Boğaziçi University

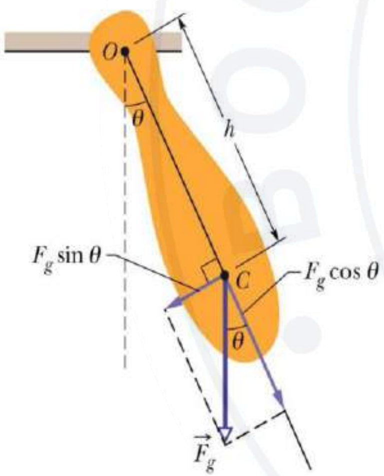
Introductory Phys Labs

ANGULAR HARMONIC MOTION

PHYL102

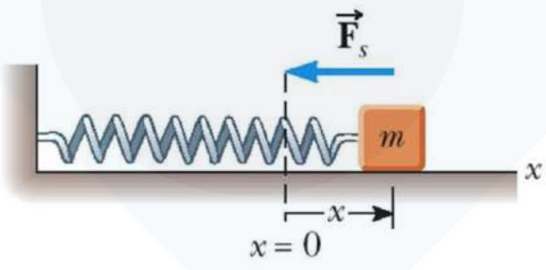
Simple harmonic motion is a special type of periodic motion where the restoring force on the moving object is directly proportional to the object's displacement magnitude and acts towards the object's equilibrium position.

Physical Pendulum



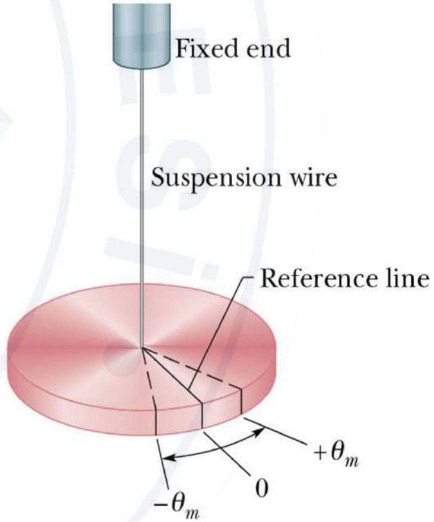
$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

Simple Harmonic Oscillator



$$T = 2\pi\sqrt{\frac{m}{k}}$$

Torsional Pendulum



$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

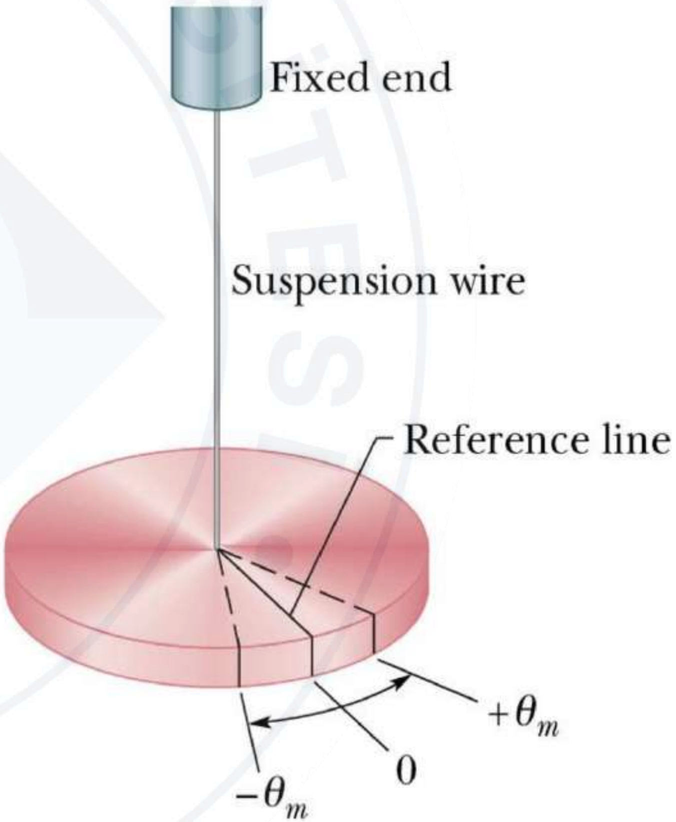
TODAY

ANGULAR SIMPLE HARMONIC MOTION

- The figure shows an angular version of simple harmonic oscillator. When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation.

