Boğaziçi University

Introductory Phys Labs

FORCE BETWEEN PARALLEL PLATES

PHYL 201

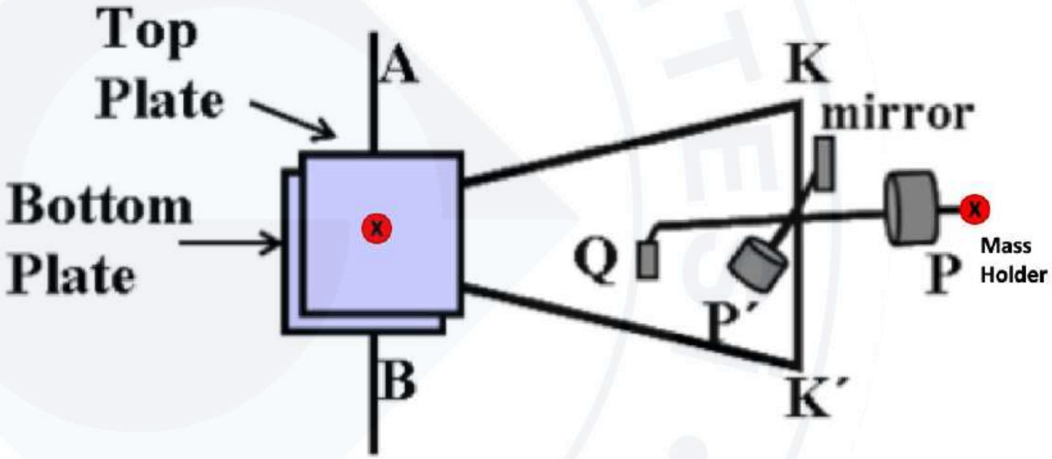
OBJECTIVE

- In this experiment, we will determine the permittivity of free space, ε_0, whose true value is 8.854×10^{-12} F/m (in SI).
- Since ε_0 is extremely tiny, we can't expect to measure it directly with ordinary lab equipment. For this purpose, we need a specially constructed apparatus (Coulomb balance).

THEORY

FORCE BETWEEN PARALLEL PLATES

The idea is to measure the force between two parallel plates, *F*, and relate it to the distance *d* between the plates. All other variables held fixed, F changes in a certain way with d.

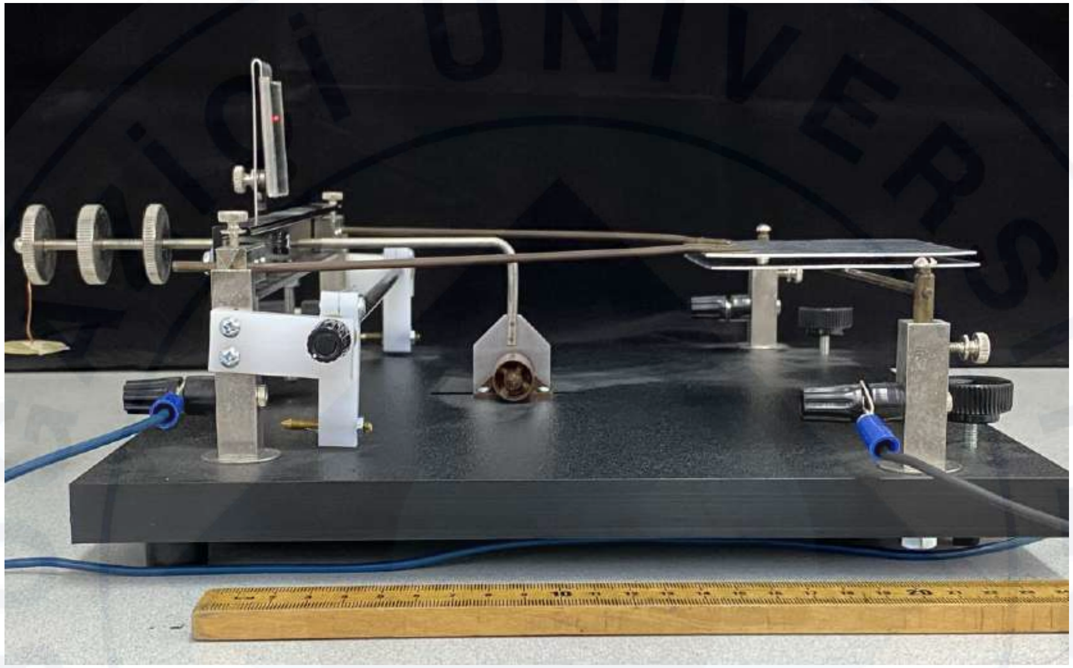


The attractive force between two parallel plates is, ideally,

$$F = \frac{e_0 A}{2d^2} V^2$$

- A: Area of the plates
- d: Distance between the plates.
- V: Potential applied across the plates.

FORCE BETWEEN PARALLEL PLATES



$$F = \frac{e_0 A}{2d^2} V^2$$

Here, A measures the area of the plates, d is their separation, V is the voltage across the plates, and ε_0 is vacuum permittivity. This force is ideal for several reasons: plates are not exactly parallel, there are fringe fields, etc. Nevertheless, by keeping the plates close together, we can well approximate this relation.

