Boğaziçi University

Introductory Phys Labs

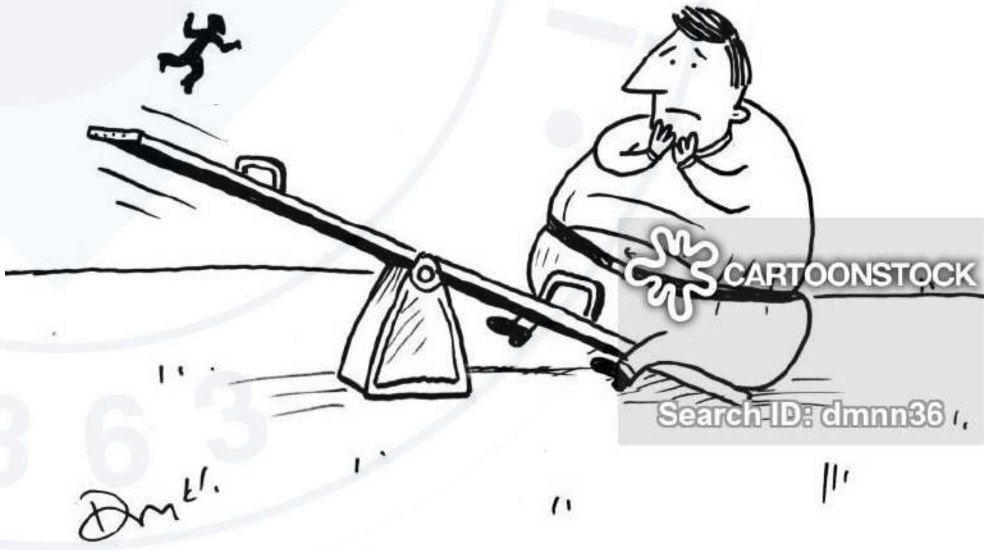
TORQUE AND ANGULAR ACCELERATION

PHYL 101

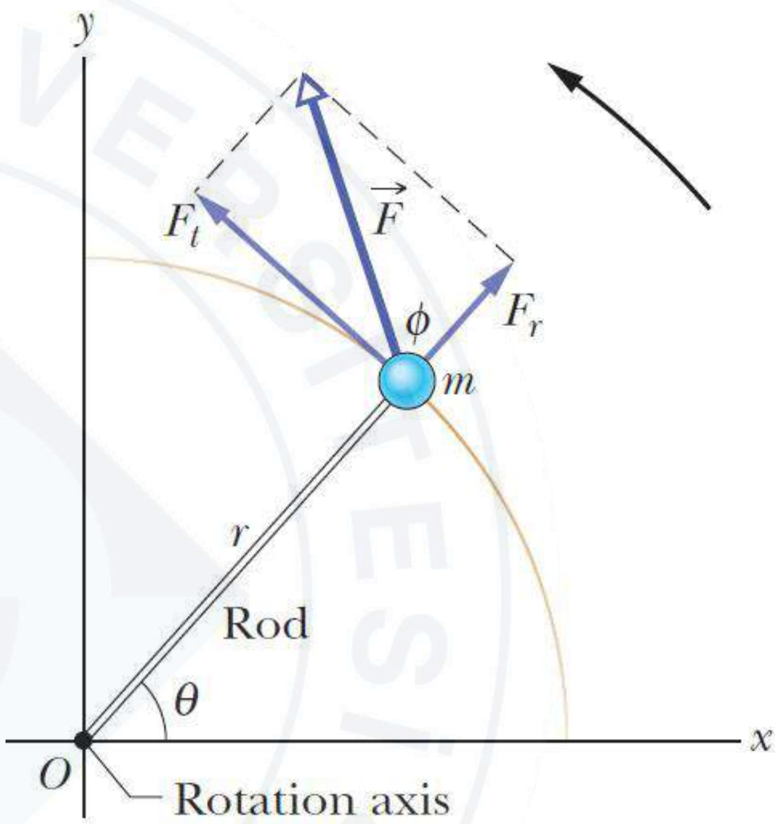
TORQUE AND ANGULAR ACCELERATION

Torque $\vec{\tau}$ can be defined as the tendency of a force to create rotation around an axis.

In this experiment, we are going to work with the same setup as the last experiment and we will analyze the torque on the system to calculate the moment of inertia I of the disk. Then, we will compare our findings with last weeks results.



THEORY



Let us start by discussing torque $\vec{\tau}$ on an object with mass m that is fixed to a rod given in the figure. Rod is rigid and free to rotate on a hinge at the origin O. Length of the rod is r and the force $\vec{F}$ is applied at an angle φ to the radial axis. Torque is defined as;

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Recalling the vector product, we can rewrite this equation as;

$$\tau = Frsin(\varphi) = F_t r$$

F_t given in the equation is the magnitude of tangential component of $\vec{F}$. Torque given here can be thought of as the ability of the force to rotate the object.

