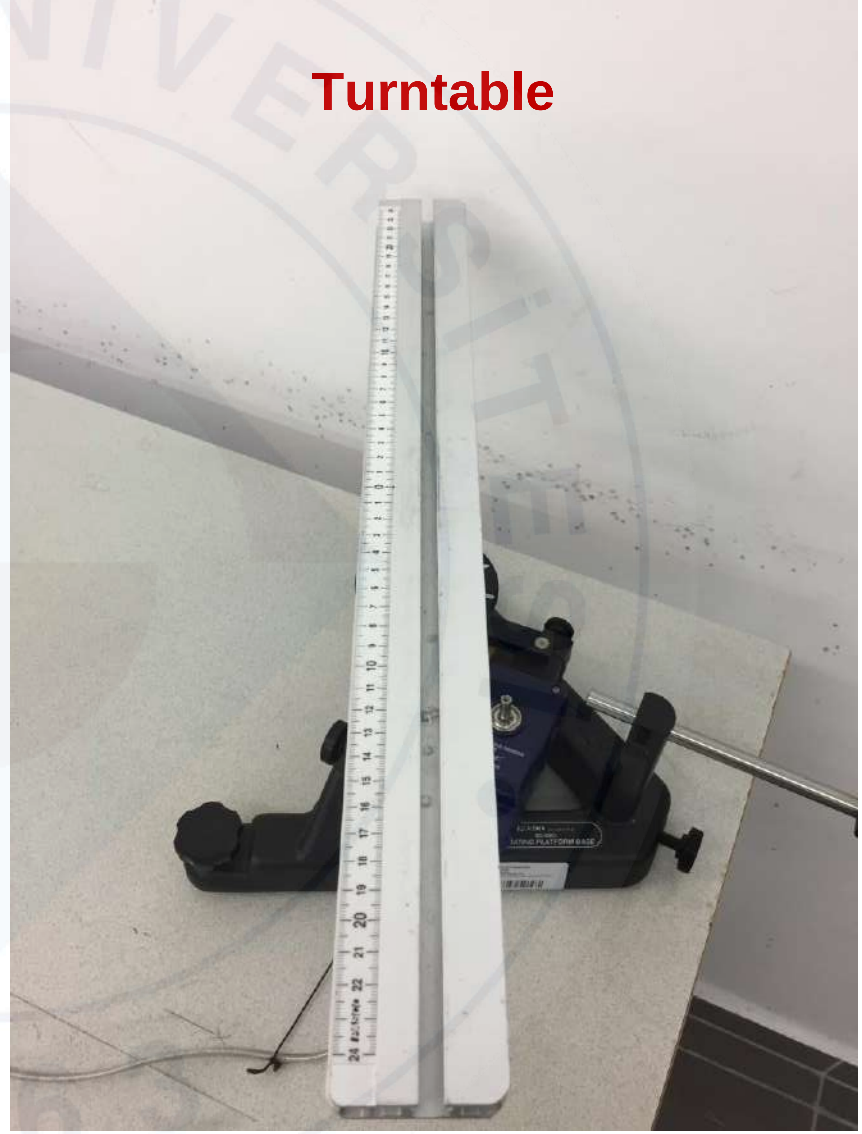
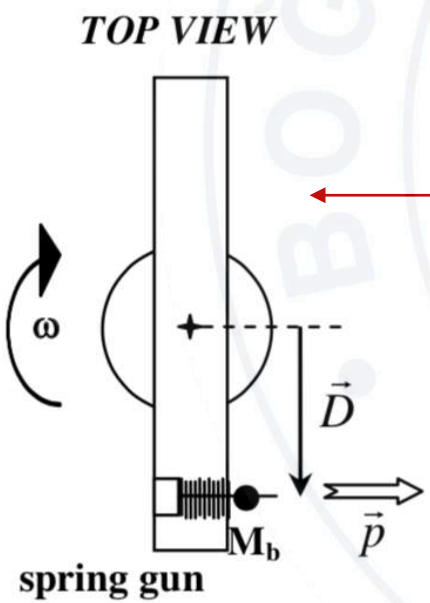
The details of the spring gun & turntable assembly