- The particle or the body undergoes small angular displacement about mean position. This results, when the body under stable equilibrium is disturbed by a small external torque.

- A *torsional pendulum* consists of a disk suspended from a wire, which is then twisted and released, resulting in an oscillatory motion.



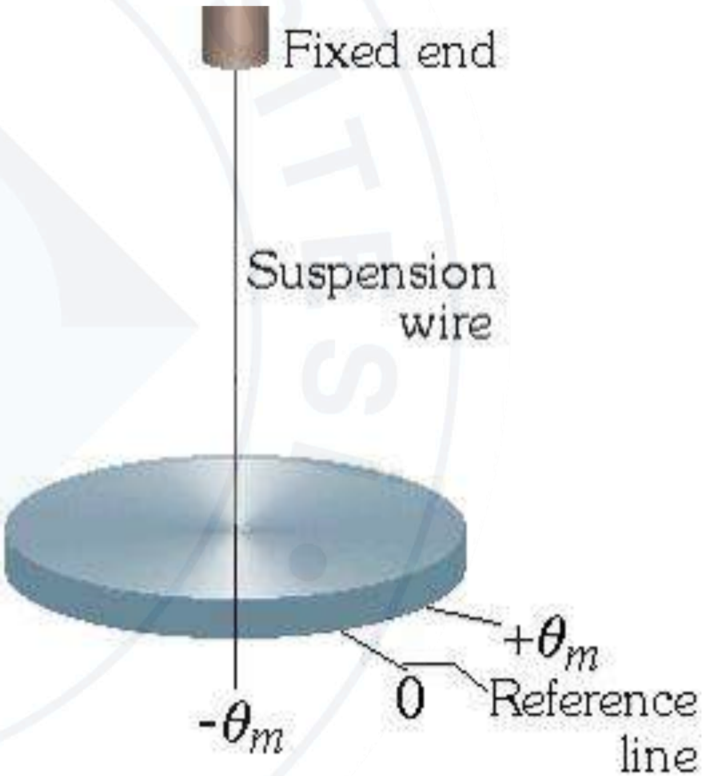
ANGULAR HARMONIC MOTION

The oscillatory motion is caused by a restoring torque which is proportional to the angular displacement,

$$\tau = I\alpha = -\kappa\theta$$

(Angular form of Hooke's law)

Where κ is the torsion constant that depends the length, diameter and material of suspension wire. **I** is the rotational inertia of the object about twisting axis and α is the angular acceleration of the object.



THEORY

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EQUATION OF MOTION:

$$\tau = I\alpha = -\kappa\theta$$

Angular acceleration $\alpha = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$

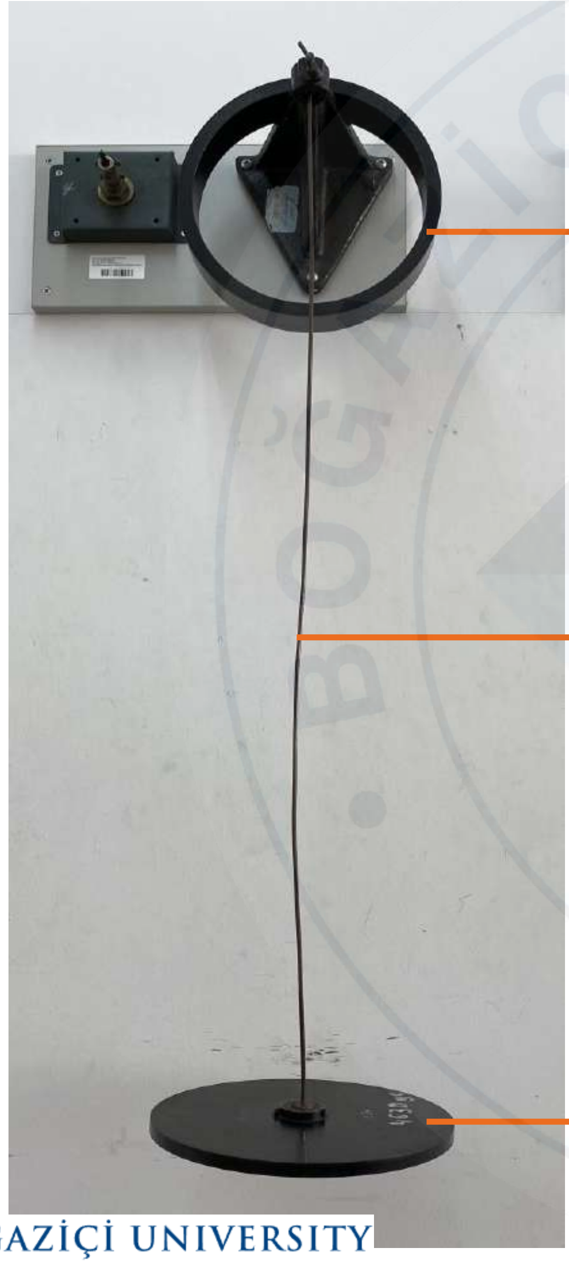
$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\kappa\theta \quad \rightarrow \quad \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\kappa}{I}\theta = 0 \quad \rightarrow \quad \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \omega^2\theta = 0$$