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Using Newton's second law of motion, we get from the last equation;

$$\tau = F_t r = m a_t r$$

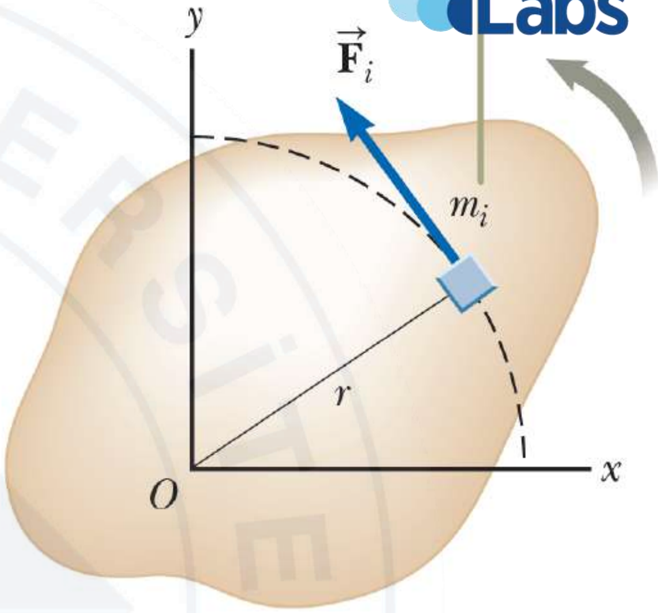
Since the motion is rotational, we can write tangential acceleration a_t in terms of angular acceleration α and radial distance of rotation to origin r;

$$\tau = m\alpha r r = (mr^2)\alpha$$

The moment of inertia is defined to be the quantity given in the parenthesis, so;

$$\boxed{\tau = I\alpha} \qquad \boxed{I = mr^2}$$

Note that I is calculated for the point particle in our specific example and it is the same formula that we got in the previous experiment . Now, let us extend our results to rigid objects.



Suppose we have a rigid object given in the figure which is free to rotate around the origin. If we could divide the object to a collection of point particles with mass m_i, using again Newton's second law, we could write;

$$\tau_i = F_i r_i = m_i a_i r_i$$

Since the object rotates as a whole, each particle indexed with i has the same α. So;

$$\tau_i = m_i r_i^2 \alpha$$

If we add torques of all these parts, we get total external torque on the object as;

$$\sum \tau_{ext} = \sum_i \tau_i = \left(\sum_i m_i r_i^2\right)\alpha$$

The sum in the parenthesis gives us I.

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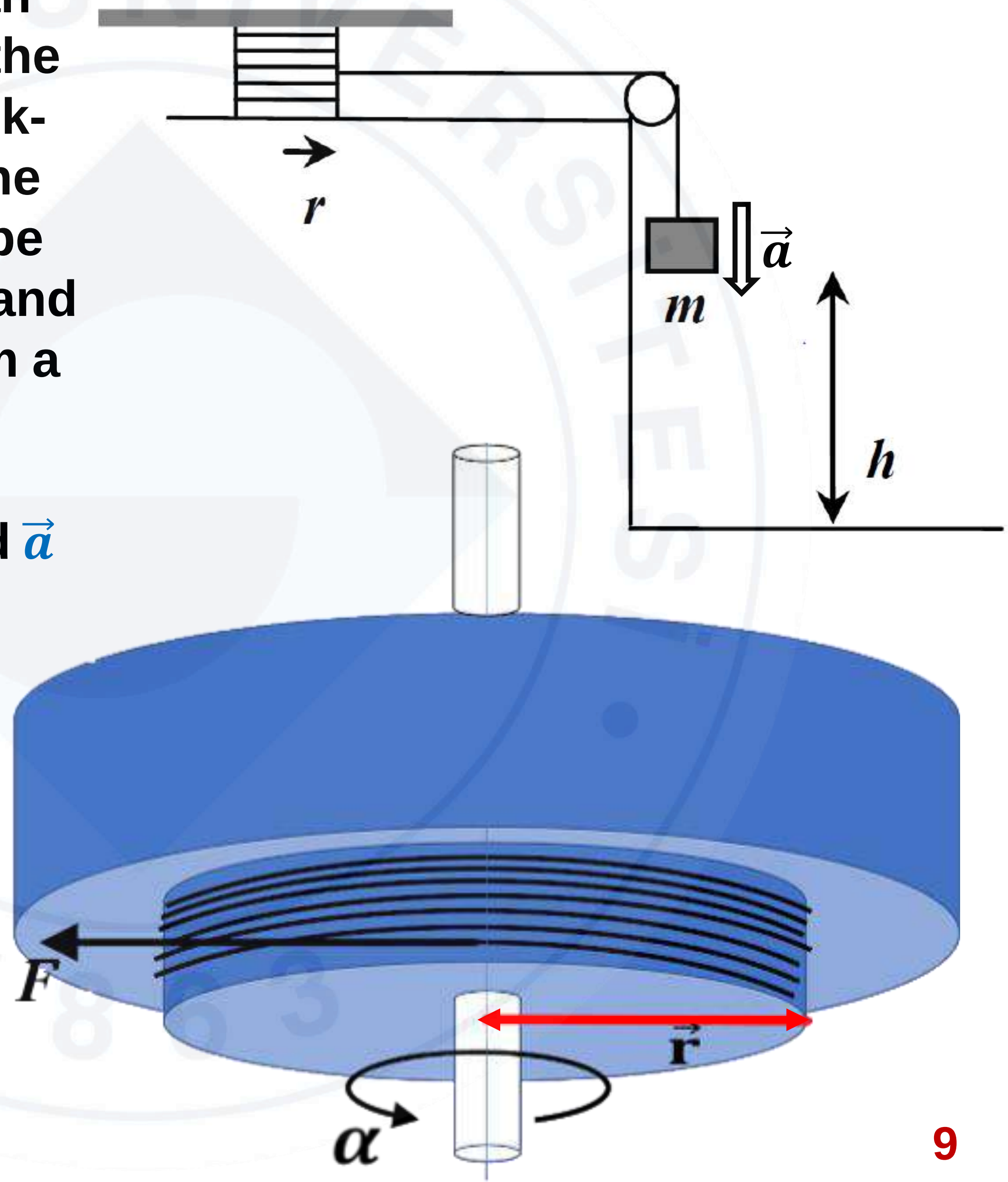
We have successfully extended our result from point particle to rigid objects and found;