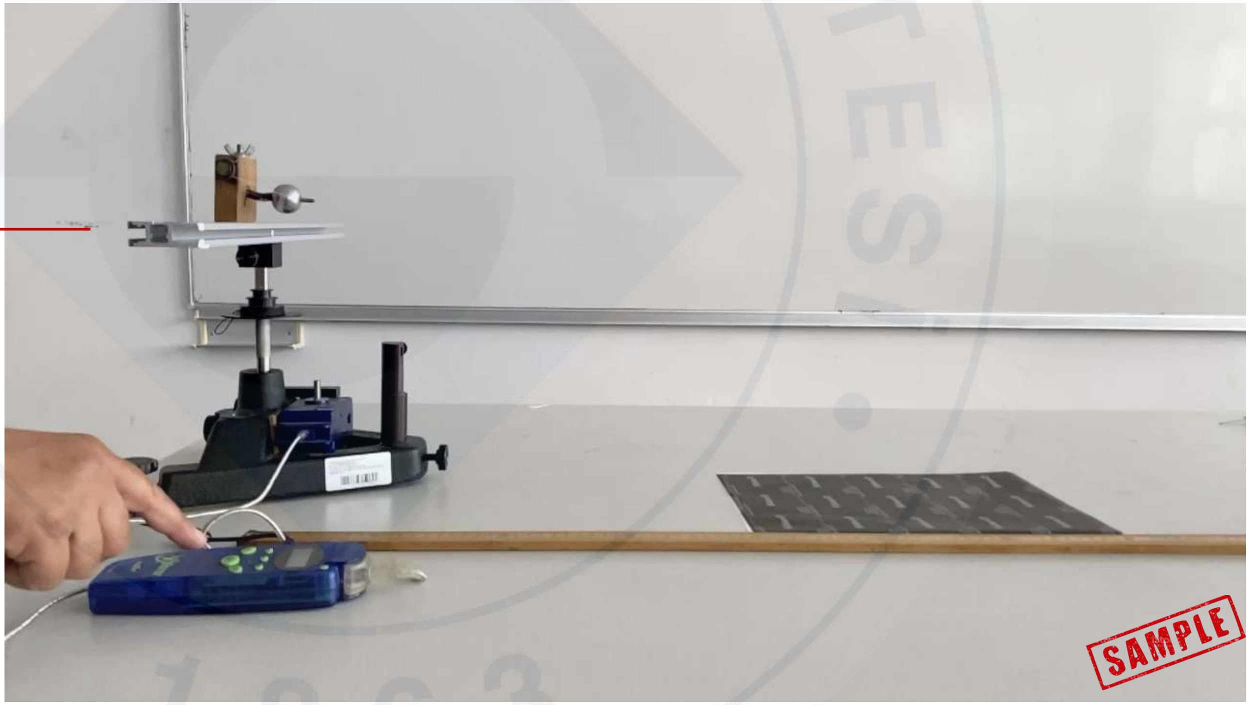
Spring gun



Turntable

We will fire the ball for 2 different positions of spring gun on the turntable.





Each of the five D values will be arranged just like the photo below:

$$M_b \, \nu \, D = I_D \, \omega$$

M_b : Mass of the ball (goal of the experiment is to find it)

ν : The initial velocity of the ball

D : The position of the spring gun on the turntable

I_D : Rotational inertia of the spring gun & turntable assembly

ω : The initial angular velocity of the assembly

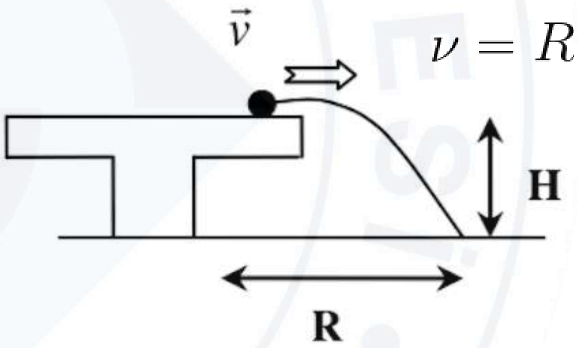
Determination of the initial velocity of the ball (v)

$$M_b \nu D = I_D \omega$$

ν is the initial velocity of the ball.

