

## SIMPLE HARMONIC MOTION

PHYL102

## SIMPLE HARMONIC MOTION

Simple harmonic motion is a special type of periodic motion where the restoring force on the moving object is directly proportional to the object's displacement magnitude and acts towards the object's equilibrium position.

## Physical Pendulum



$$
T=2 \pi \sqrt{\frac{I}{M g h}}
$$

Previous Experiment

Simple Harmonic Oscillator

$T=2 \pi \sqrt{\frac{m}{k}}$
TODAY

Torsional Pendulum


$$
T=2 \pi \sqrt{\frac{I}{\kappa}}
$$

5th Experiment

## SIMPLE HARMONIC MOTION

Simple harmonic motion is governed by a restorative force. For a spring-mass system, such as a block attached to a spring, the spring force is responsible for the oscillation. The force required to stretch such a system is directly proportional to the extension of the spring

## Hooke's law

$$
F=-k x
$$

Frequency of oscillation

$$
\omega=\sqrt{\frac{k}{m}}
$$


e 2006 Brooks/Cole - Thomson

Period of oscillation

$$
T=\frac{2 \pi}{\omega} \quad \rightarrow \quad T=2 \pi \sqrt{\frac{m}{k}}
$$

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## THEORY

## SIMPLE HARMONIC MOTION

SETUP:
The system shown in the figure below will be exhibiting a periodic motion due to the variable restoring force in the spring.


Net Forces

$$
T-k x=m_{c} a \quad m_{0} g-T=m_{0} a
$$

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## SIMPLE HARMONIC MOTION

EQUATION OF MOTION:
$T-k x=m_{c} a$
$m_{0} g-T=m_{0} a$
$m_{0} g-k x=\left(m_{0}+m_{c}\right) a=m_{\text {total }} a$
$m_{0} g-k x=m_{t} \frac{\mathrm{~d}^{2} x}{\mathrm{~d} t^{2}}$
$\frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}+\frac{k}{m_{t}} x=\frac{m_{0} g}{m_{t}} \quad$ Period of oscillation is given by $\omega^{2}=\frac{k}{m_{t}}$
$\frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}+\omega^{2} x=\frac{m_{0} g}{m_{t}}$
Solution of this equation will be :
$x(t)=\frac{m_{0} g}{k}-A \cos (\omega t+\delta)$

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## SIMPLE HARMONIC MOTION

Derivative of the position with respect to time will yield the velocity as a function of time and the second derivative will give us the acceleration:

$$
\begin{aligned}
& x(t)=\frac{m_{0} g}{k}-A \cos (\omega t+\delta) \\
& \frac{\mathrm{d} x(t)}{\mathrm{d} t}=v(t)=A \omega \sin (\omega t+\delta) \quad \rightarrow \quad v_{\max }=A \omega \\
& \frac{\mathrm{~d}^{2} x(t)}{\mathrm{d} t^{2}}=a(t)=A \omega^{2} \cos (\omega t+\delta) \quad \rightarrow \quad a_{\max }=A \omega^{2}
\end{aligned}
$$

Notice that when the magnitude of the velocity reaches its maximum the acceleration becomes zero and vice versa.

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## APPARATUS

## SIMPLE HARMONIC MOTION

Position sensor stands just behind the car. Data Logger is attached to position sensor to record the data.


## EXPERIMENT

## SIMPLE HARMONIC MOTION

OBJECTIVE:

To observe simple harmonic motion in the system that exhibits a periodic motion due to variable restoring force in the spring

To calculate physical parameters associated with simple harmonic motion like $\boldsymbol{\omega}, \boldsymbol{k}, \boldsymbol{T}, \boldsymbol{v}_{\max }$ in order to find the mass of the car by measuring position of the car at $\Delta t=0.1$ second intervals.


## SIMPLE HARMONIC MOTION

The position sensor is placed at a certain distance far from the car. The data logger is started at the desired rate, 10 per second, and the car is released. Position of the car in each 0.1 second is recorded in the data logger.


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## SIMPLE HARMONIC MOTION

The car first accelerates $(\boldsymbol{m g} \boldsymbol{>} \boldsymbol{k} \boldsymbol{x}$ ), attains its maximum velocity where $\boldsymbol{m} \boldsymbol{g}=\boldsymbol{k} \boldsymbol{x}$, then decelerates ( $\boldsymbol{m g} \boldsymbol{<} \boldsymbol{k} \boldsymbol{x}$ ) and finally stops to come back.