$$\boxed{I = \sum_i m_i r_i^2} \Rightarrow \boxed{\sum \tau_{ext} = I_{total}\alpha}$$

We will be using the formula on the right-hand side to start our analysis after we see our experimental setup.

The formula we have derived in the last experiment will also be derived by torque analysis in the upcoming slides.

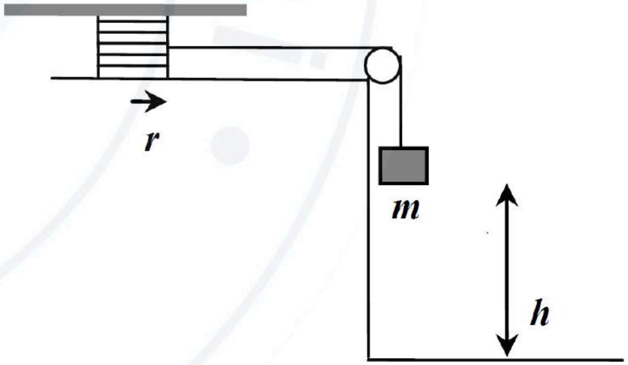
Now, we will see how we can use this setup to calculate the moment of inertia of the disk-drum system. As given in the sketch on the right side, rope is wound around the drum and the mass m *is* released from a height of h. Here, r is the radius of the drum that we wound our rope around and $\vec{a}$ is the acceleration of mass.



Here, F is the tension throughout the rope and α is the angular acceleration.

What to measure: Radius of the drum r and mass on the mass holder m for both experimental calculations. Height of mass holder from the floor h and time for descent T for the ***first experimental calculations***. Angular position of the drum $\theta(t)$ via data logger for the ***second experimental calculations***.

What to calculate : Angular velocity $\omega(t)$ and angular acceleration α



Experimental findings : Moment of inertia of I_1 and I_2 of the rotating part from two experimental calculations

Tension F can be defined as;

$$F = mg - ma$$

Since F is perpendicular to r , we can write torque τ as;

$$\tau = rF = r(mg - ma)$$

We have found out in the previous slides that $\tau = I\alpha$, so we get;

$$I\alpha = r(mg - ma)$$

Substituting $a = \alpha r$ and leaving I alone in the left-hand side;

$$I = \frac{r(mg - m\alpha r)}{\alpha} \Rightarrow \boxed{I_1 = \frac{mgr}{\alpha} - mr^2}$$

We are going to use $g = 9.81\, m/s^2$ and measure r, m and α to calculate I_1. Subscript is to differentiate two different experimental results we are going to compare. We will derive α from data logger recordings, while others will be straightforward measurements.

Now, let us use $\alpha = a/r$ in the last equation.

$$I = \frac{mgr}{(a/r)} - mr^2 = mr^2\left(\frac{g}{a} - 1\right)$$

Let us denote T to be the time of descent of the mass. Since the mass experiences a free fall from rest with acceleration a, we can use $h = \frac{1}{2}aT^2$ or equivalently $a = \frac{2h}{T^2}$ to get;

$$\boxed{I_2 = mr^2\left(\frac{gT^2}{2h} - 1\right)}$$

Recall that this is the result we have found last week by conservation of energy. So, we will measure m, r, T, h and use $g = 9.81\ m/s^2$ to calculate experimental value of I_2, second experimental way of measuring moment of inertia of the object.