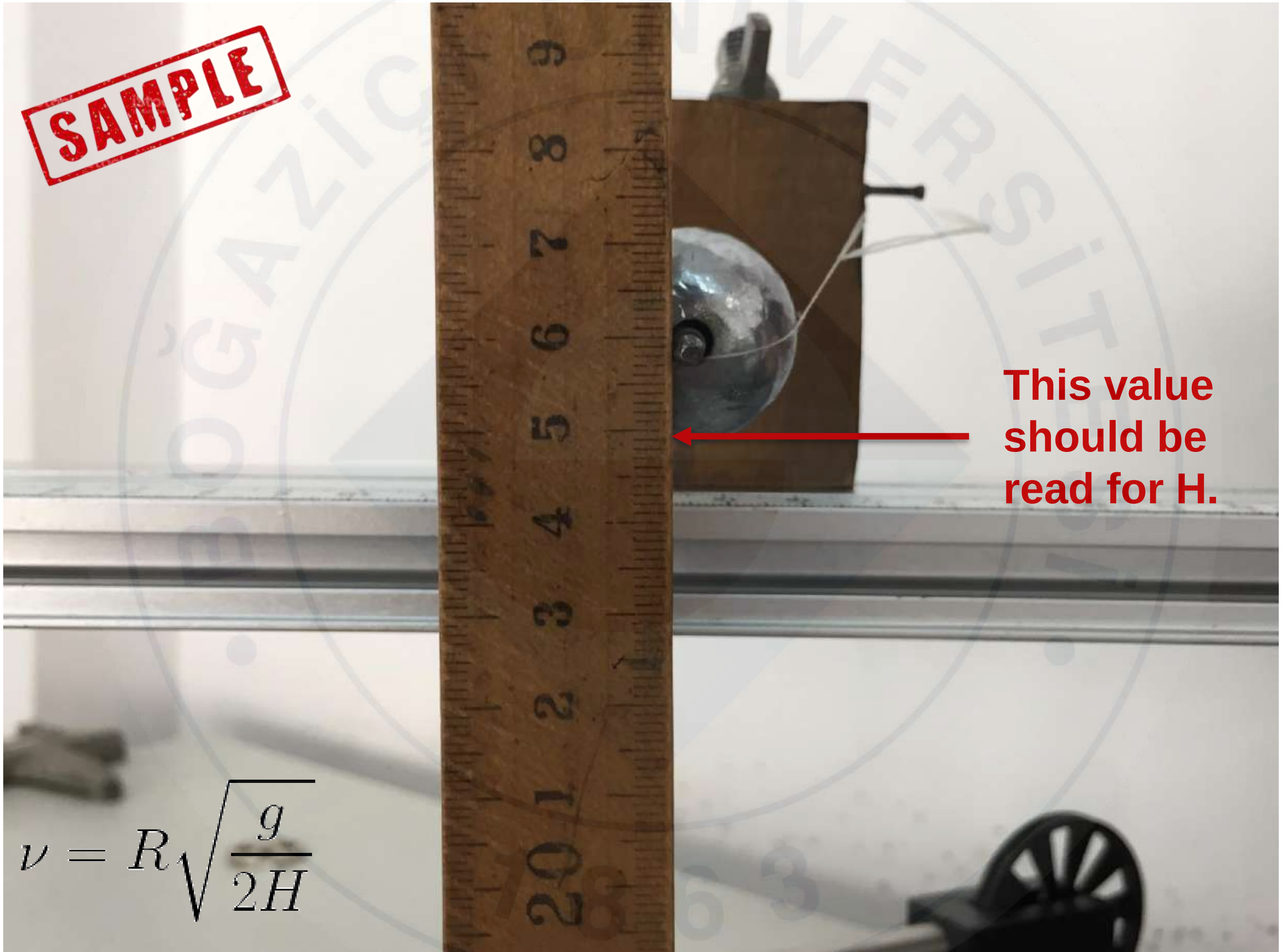
The initial velocity of the ball can be determined by measuring the range and the initial height of the ball.

