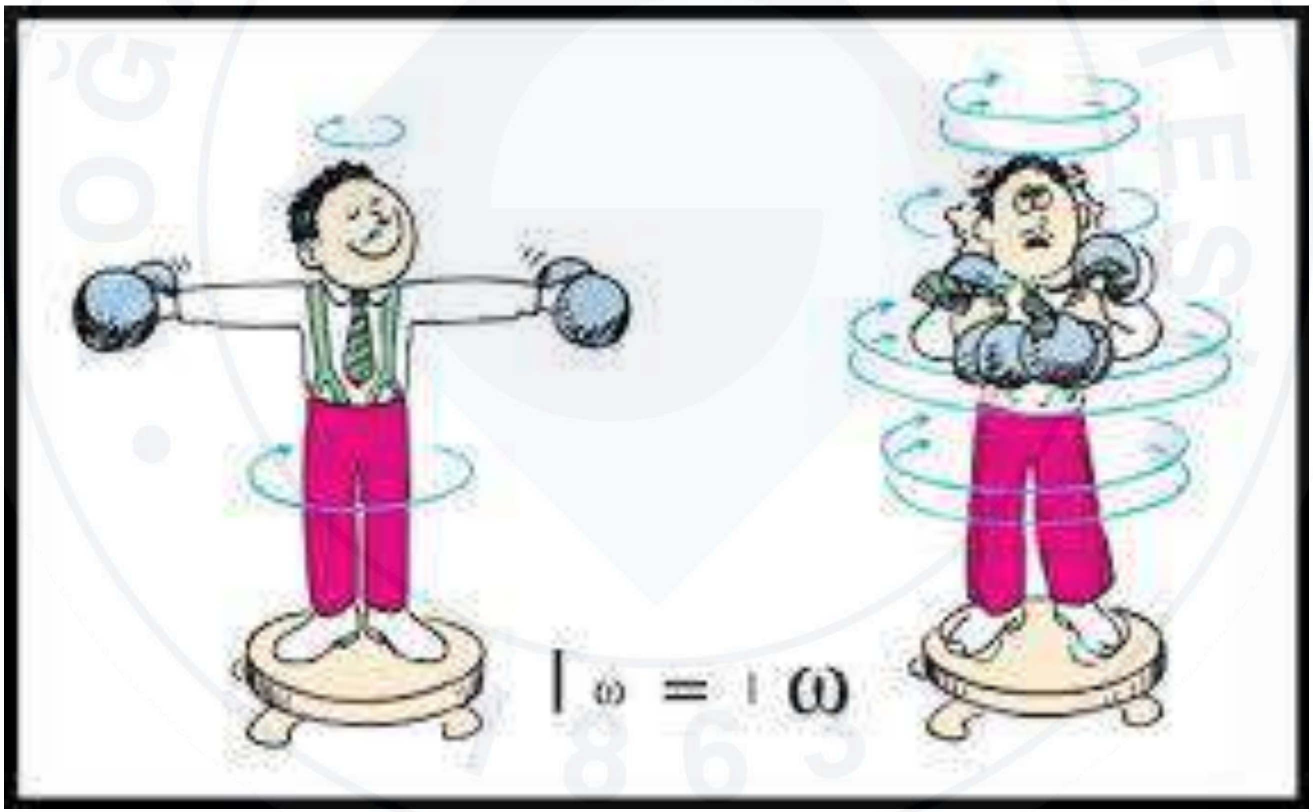
Boğaziçi University

Introductory Phys Labs

ROTATIONAL INERTIA

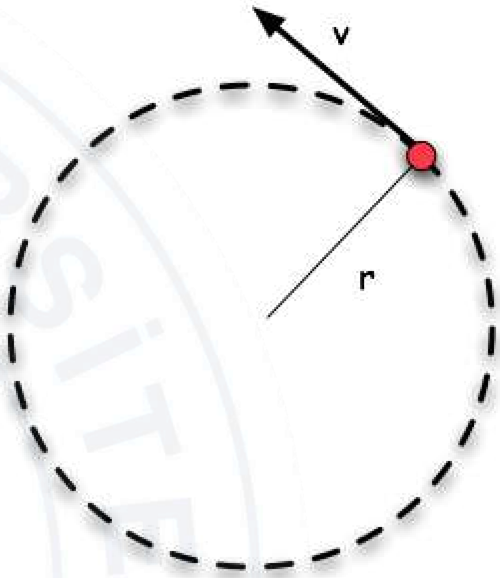
PHYL 101

Rotational inertia *I*, or more commonly known as moment of inertia, is a useful definition when dealing with objects rotating around a fixed axis. It quantifies the resistance of an object to angular acceleration just as mass quantifies the resistance of an object to linear acceleration.