We will be getting the angular position θ time series from data logger. The data logger is set to take 2 samples per second, so the time interval between two data points will be $\Delta t = 0.5s$. So, the average angular velocity ω_{avg} in the interval between two data points can be defined as;

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

Note that the time when the instantaneous angular velocity ω is equal to ω_{avg} in the midpoint of the time interval since the motion has constant angular acceleration. We could make the calculations to see this in a system starting from rest with $\theta(0) = 0$ as;

$$\Delta\theta = \frac{\alpha}{2}\left(t_2^2 - t_1^2\right) = \frac{\alpha}{2}(t_2 + t_1)(t_2 - t_1) = \frac{\alpha}{2}(t_2 + t_1)\Delta t$$

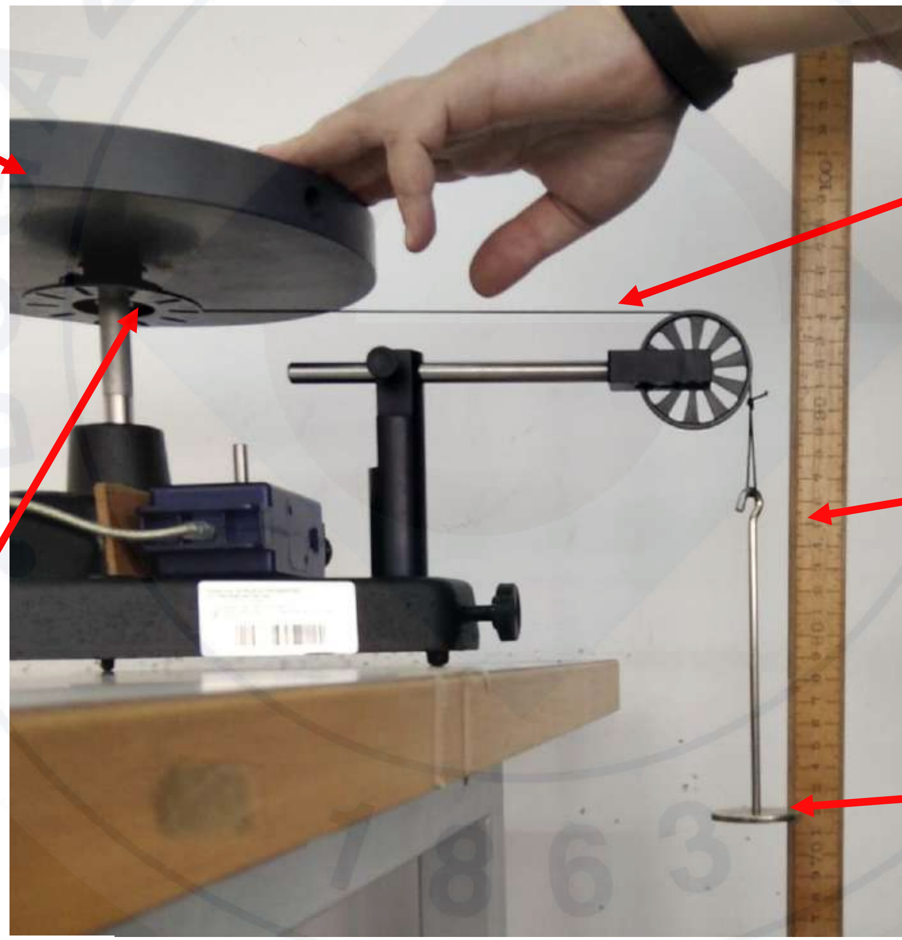
$$\Rightarrow \quad \omega_{avg} = \frac{\Delta\theta}{\Delta t} = \alpha\frac{(t_2+t_1)}{2} = \omega\left(\frac{t_2+t_1}{2}\right)$$

APPARATUS

During this experiment, we are going to calculate moment of inertia of a disk and a ring using the setup below. Then, we will compare our results with theoretical calculations of these objects.

Disk is placed on top of the ***drum*** and will rotate around an axis that goes through its center of mass ***CM***

This is the ***drum*** on which we will place the ***disk*** and the ***ring***



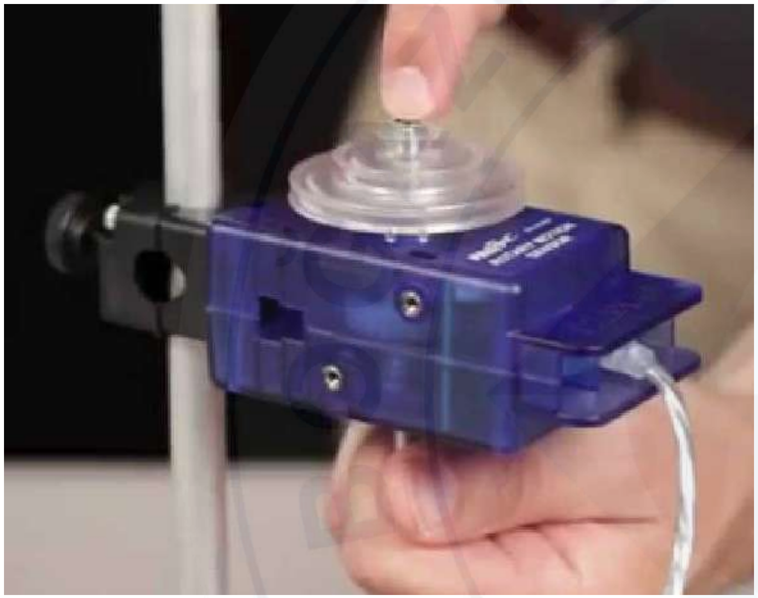
The rope is wound around the ***drum*** and a mass is hanged to the end of it