Where $\omega^2 = \frac{\kappa}{I}$ $\quad \omega = \sqrt{\frac{\kappa}{I}}$ $\quad T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

APPARATUS

ANGULAR HARMONIC MOTION



Ring

$$I_{ring} = \frac{1}{2} M\left[\left(\bar{R}_{inner}\right)^2 + \left(\bar{R}_{outer}\right)^2\right]$$

Torsional wire

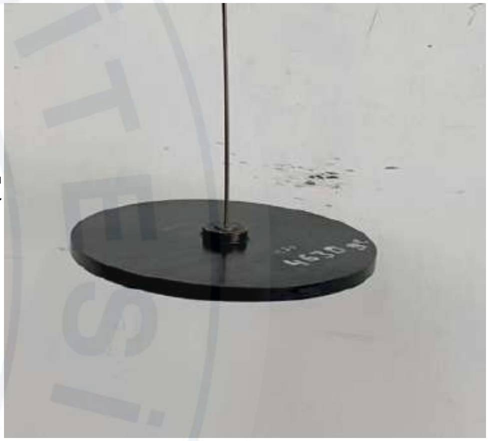
Disk

$$I_{disk} = \frac{MR^2}{2}$$

EXPERIMENT

OBJECTIVE: To find the moment of inertia of the ring and compare result with theoretical value

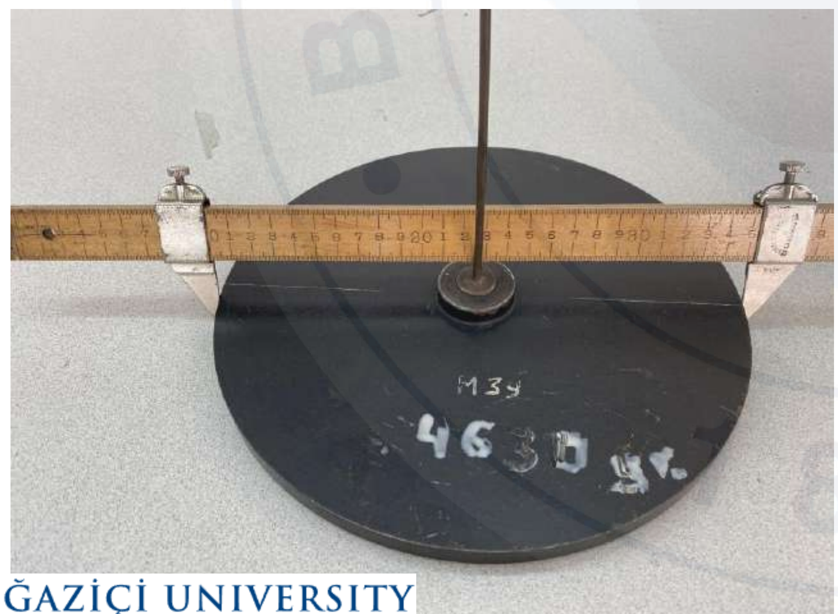
- ***PART I :***
  - Measure the period of disc
  - Calculate the moment of inertia of the disc
  - Find the torsion constant κ