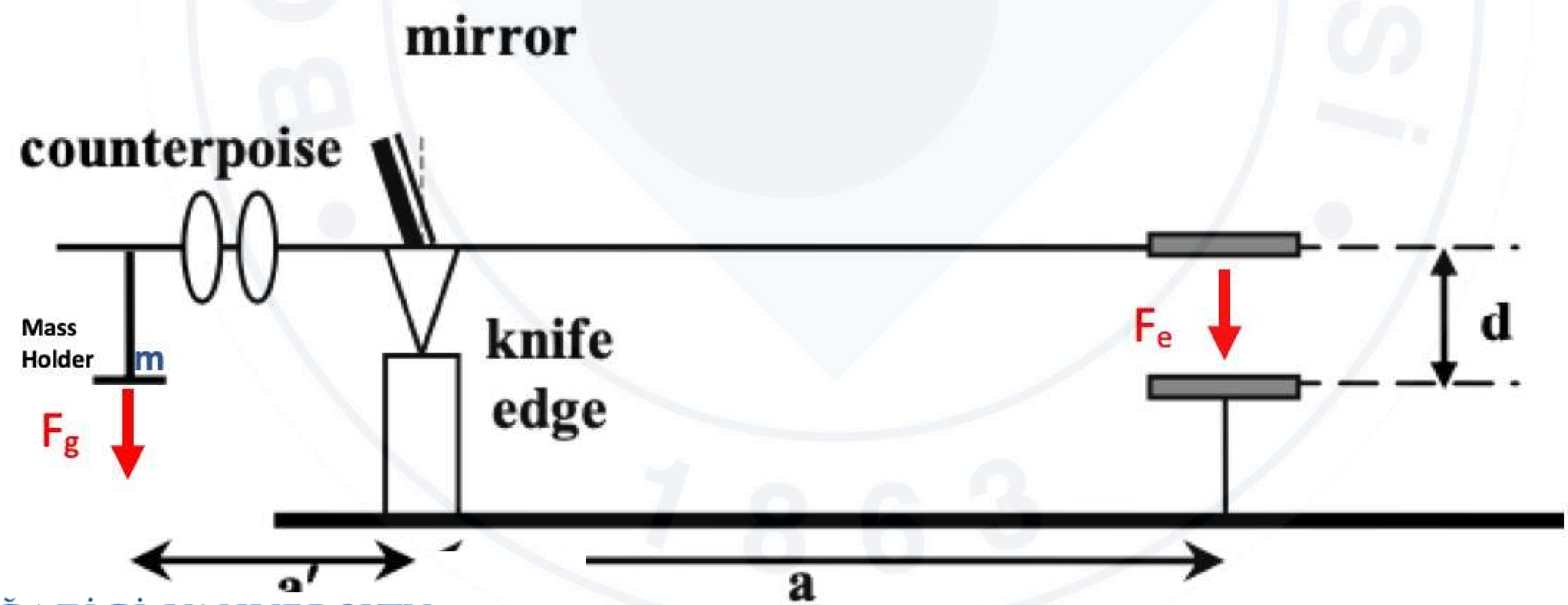
FORCE BETWEEN PARALLEL PLATES

When we mechanically balance the electric force with the gravitational force on the ends of the apparatus, that is when we equate the two torques, we get the relation

$$\Sigma \tau = F_g a' - F_e a = 0$$

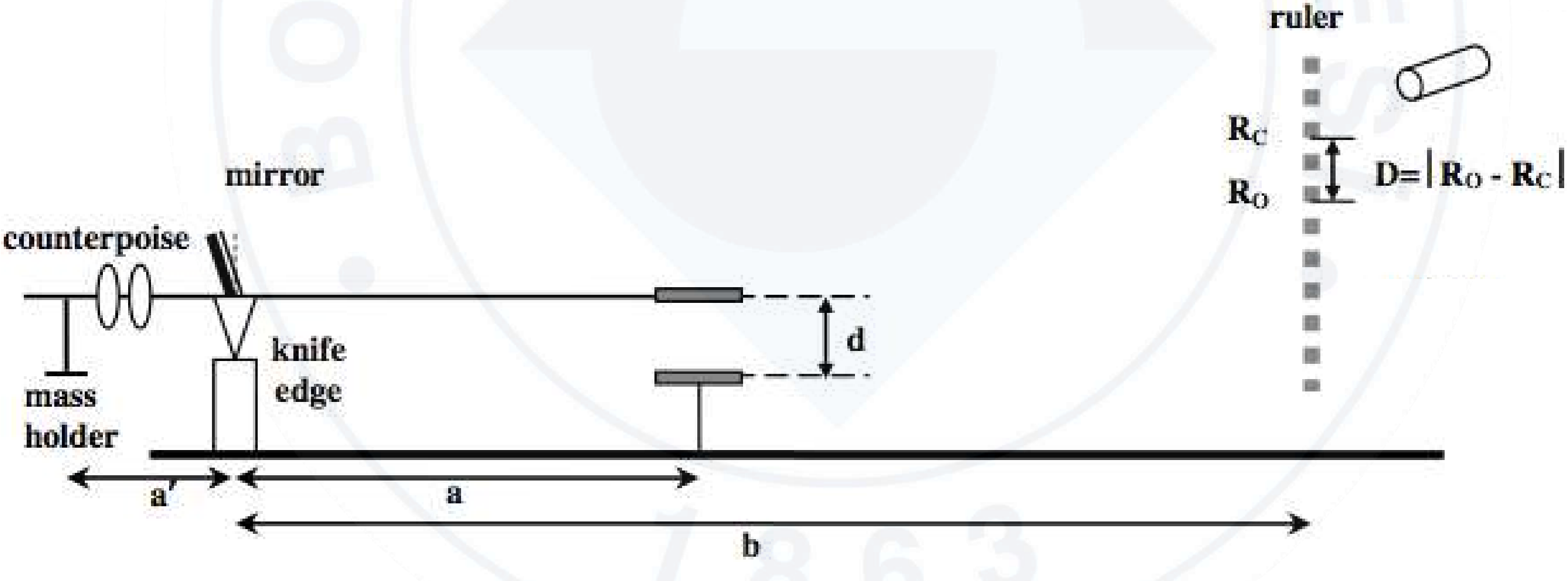
$$F_g a' = F_e a,$$

where $F_g = mg$ is the gravitational force due to mass m on the holder, F_e is the electrostatic force between the plates, a′ and a are the moment arms as can be seen in the diagram below. Knowing F_g, we can determine F_e.



Then, writing F_e as kV^2, we see a linear relationship between F_e and V^2 with slope k. In this experiment we make use of this linear relationship to determine k, and in turn ε_0.