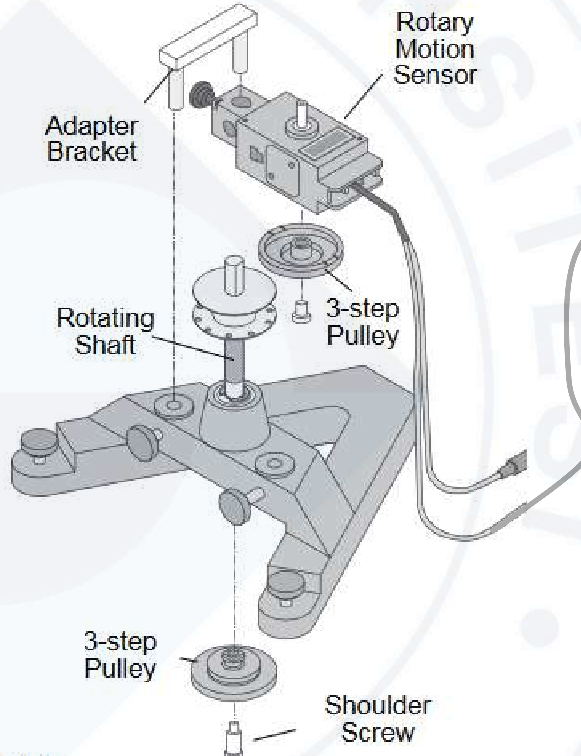
Ruler is here to measure the height of the masses from the ground ***h***

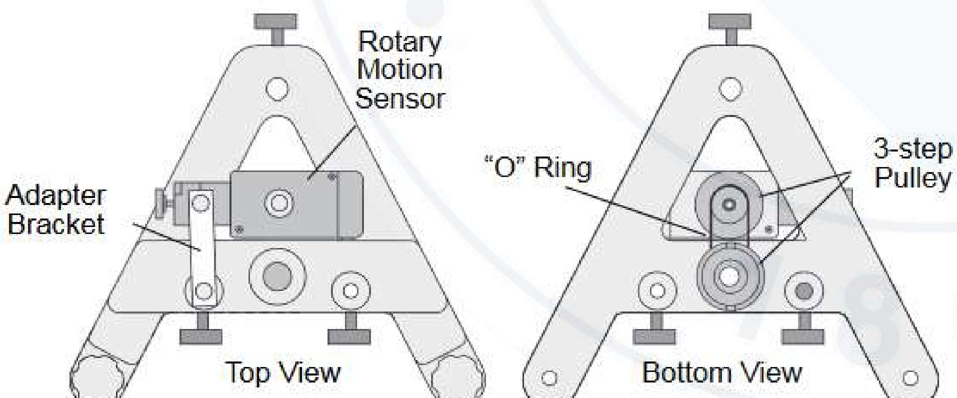
The ***mass holder*** is hanged at the end of the rope and it does not have any additional weights stacked up on it in this picture. Total mass here will be called ***m***

Additional apparatus will be the data logger and rotationary sensor connected to drum.

The data logger will be used to get angular position time series.