$$\nu = R\sqrt{\frac{g}{2H}}$$

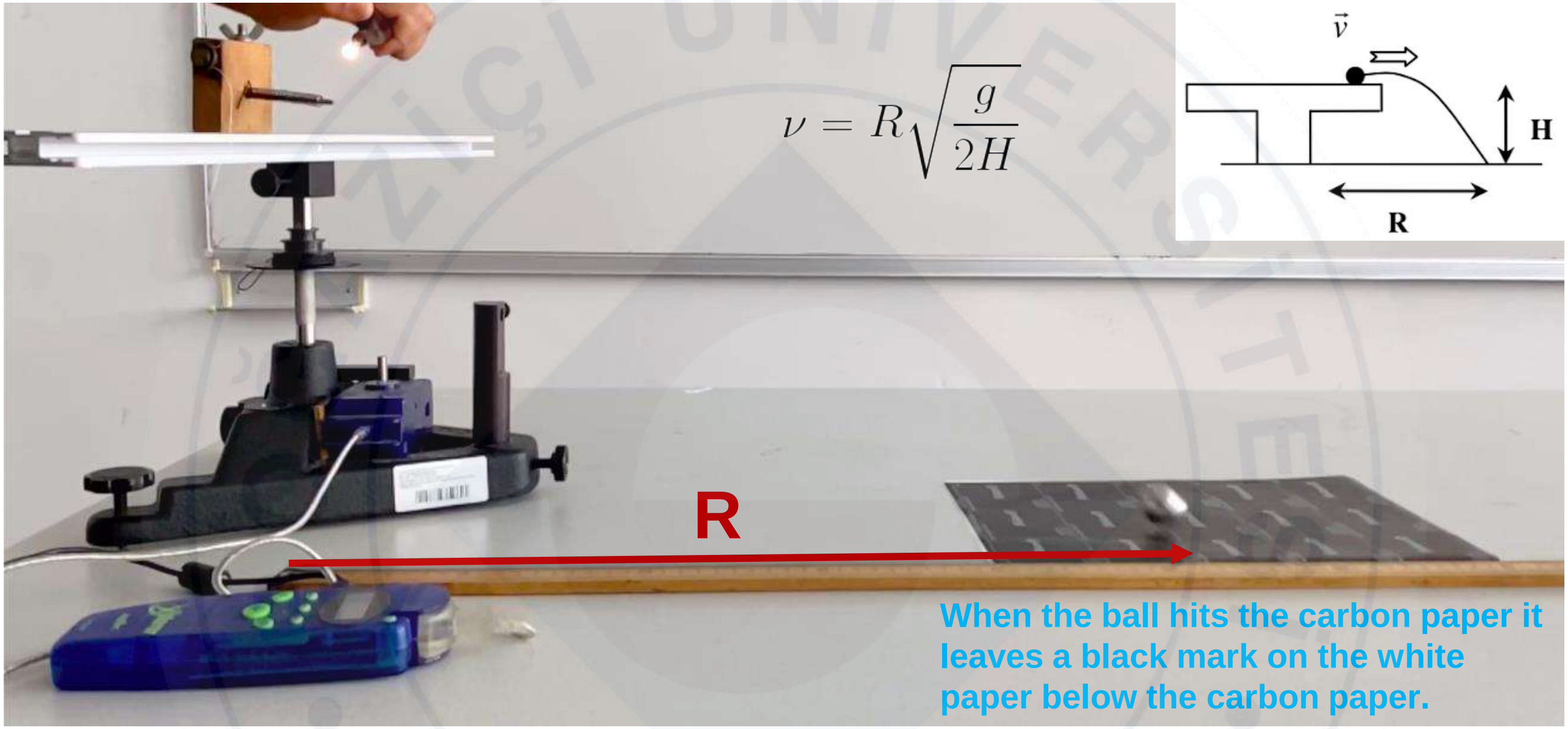


ν is the initial velocity of the ball.