- ***PART II :***
  - Measure the period of the disc and the ring together
  - Calculate the total moment of inertia of (disc + ring)
  - Find the experimental value of the moment of the inertia of the ring

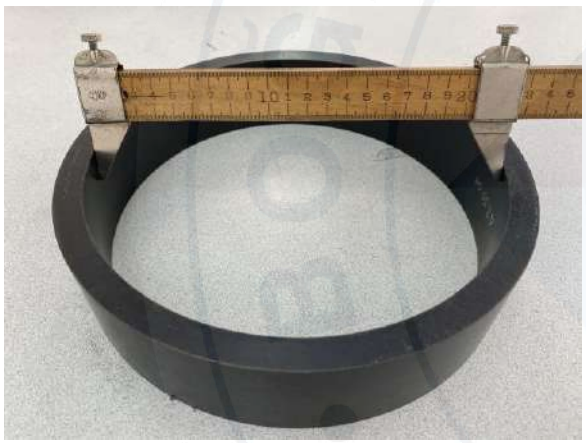
Theoretical value of the moment of inertia of the Disc:

$$I_{disk} = \frac{MR^2}{2}$$



Theoretical value of the moment of inertia of the Ring:

$$I_{ring} = \frac{1}{2} M \left[\left(\overline{R}_{inner} \right)^2 + \left(\overline{R}_{outer} \right)^2 \right]$$



PART I

Use your phone's stopwatch to measure the time, *t*, for the disc to complete 50 oscillations, and determine the mean period of oscillation *T*.

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad \rightarrow \quad I_{disc} = \frac{MR^2}{2}$$

compute κ torsion constant of the rod.

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PART II

Place the ring whose moment of inertia is unknown, on the disc. Use your phone's stopwatch to measure the time to complete 50 oscillations, and determine the mean period of oscillation *T*.

$$T = 2\pi\sqrt{\frac{I_{total}}{\kappa}}$$

Compute the sum of the moment of inertias of the disc and the ring.