When the car is at maximum velocity:

$$
\mathrm{m}_{0} \mathrm{~g}=\mathrm{kx}_{\mathrm{eq}} \quad \mathrm{k}=\mathrm{m}_{0} \mathrm{~g} / \mathrm{x}_{\mathrm{eq}}
$$

$$
\begin{aligned}
& x_{0}, t=0.0 \mathrm{~s} \\
& x_{0}+x_{1}, t=0.1 \mathrm{~s} \\
& x_{0}+x_{2}, t=0.2 \mathrm{~s} \\
& x_{0}+x_{3}, t=0.3 \mathrm{~s} \\
& x_{0}+x_{4}, t=0.4 \mathrm{~s} \\
& x_{0}+x_{5}, t=0.5 \mathrm{~s} \\
& x_{0}+x_{6}, t=0.6 \mathrm{~s} \\
& x_{0}+x_{7}, t=0.7 \mathrm{~s} \\
& x_{0}+x_{8}, t=0.8 \mathrm{~s} \\
& x_{0}+x_{9}, t=0.9 \mathrm{~s} \\
& x_{0}+x_{10}, t=1.0 \mathrm{~s} \\
& x_{0}+x_{11}, t=1.1 \mathrm{~s}
\end{aligned}
$$

## SIMPLE HARMONIC MOTION

Data Logger is attached to the position sensor and it records the position of the car at $\Delta t=0.1 \mathrm{~s}$ intervals.

The first few data values will be unchanged because although Data Logger collects the data, car is not released, it is not moving. This data is $\mathrm{X}_{0}$.

Stop taking data when the car turns back.


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- Determine the initial distance $\mathrm{x}_{0}$ from data logger
- Subtract $\mathrm{x}_{0}$ from each data logger reading value to determine real displacement of the car, x .

- Determine $\Delta x$ for each interval
- Calculate the average velocity for each interval. $\Delta t=0.1 \mathrm{~s}$


## SIMPLE HARMONIC MOTION

- Plot the average velocity versus time and the total displacement versus time curves.
- Mark your velocities at the middle of each time interval.
- From the velocity versus time graph,
- determine the maximum velocity, $\mathrm{v}_{\text {max, }}$ which corresponds to zero acceleration
- the corresponding time $t_{e q}$
- the period, T.
- From the displacement versus time graph, determine the equilibrium displacement $x_{\text {eq }}$ corresponding time $\mathrm{t}_{\text {eq. }}$

What you need to determine $\mathrm{k}, \mathrm{T}, \omega, \mathrm{A}, \mathrm{x}_{\max }, \mathrm{a}_{\max }, \mathrm{m}_{\mathrm{tot}}, \mathrm{m}_{\mathrm{car}}$

$$
\begin{aligned}
& x(t)=\left(m_{0} g / k\right)-A \operatorname{Cos}(\omega t+\delta), \\
& \quad \text { where } \operatorname{Cos}(\omega t+\delta)=0 \text {, then } x(t)=x_{e q}=m_{0} g / k \\
& \quad \text { where } \operatorname{Cos}(\omega t+\delta)=-1, \text { then } x(t)=x_{\max }=\left(m_{0} g / k\right)+A
\end{aligned}
$$

$\mathrm{v}(\mathrm{t})=\mathrm{A} \omega \operatorname{Sin}(\omega \mathrm{t}+\delta)$,
where $\operatorname{Sin}(\omega \mathrm{t}+\delta)=1$, then $\mathrm{v}(\mathrm{t})=\mathrm{v}_{\max }=\mathrm{A} \omega$

$$
\begin{aligned}
& \mathrm{a}(\mathrm{t})=\mathrm{A} \omega^{2} \operatorname{Cos}(\omega \mathrm{t}+\delta), \\
& \quad \text { where } \operatorname{Cos}(\omega \mathrm{t}+\delta)=1 \text {, then } \mathrm{a}(\mathrm{t})=\mathrm{a}_{\max }=\mathrm{A} \omega^{2}
\end{aligned}
$$

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