$$\nu = R\sqrt{\frac{g}{2H}}$$

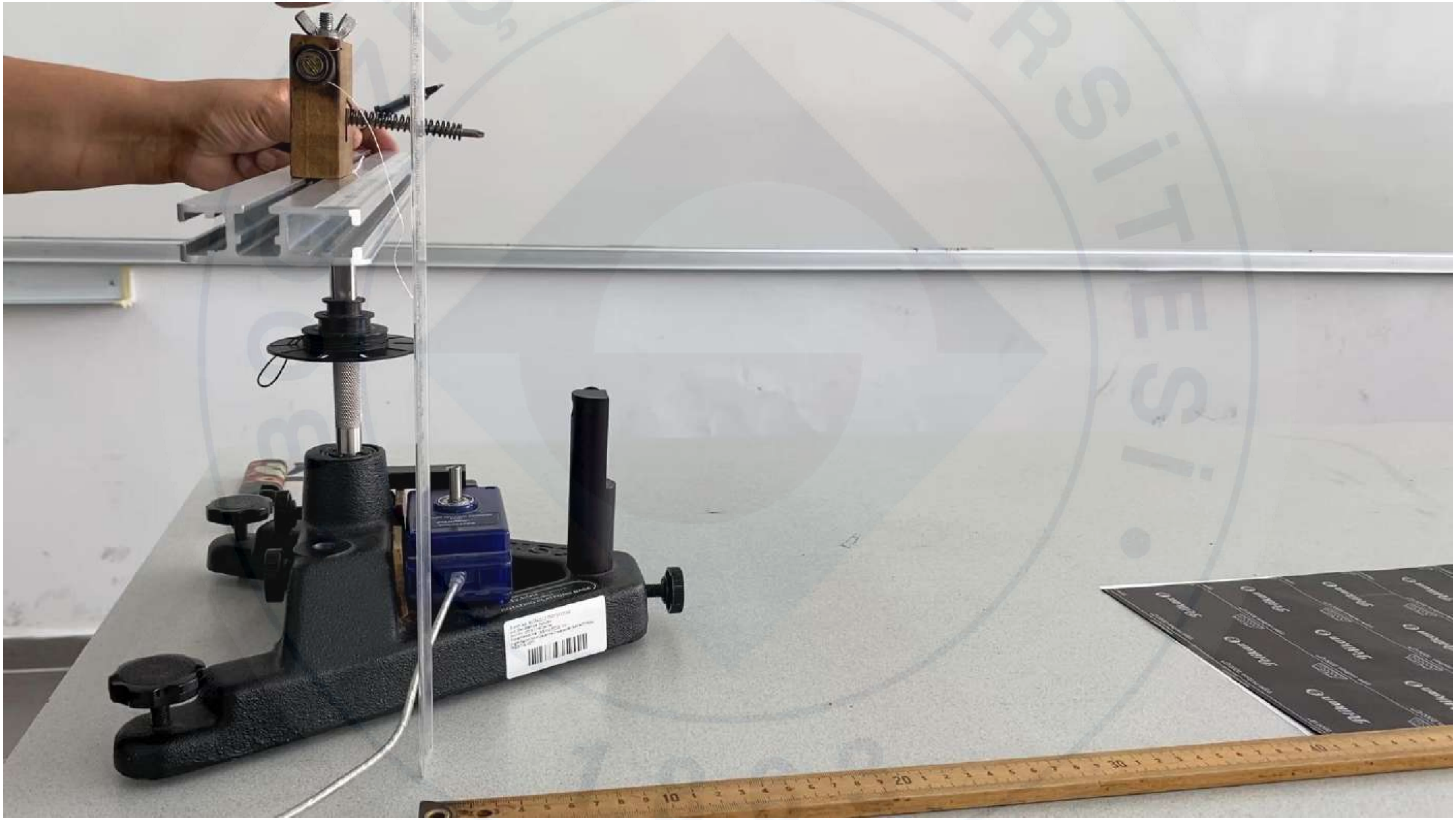


$$\nu = R\sqrt{\frac{g}{2H}}$$



We can find the range (R) of the ball using our ruler on the table.

After firing the ball, the mark on the paper is checked:



$$\nu = R\sqrt{\frac{g}{2H}}$$

We will fire the ball for 2 different values of D. For each D value, we need to calculate the initial velocity of the ball using the equation above. Then, we can fill the table below.

D ( )	R ( )	Velocity of the ball v ( )

Determination of the rotational inertia of the spring gun & turntable assembly (I_D)

Remember, our ultimate aim is to find the mass of the ball M_b using the equation below. The next step is about I_D .

$$M_b \, v \, D = I_D \, \omega$$

Rotational inertia of the spring gun & turntable assembly for the distance D of spring gun from the center of the turntable

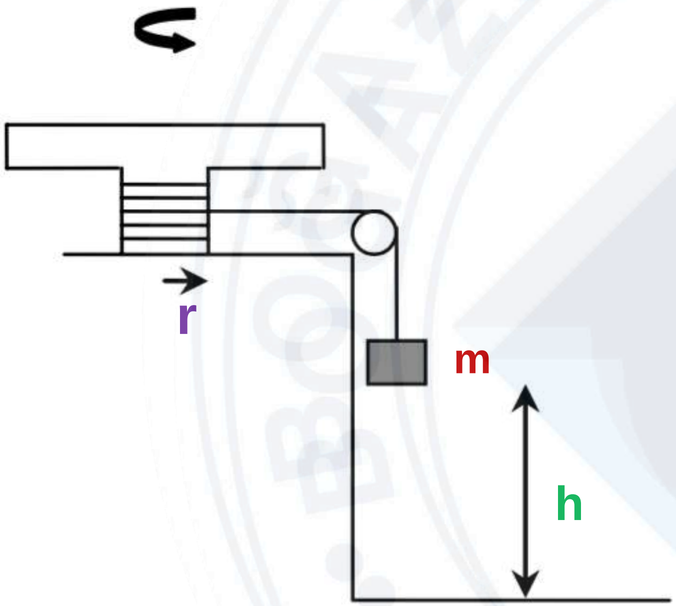
By the parallel axes theorem:

$$I_D = I_{SPRINGGUN}^{CM} + M_{gun}\,D^2$$

Rotational inertia of the spring gun & turntable assembly when the spring gun is at the center (D=0).