PART II

$$T = 2\pi\sqrt{\frac{I_{total}}{\kappa}}$$

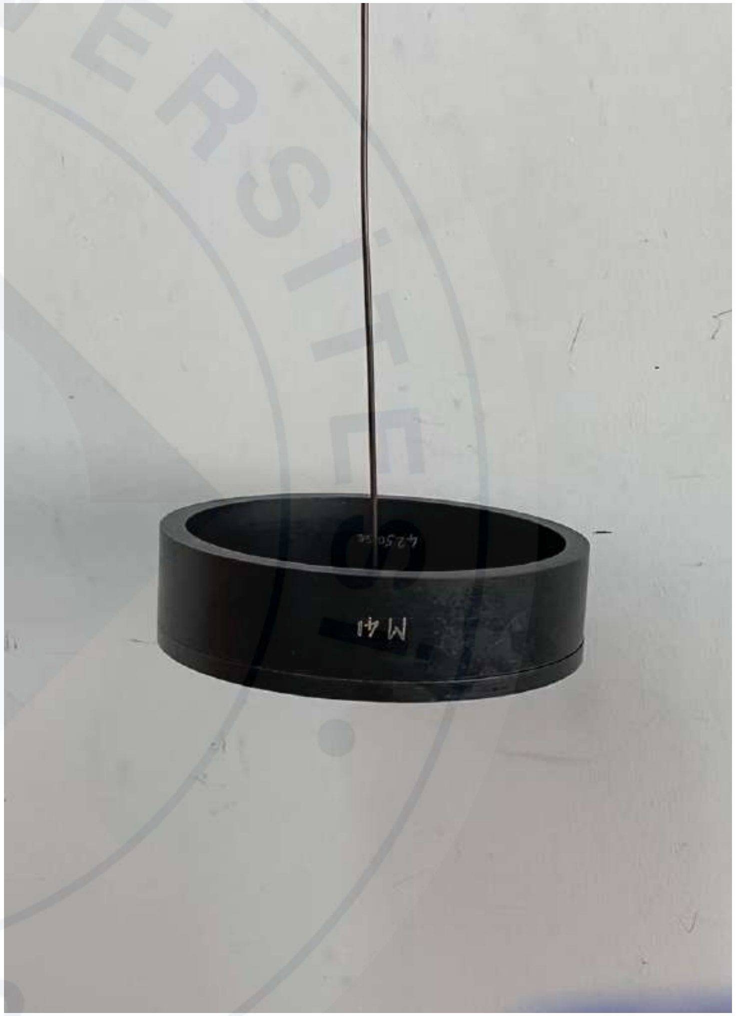
Total moment of inertia is equal to

$$I_{total} = I_{disc} + I_{ring}$$

Experimental value of the moment of inertia of the **ring**:

$$I_{ring} = I_{total} - I_{disc}$$

Compute the theoretical value of moment of inertia and determine the percentage error.



Part 1: Moment of Inertia of the Disk

Take the data and fill in the page for the DISC.

Description / Symbol			Value & Unit	# of Significant Figures
Time for 50 oscillations	t	=	....	....
Time for one oscillation	T	=	....	....
Diameter of the disc	D_{disc}	=	....	....
Radius of the disc	R_{disc}	=	....	....
Mass of the disc	M_{disc}	=	....	..............…

Take the data, fill in the page for the Disc + Ring.

Description / Symbol			Value & Unit	# of Significant Figures
Time for 50 oscillations	t^*	=	....	....
Time for one oscillation	T^*	=	....	....
Outer diameter of the ring	D_{outer}	=	....	....

Description / Symbol			Value & Unit	# of Significant Figures
Outer Radius of the ring	R_{outer}	=	....	....
Inner diameter of the ring	D_{inner}	=	....	....
Inner Radius of the ring	R_{inner}	=	....	....
Mass of the ring	M_{ring}	=	....	......

ANGULAR HARMONIC MOTION

- Use theoretical formula to calculate $I_{disc.}$
- Use period formula for torsion pendulum, calculate torsion constant.
- Use period formula and calculated torsion constant, determine $I_{total}=I_{disc}+I_{ring}$
- Find I_{ring}

CALCULATIONS:

Description		Calculations (show each step)	Result
Moment of Inertia of the disk (theoretical) I_{disc}	=	....	....
Torsion constant of the rod (emprical) κ	=	....	....

Description		Calculations (show each step)	Result
Total Moment of Inertia of the disk and the ring (emprical) I_{total}	=	....	....
Moment of Inertia of the Ring (emprical) $I_{ring\text{-}EV}$	=	....	....
Theoretical value of the Moment of Inertia of the ring $I_{ring\text{-}TV}$	=	....	....

% Error for the Moment of Inertia of the object: ....

CALCULATIONS:

- Use theoretical formula to calculate $I_{disc.}$
- Use period formula for torsion pendulum, calculate torsion constant.

Description		Calculations (show each step)	Result
Moment of Inertia of the disk (theoretical) I_{disc}	=	$I_{disc} = \frac{MR^2}{2}$	....
Torsion constant of the rod (emprical) κ	=	$T = 2\pi\sqrt{\frac{I}{\kappa}}$	....

- Use period formula and calculated torsion constant, determine $I_{total}=I_{disc} + I_{ring}$
- Subtract I_{disc} and find I_{ring}.
- Use theoretical formula and calculate I_{ring}.

Description	Calculations (show each step)	Result
Total Moment of Inertia of the disk and the ring (emprical) I_{total} =	$T = 2\pi\sqrt{\frac{I_{total}}{\kappa}}$ ↑ Find I_{total}	
Moment of Inertia of the Ring (emprical) $I_{ring\text{-}EV}$ =	$I_{ring} = I_{total} - I_{disc}$	
Theoretical value of the Moment of Inertia of the ring $I_{ring\text{-}TV}$ =	$I_{ring} = \frac{1}{2}M\left[\left(\bar{R}_{inner}\right)^2 + \left(\bar{R}_{outer}\right)^2\right]$	

% Error for the Moment of Inertia of the object: ....