TORQUE AND ANGULAR ACCELERATION - Apparatus

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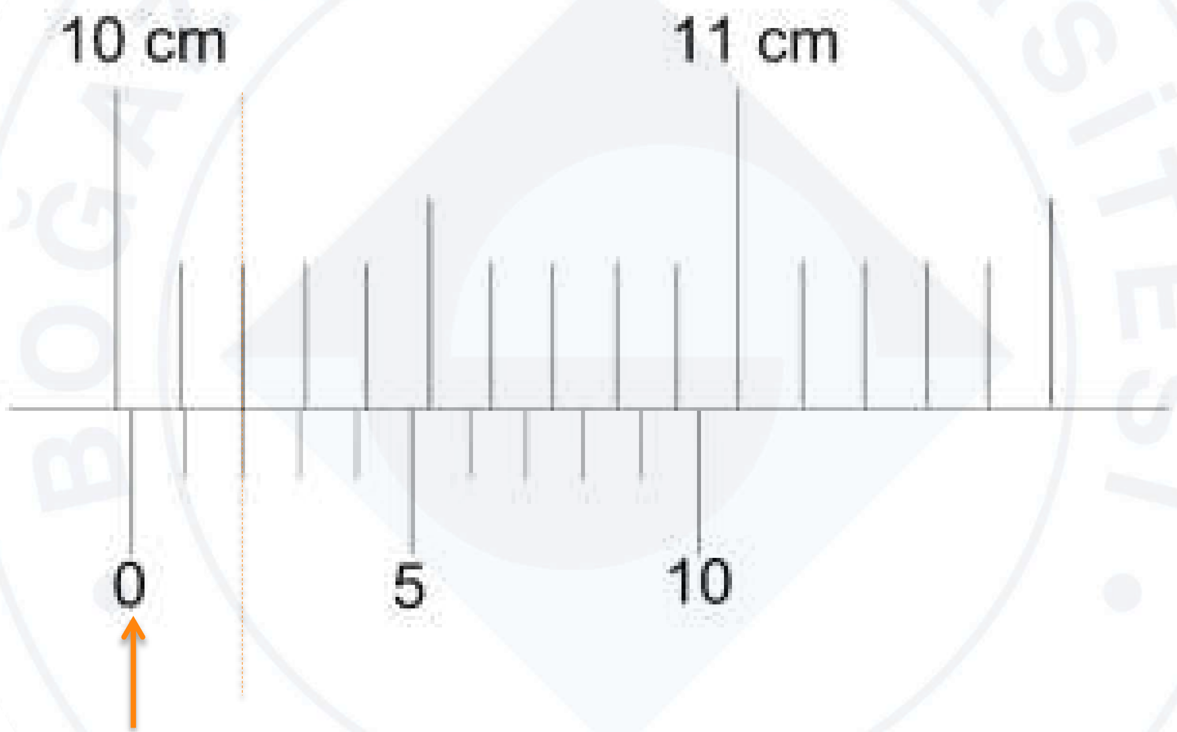
Data logger is started as the disk is released. You will record the data of angular position $\theta(t)$ of the disk in the from the Data Logger screen.

Video1

Vernier Reading Example:



Main number: 10
One decimal: 0
10.0

The aligned line corresponds to 2.
0.02

RESULT: 10.02 cm

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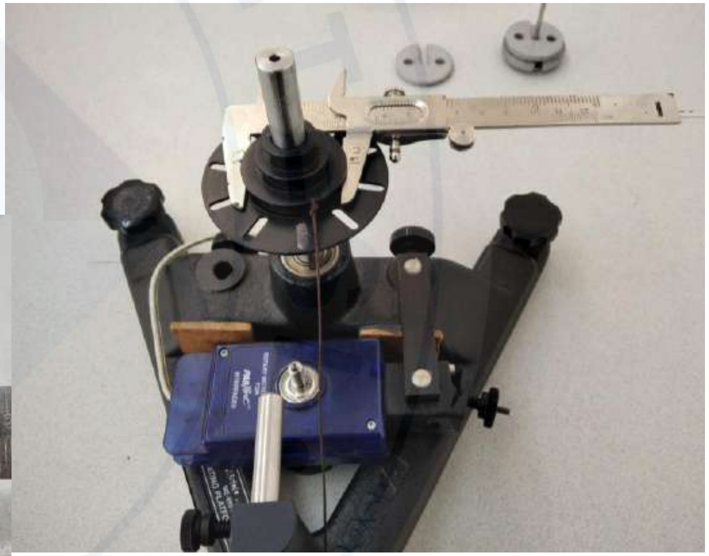
EXPERIMENT

At page 105 of your lab books, you will fill d, r values

SAMPLE

Description / Symbol			Value & Unit
Diameter of the drum	d	$=$	. . . .
Radius of the drum	r	$=$	Will be derived from d

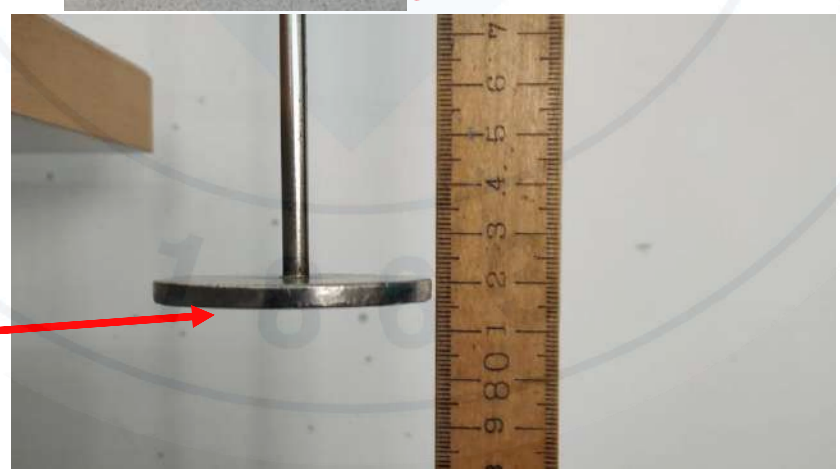


TORQUE AND ANGULAR ACCELERATION – Experiment

At page 105 of your lab books, you will fill m and h values.