Spring gun is at the center of the turntable

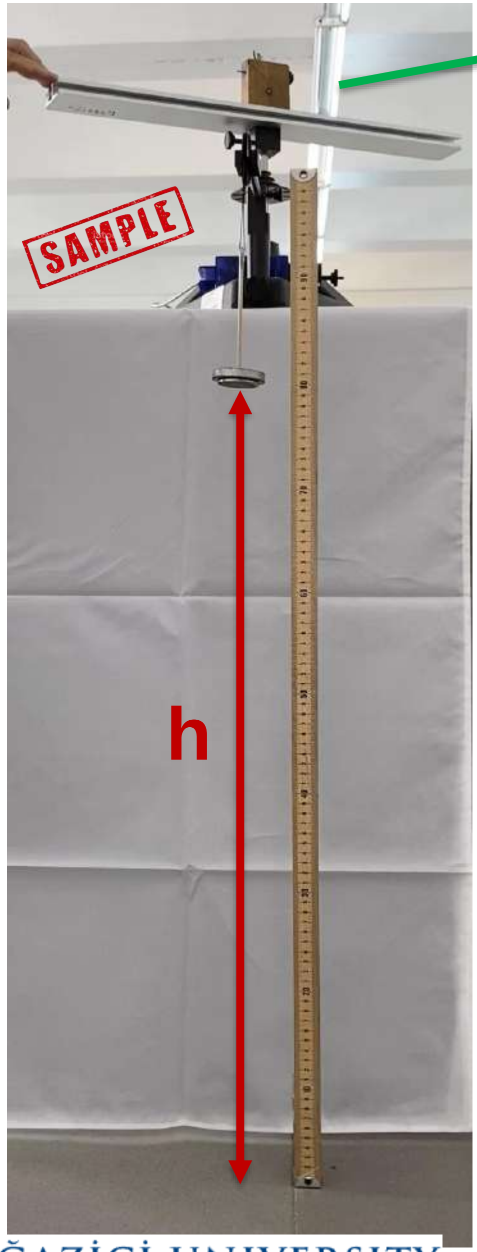
We can find $I^{CM}_{SPRINGGUN}$ **as we did in the Rotational Inertia experiment.**



We will release a mass m with the mass hanger from the height h and measure the time of descent t.

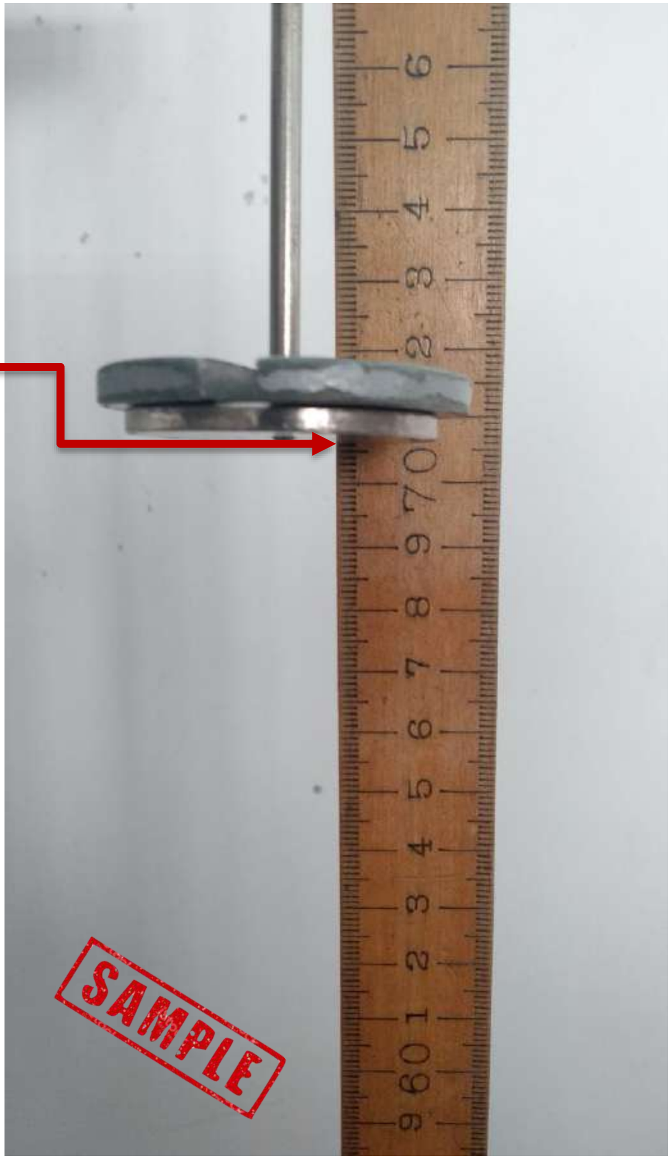
$$I^{CM}_{SPRINGGUN} = mr^2[\frac{gt^2}{2h} - 1]$$

m : mass on the mass holder
r : radius of the drum
t : time for descent
h : height of the mass holder from the floor