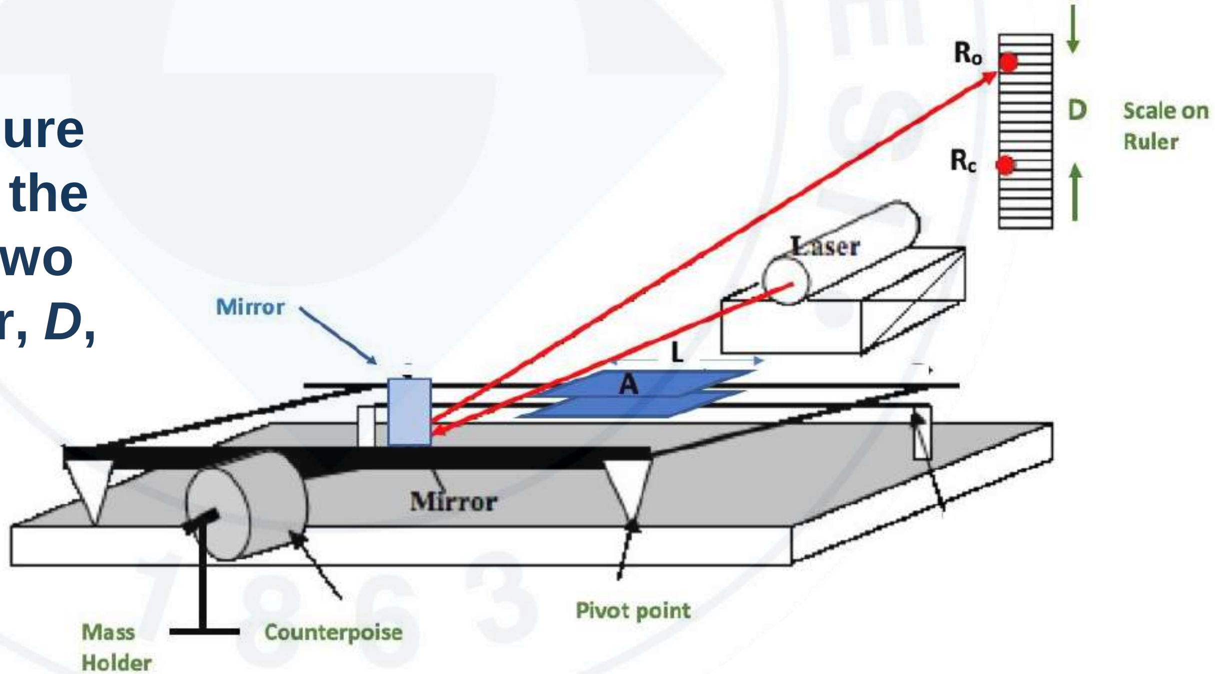
$$e_0 = k\frac{2d^2}{A}$$



To determine *d*, we don't use an ordinary ruler for two reasons. First, we don't want to disturb the delicate balance, and second, we want better precision. Thus, we determine *d* indirectly by means of a **laser-mirror-ruler** system. The laser targets the mirror fixed on the balance, and reflects back and falls on the ruler attached to the laser. Two positions of the plates correspond to two readings of the laser spot on the ruler.

One can show using pure geometry that *d* and the difference between two readings on the ruler, *D*, are related as