Description / Symbol			Value & Unit
Mass on the mass holder	m	=	....
Height of the mass holder from the floor	h	=	

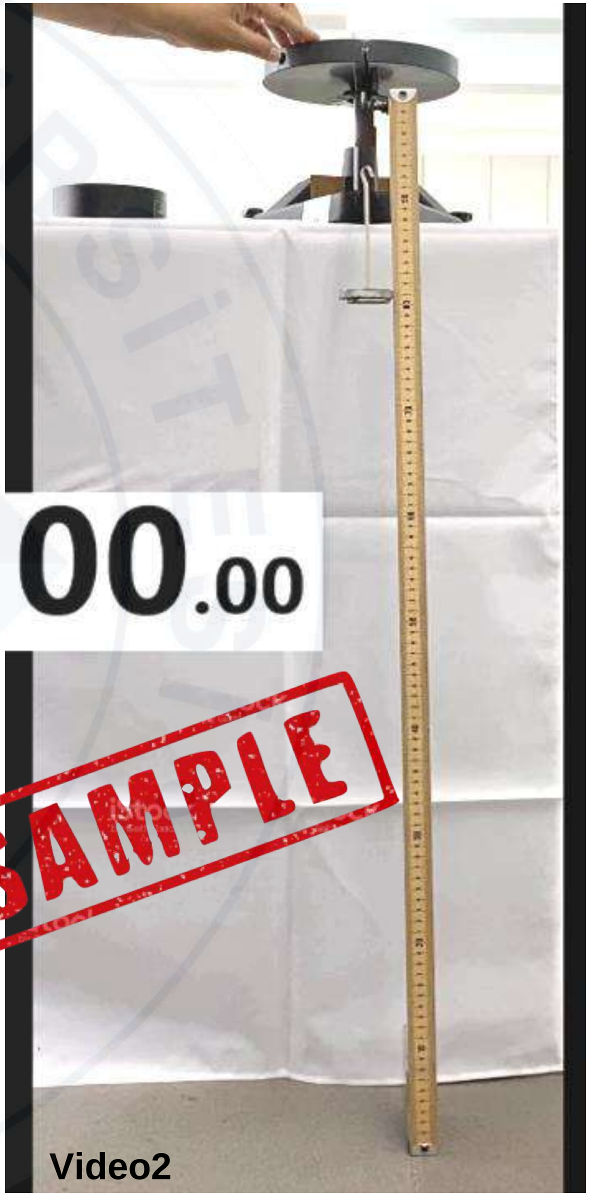
Precision: 1g



Read h from the bottom of mass hanger

At page 105 of your lab books, you will fill T value. You will measure T with your cell phone.

Description / Symbol		Value & Unit
Time for descent	T =	....



Next, we move on to page 107.

- 2nd column Δt is the time interval between data points and it is always 0.5 seconds.
- 3rd column t will be filled as 0.0, 0.5, 1.0 and so on. Here, we will assume that data loggers time keeping is more precise than any other measurements we take, so we will ignore it for significant figure estimation.
- 4th column θ is read from data logger. This will be explained in an upcoming slide.

Fill units here

Number of Intervals	Δt ( )	t ( )	Θ ( )	$\Delta\Theta$ ( )	$\omega_{average} = \Delta\Theta / \Delta t$ ( )
	0.5	0.0	θ_1		
1	0.5	0.5	θ_2		
2	0.5	1.0	θ_3		
3	0.5	1.5	θ_4		
4	…	…	…		

Now, we move on to the calculation parts of the table.

- 5th column $\Delta\theta$ is the difference between two successive data points
- 6th column will be calculated by $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$

Fill units here

Number of Intervals	Δt ( )	t ( )	Θ ( )	$\Delta\Theta$ ( )	$\omega_{average} = \Delta\Theta / \Delta t$ ( )
	0.5	0.0	θ_1		
1	0.5	0.5	θ_2	$\theta_2 - \theta_1$	$(\theta_2 - \theta_1)/\Delta t$
2	0.5	1.0	θ_3	$\theta_3 - \theta_2$	$(\theta_3 - \theta_2)/\Delta t$
3	0.5	1.5	θ_4	$\theta_4 - \theta_3$	$(\theta_4 - \theta_3)/\Delta t$
4	…	…	…	…	…

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On the Data logger screen, you will see recordings like the one given here. Starting from the first data you see, record the values sequentially to the 4th column θ.

Results with negative values mean that the rotation is in the negative direction and has no importance to what we are doing, so you may record all to be positive.