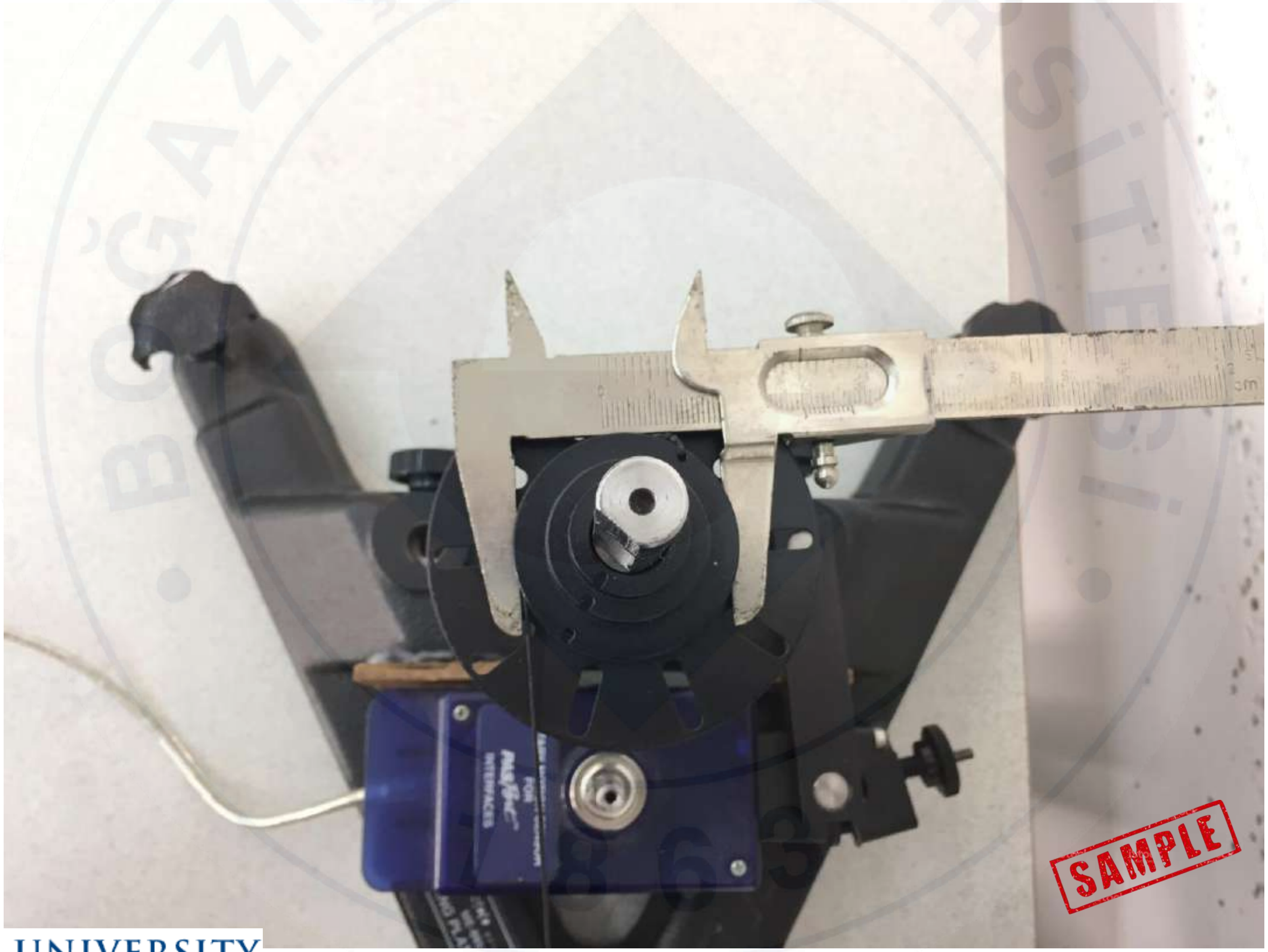


The spring gun is at the center

The height h can be found using the ruler



The radius of the drum can be calculated using the measurement of the diameter of the drum using vernier scale.



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As we know m, r, h, we can measure time for descent (t) with a stopwatch and find the rotational inertia of the spring gun & turntable assembly when the spring gun is at the center:

$$I^{CM}_{SPRINGGUN} = mr^2[\frac{gt^2}{2h} - 1]$$


M_{gun}
Precisa 3100D
72,3
T
MODE
ON/OFF
NET
SWISS QUALITY
SAMPLE

The next step is calculating I_D for the D values we fired the spring gun using the formula below and filling I_D values on the table:

$$I_D = I^{CM}_{SPRINGGUN} + M_{gun} D^2$$

D ( )	*ω* ( )	I_D ( )	M_b ( )
$\sum_{i=1}^{2} M_b^i =$			

Determination of the initial angular velocity of the spring gun & turntable assembly (ω)

Going back to our main equation, so far what we have done:

$$M_b \nu D = I_D \omega$$

Initial velocity of the ball as it's fired for each D value

$$\nu = R\sqrt{\frac{g}{2H}}$$

5 different positions of the spring gun on the turntable

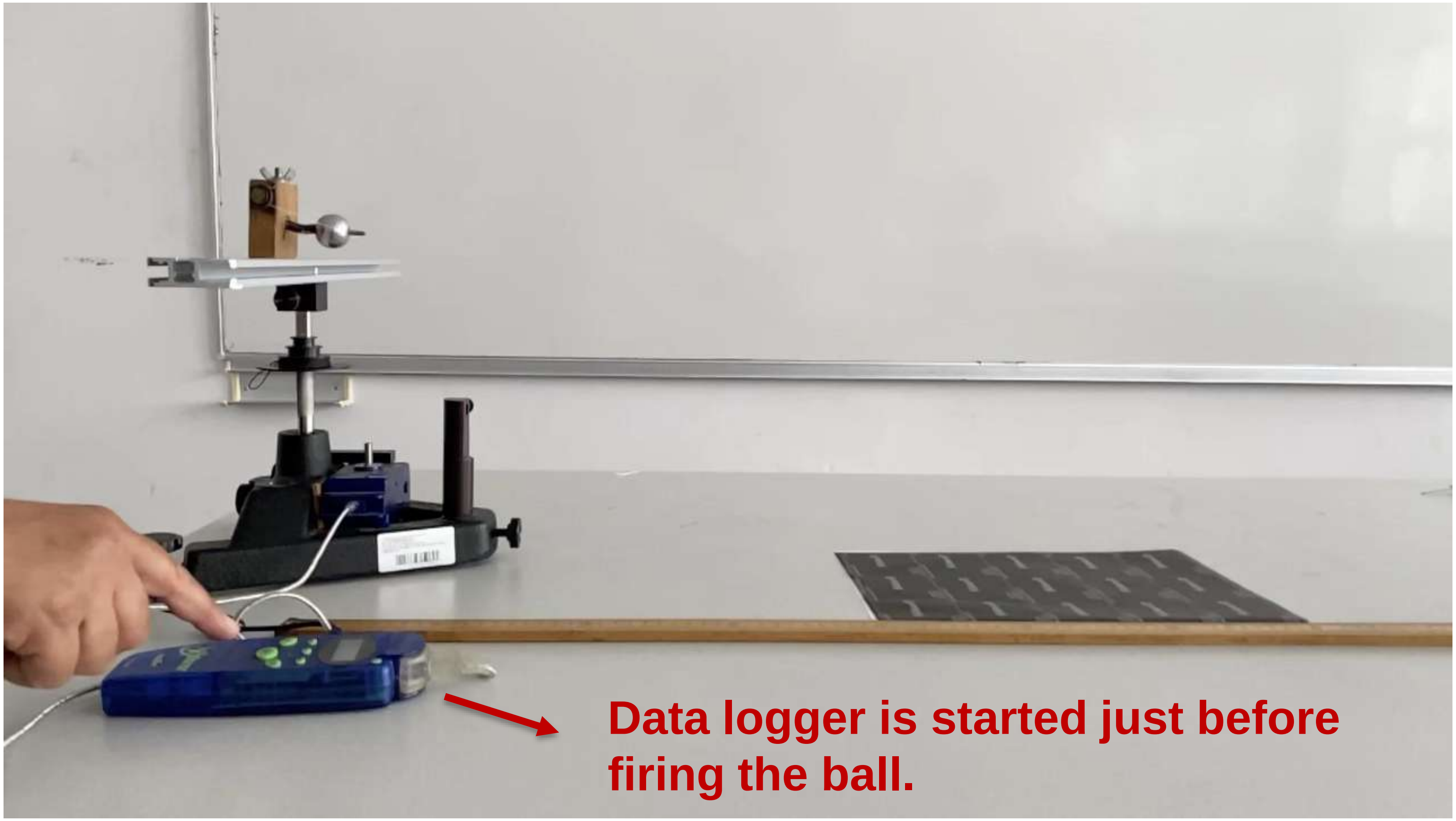
Rotational inertia of the spring gun & turntable assembly

$$I_D = I^{CM}_{SPRINGGUN} + M_{gun} D^2$$

$$I^{CM}_{SPRINGGUN} = mr^2[\frac{gt^2}{2h} - 1]$$

To calculate M_b for each D value, we need the initial angular velocity (ω) values of the string gun & turntable assembly.

We can read the initial angular velocity of each shot of the ball using the data logger.



Data logger is started just before firing the ball.

After reading the initial angular velocity for each D value, we can fill the ω values on the table below:

D ( )	ω ( )	I_D ( )	M_b ( )
$\sum_{i=1}^{2} M_b^i =$			

D ( )	ω ( )	I_D ( )	M_b ( )
$\sum_{i=1}^{2} M_b^i =$			

Having filled the ω and I_D values for each D on the table above, now we can calculate mass of the ball (M_b) for each D using our main equation below:

$$M_b \, v \, D = I_D \, \omega$$

Then we can find the average value of mass of the ball, M_b.

RESULTS:

Average mass of the ball M_b = ...

Lastly, we will compare this value with the actual mass of the ball:

% Error for M_b = ...


SAMPLE
Precisa 3100 D
74.8
100%
50%
0%
T
MODE
ON/OFF
NET
SWISS QUALITY