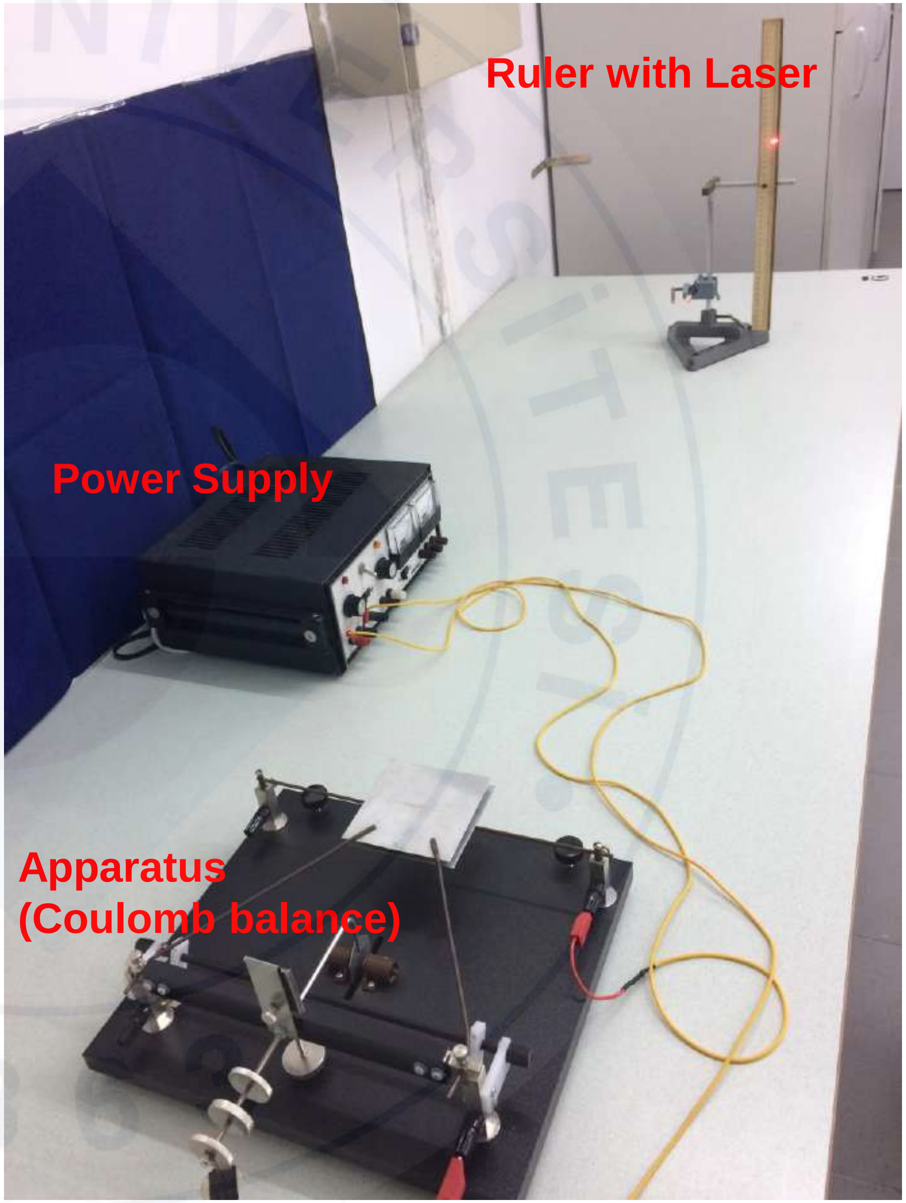
$$d = Da / 2b$$



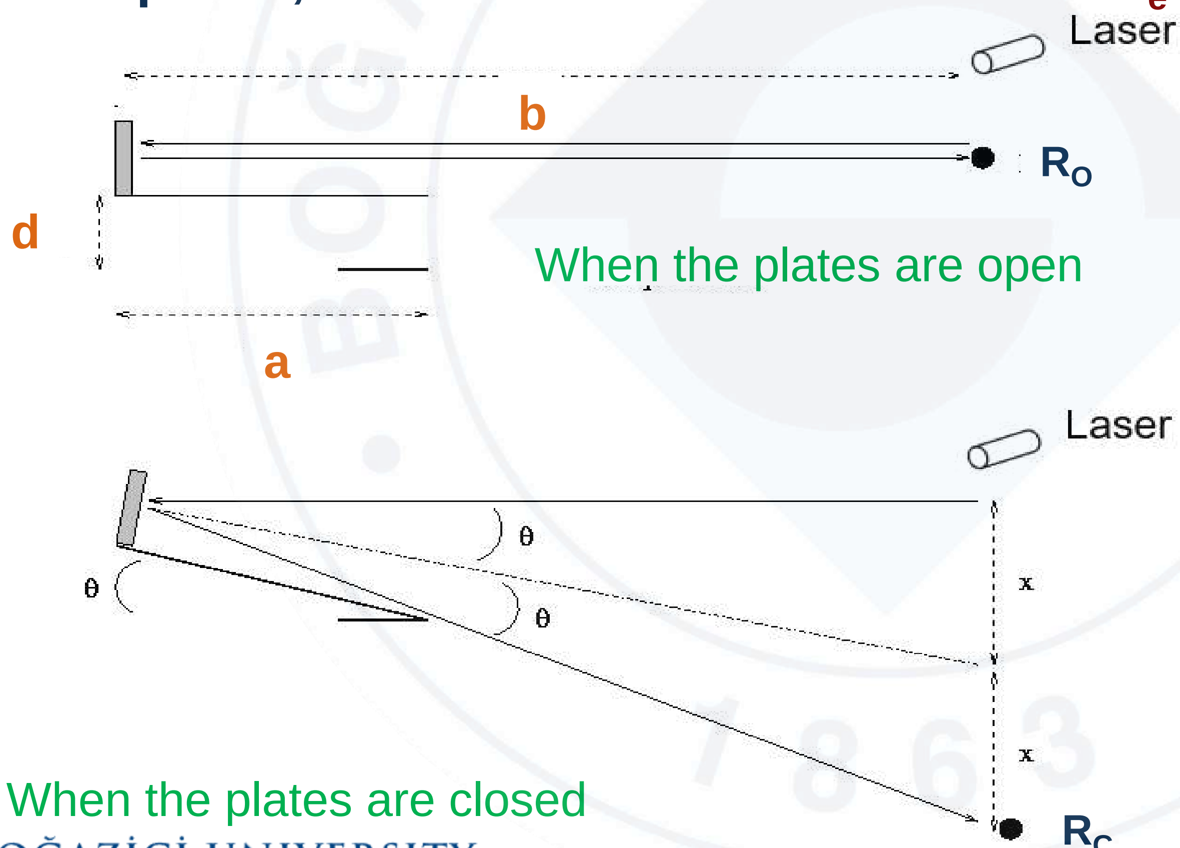
SETUP AND EXPERIMENT

In this experiment, we will use

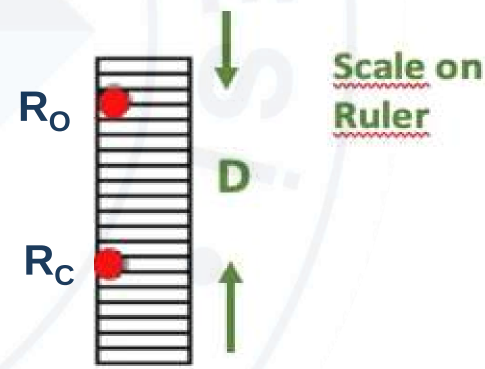
- High voltage power supply,
- A specially constructed apparatus (Coulomb balance) with a mirror on,
- A vertical ruler with a laser pointer attached,
- Small weights, in milligram order (will be shown).



The laser beam hits the mirror on the apparatus, reflects, and falls back on the vertical scale. Reading the position of the laser spot on the scale, we can indirectly determine the separation between the plates, *d*. Caution: d must be small for $F_e = kV^2$ relation to hold.



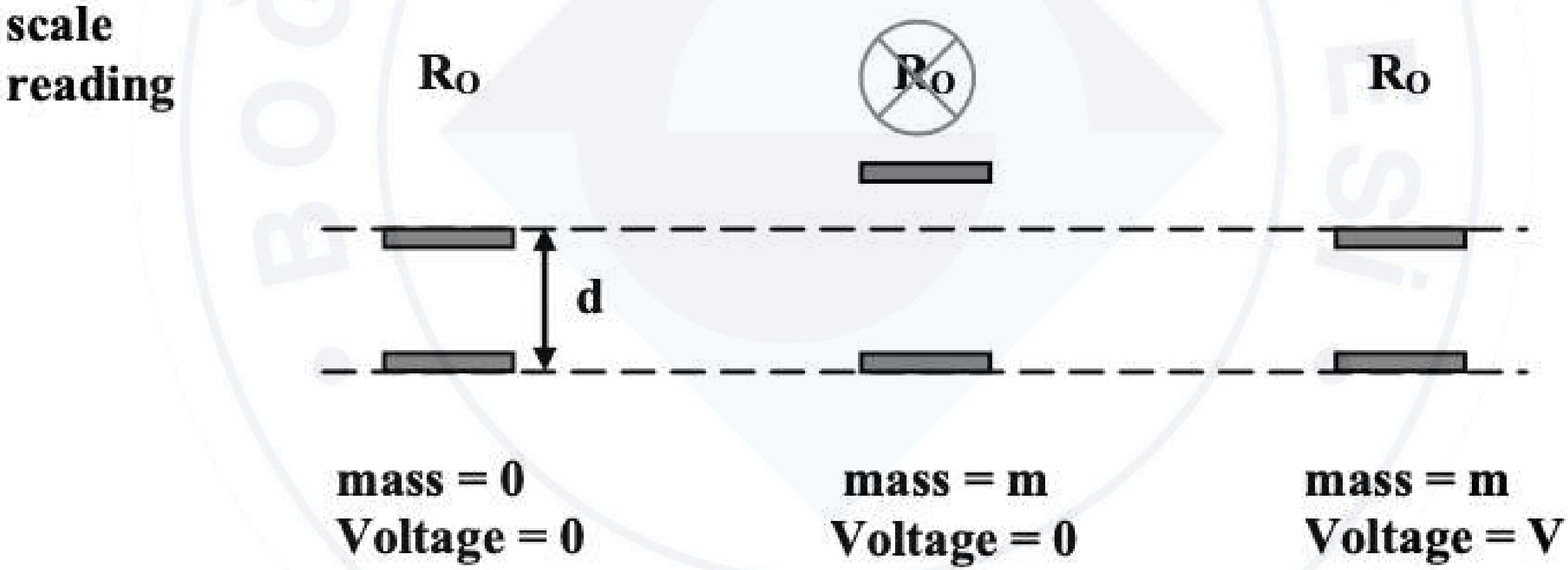
Laser spots on ruler:



$$D = |R_O - R_c|$$

d = Da / 2b

After we balance the apparatus, we put some mass *m* on the holder, and try to restore the original balance by applying some potential *V* across the plates.