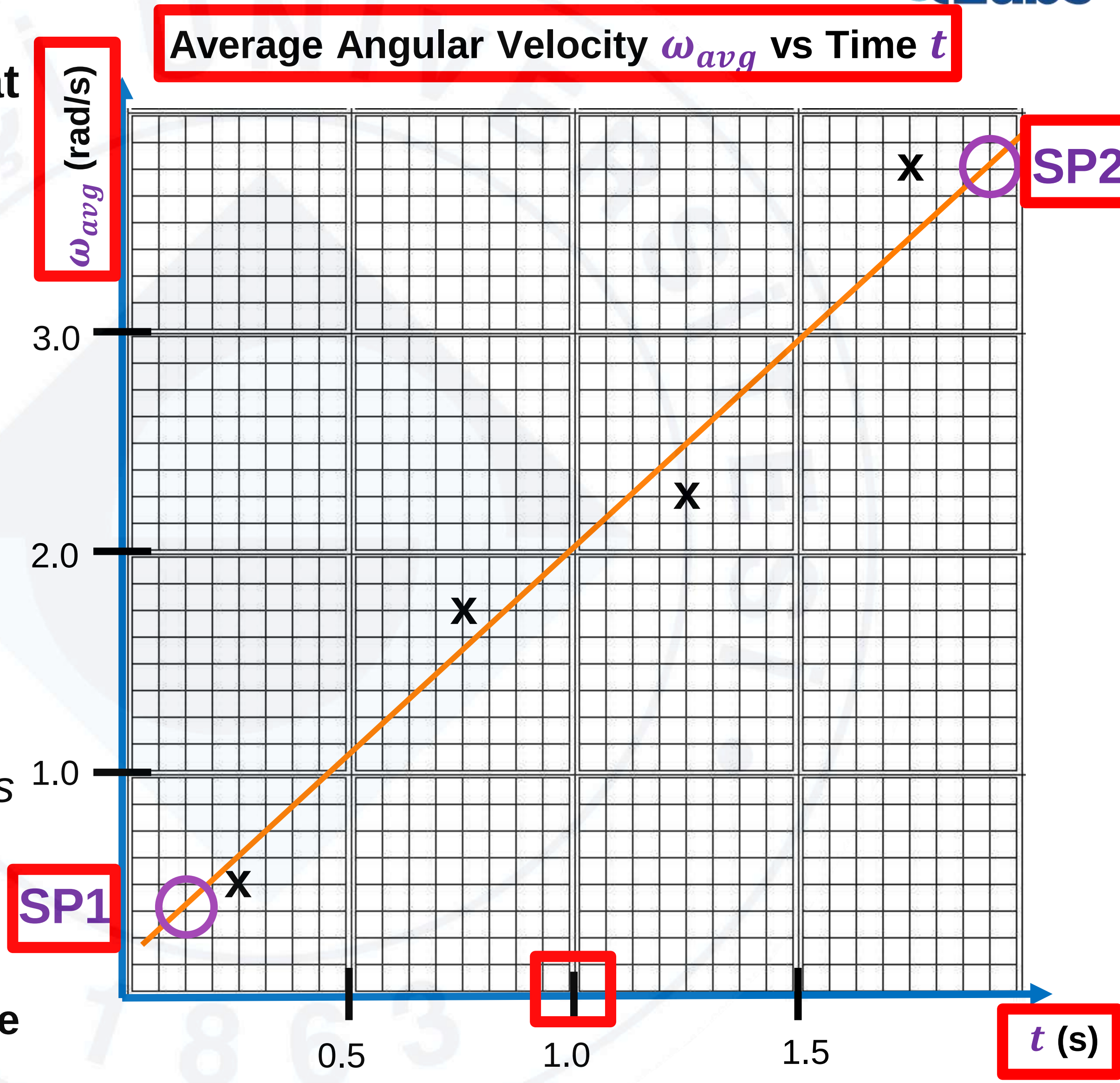
Fill units

t	Θ
( )	( )
0.0	0.071
0.5	0.523
1.0	1.409
1.5	2.717
…	…

Video3

TORQUE AND ANGULAR ACCELERATION – Experiment

- Scale your graph so that it covers most of the graph paper
- Add a title
- Add axes labels along with the units
- Add ticks
- Draw a best fit line.
- Select two slope points SP_1 and SP_2 on your best fit line. *(These should <u>not</u> be one of your data points and should be as apart from each other as can be to have a more significant slope value.)*
- Write down x and y axis values of these slope points to the part below the graph.



x : Data points

At last, we move on to this page.

Description / Symbol		Calculations (show each step)	Result	Dimension
SLOPE	=	$\dfrac{y_{SP2} - y_{SP1}}{x_{SP2} - x_{SP1}}$	………………	………………
Angular Acceleration α	=	**SLOPE value you have found above**	………………	………………
Moment of Inertia $I = \dfrac{mgr}{\alpha} - mr^2$	=	**Calculate using r, m, α and use $g = 9.81\ m/s^2$**	………………	………………
$I = mr^2\left[\dfrac{gT^2}{2h} - 1\right]$	=	**Calculate using m, r, t, h and use $g = 9.81\ m/s^2$**	………………	………………

%difference for I: $= \dfrac{\lvert I_{th} - I_{exp}\rvert}{I_{th} + I_{exp}} \times 100$