FORCE BETWEEN PARALLEL PLATES

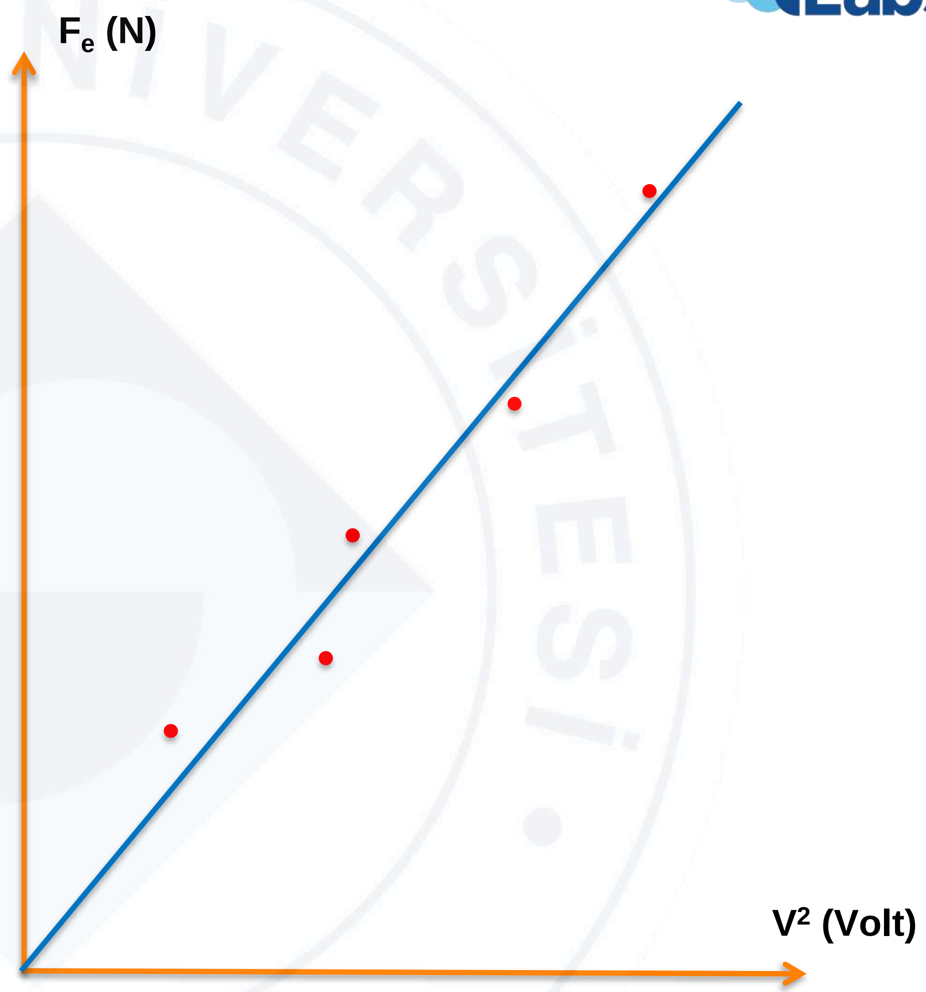
When balance is restored, F_e, can be written as,

F_e = mg (a’ / a),

which is also equal to

$F_e = kV^2$

If we plot F_e against V^2, we expect to get a straight line with slope k. Then we can extract $\varepsilon_{0,EV}$ from k (EV stands for Evaluated Value).

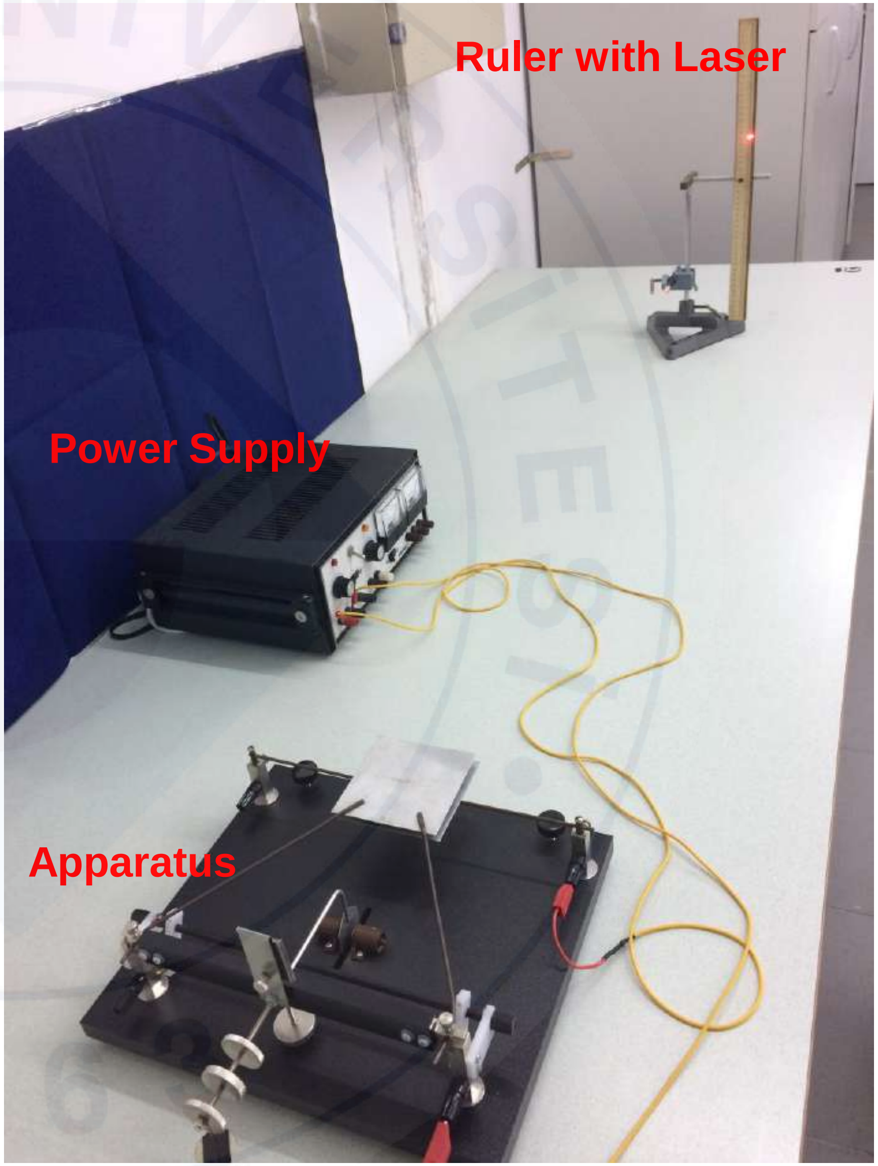


$$\varepsilon_{0,EV} = slope \times \frac{2d^2}{A}$$

SETUP PARAMETERS

An overview

What to measure: **Length of the lever arm, a; Lever arm for the weight, a'; Distance from the mirror scale to the ruler, b; Length of the square plate, L; Reading when the plates are open, R_o; Reading when the plates are closed, Rc;**



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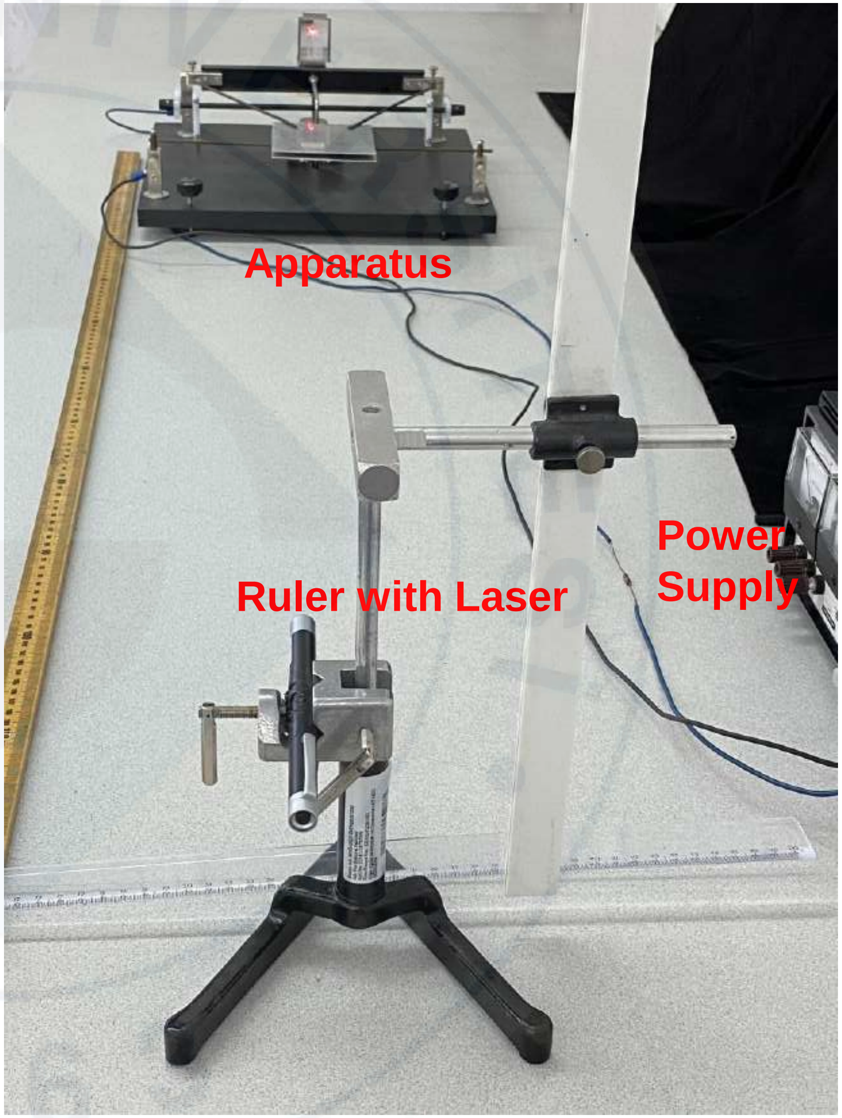
An overview

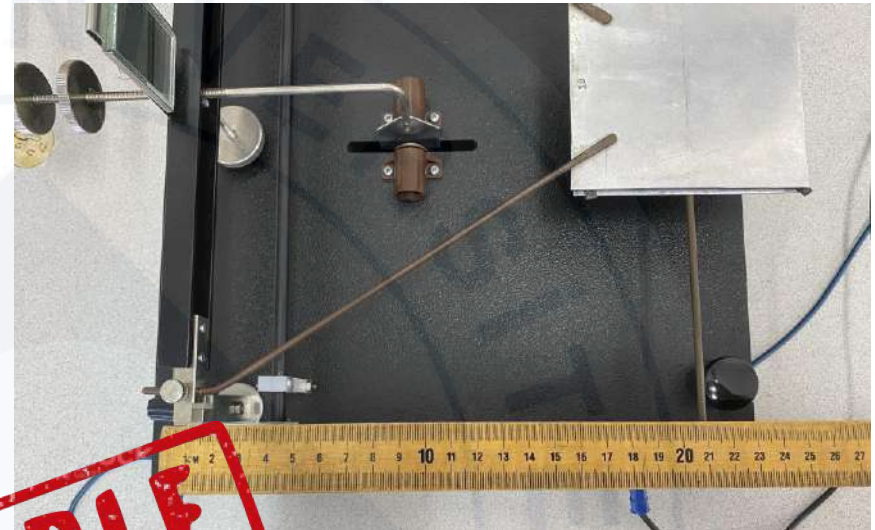
What to calculate: **Area of the plate, A; Difference in readings, D = $|R_c - R_o|$; Separation between the plates, d = D x (a/2b).**



Length of the Lever Arm, a

Read the length of the lever arm, a, to the millimeter.

Record your reading in meters.

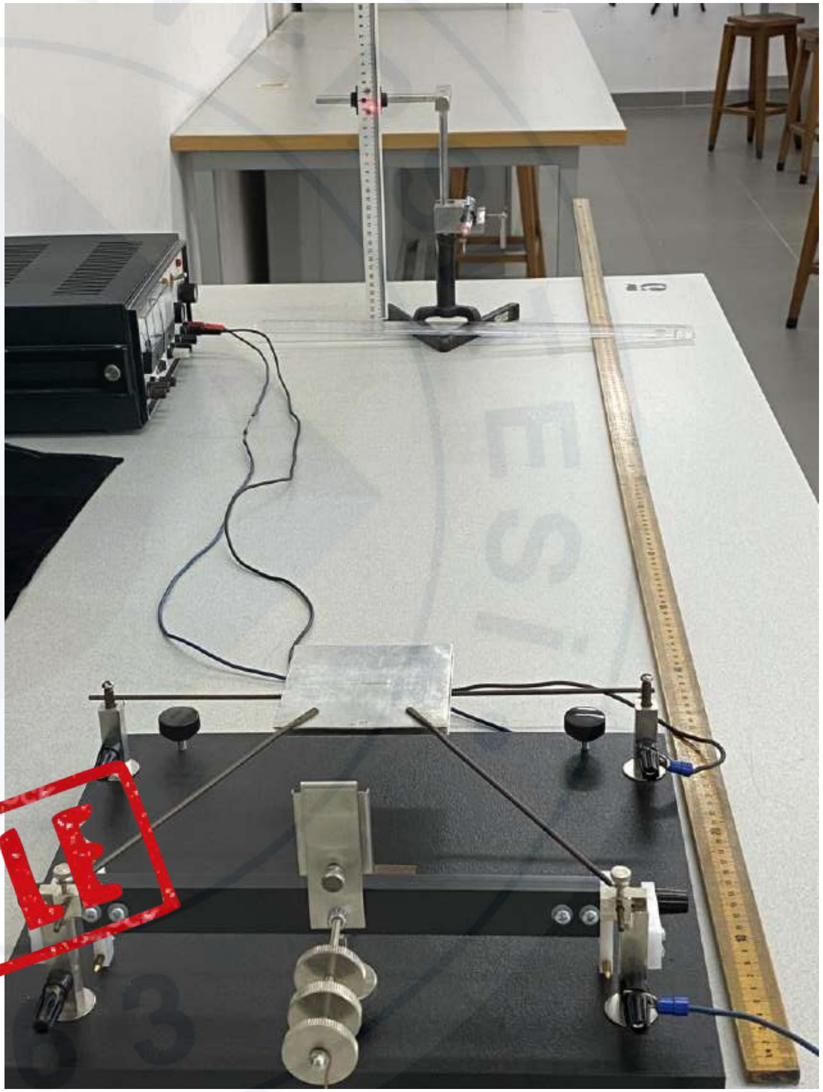


Lever Arm for the weight, a’

Read the length of the lever arm for the weight, a’, to the millimeter. Record your reading in meters.

Distance from the mirror scale to the ruler, b

Read the distance from the mirror scale to the ruler, b, to the millimeter. Record your reading in meters.

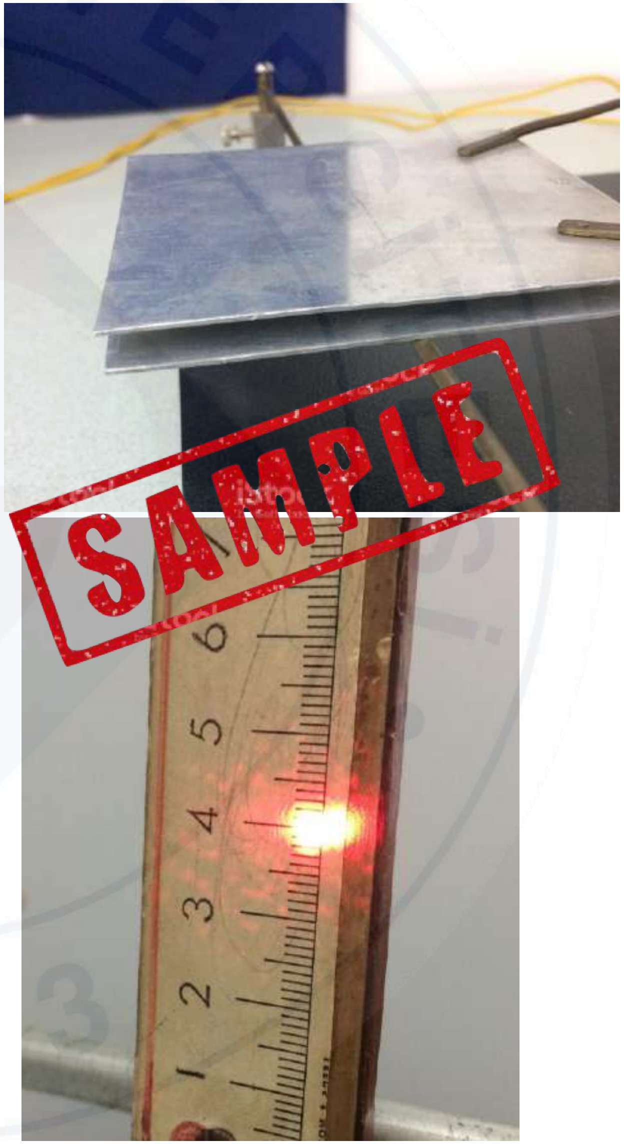


Length of the Plate, L

Read the length of the plate, L, to the millimeter. Record your reading in meters.

Reading when the Plates are Open, R_o

Read where the spot hits. Since the spot covers a few millimeters, the best estimate will be the arithmetic average of its bounds. You may either read where the spot fades and take their average, or estimate the midpoint of the spot and record it.



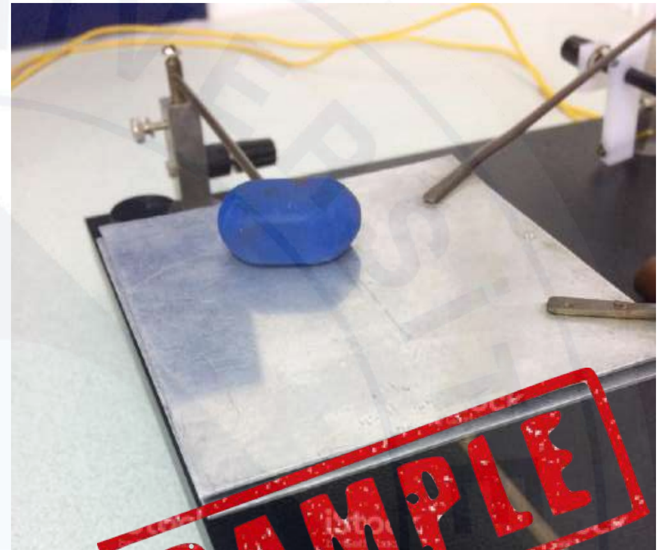
FORCE BETWEEN PARALLEL PLATES

Reading when the Plates are Closed, R_c

We put an object to close the plates first.

Read where the spot hits. Since the spot covers a few millimeters, the best estimate will be the arithmetic average of its bounds. You may either read where the spot fades and take their average, or estimate the midpoint of the spot and record it.

Caution: Here the spot hits below 0. You should record it as -0.8cm, for example.

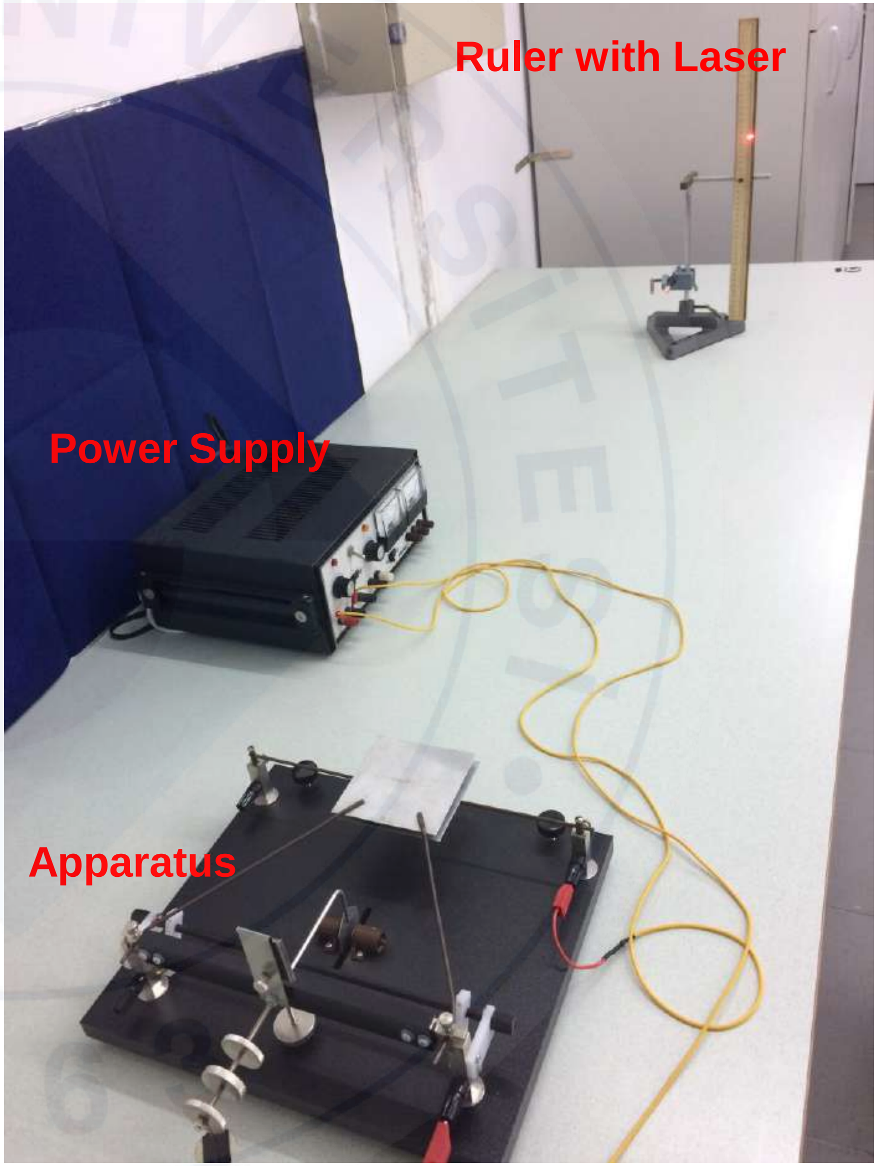


BREAKING AND RESTORING BALANCE

An overview

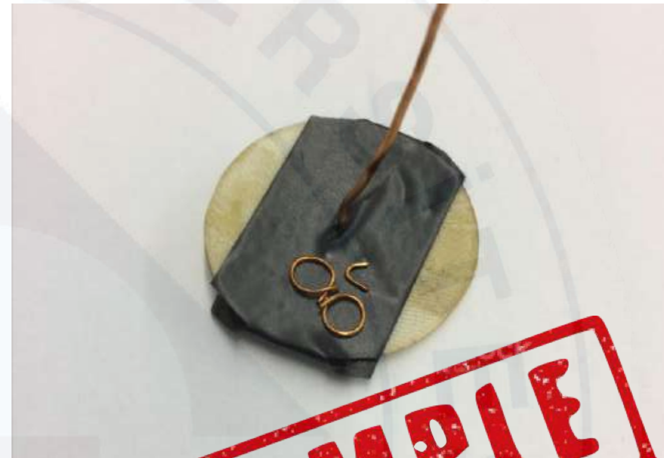
What to read: **Masses on the holder, m; Applied potential, V.**

What to calculate: **The electric force between the plates, $F_e = mga'/a$; Square of the potential, V^2.**



Breaking the Balance

We now hang a certain amount of mass, such as shown on the right, and break the balance, as you can see.



v: 5mg (each)

U: 10mg (each)

O: 20mg (each)

These values are significant to the last digit.

Restoring the Balance

Then we apply some voltage across the plates, so that they pull on each other and restore the balance.

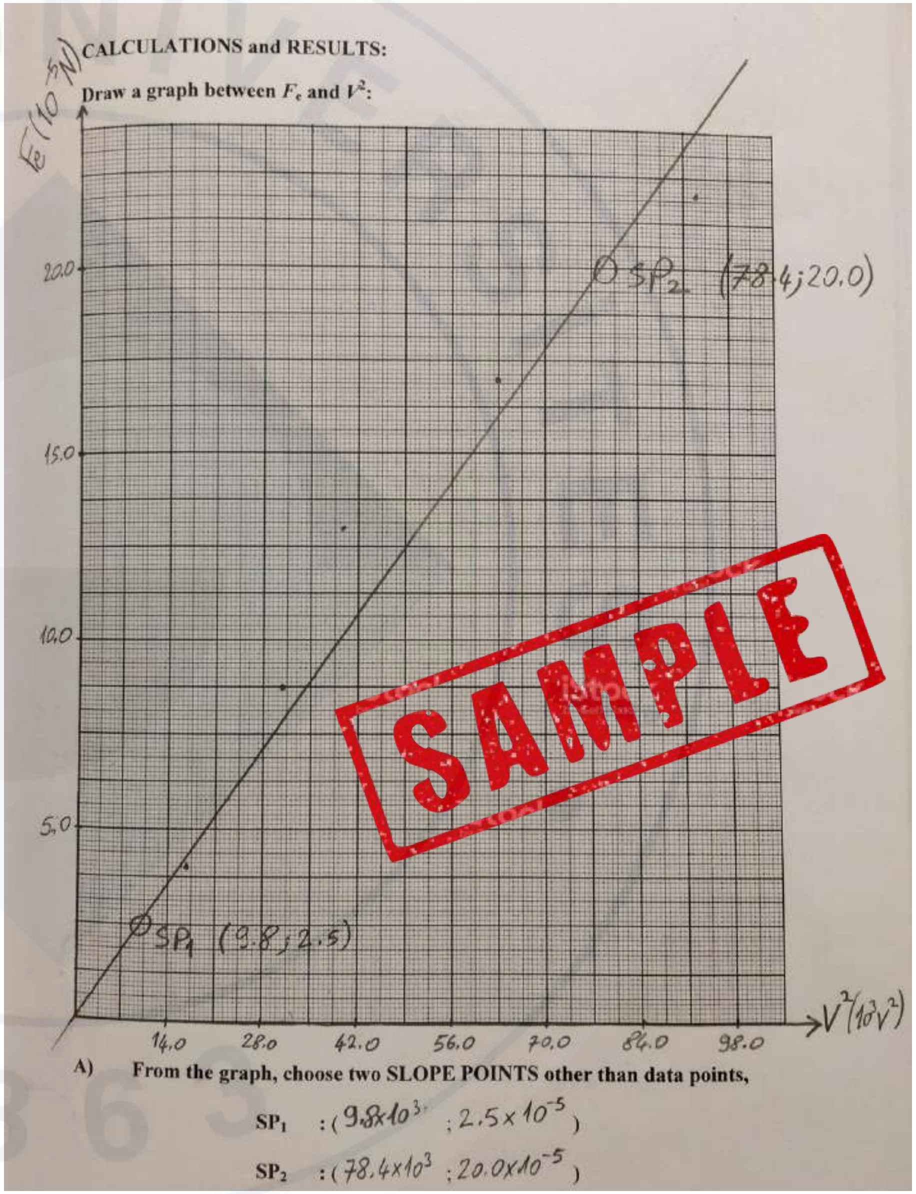
Note that the spot hits exactly where the initial balance was, R_o.

Caution: Read the red scale.

We break and then restore the balance 5 times with five different masses and 5 corresponding voltages.

For each pair m and V, We calculate $F_e = mga'/a$, and V^2.

Then we plot F_e against V^2 on a grid, fit the plot to a straight line, and determine its slope.



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From this slope, we determine $\varepsilon_{0,EV}$ as

$$\varepsilon_{0,EV} = slope \times \frac{2d^2}{A}$$

Finally, we determine the percentage error for ε_0, by comparing the true value and the experimental value of it.