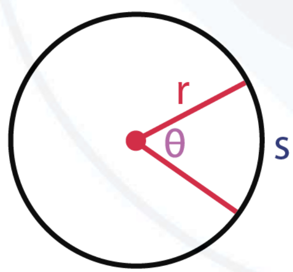
THEORY

Let us start by discussing rotational energy K_R of a point particle rotating around an axis with velocity v at a fixed radial distance r as given in the figure. Let us denote the mass of the particle m. Then K_R is simply;



$$K_R = \frac{1}{2} m v^2$$

Now, recall the calculation of arc length s on a circle with radius r;



$$s = r\theta$$

Keeping in mind that r is constant, we take the time derivative of both sides. Recall that time derivative of s and θ are velocity v and angular velocity ω.

$$v = r\omega$$

If we use the last equation to replace v with ωr in the kinetic energy equation we get;

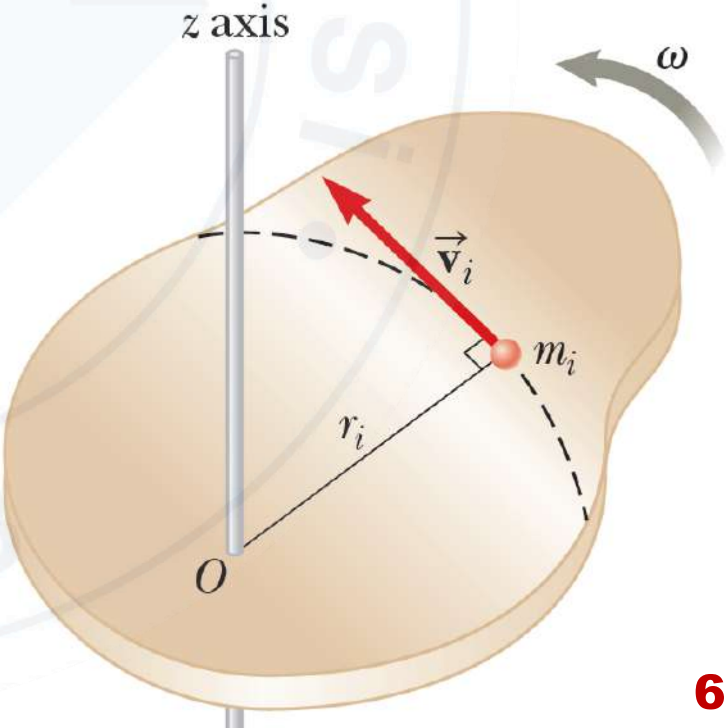
$$K_R = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}(mr^2)\omega^2$$

Here, we define the moment of inertia as the quantity in parenthesis as;

$$\boxed{I = mr^2} \quad \Rightarrow \quad K_R = \frac{1}{2}I\omega^2$$

Note that I is calculated for a point particle in our specific example.

To generalize this result to arbitrary rigid objects, we can start by thinking of them as combinations of small point-like particles indexed with i as shown in the figure.



The rotational energy of i^{th} particle is simply;

$$K_R^i = \frac{1}{2} m_i v_i^2$$

Since the object rotates as a whole, each particle indexed with i has the same ω. So;

$$K_R^i = \frac{1}{2} m_i r_i^2 \omega^2$$

Let us finally write the total rotational kinetic energy of the object;

$$K_R^{total} = \sum_i K_R^i = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

The sum in the parenthesis gives us I of the whole object as;

$$\boxed{I = \sum_i m_i r_i^2} \Rightarrow K_R^{total} = \frac{1}{2} I \omega^2$$

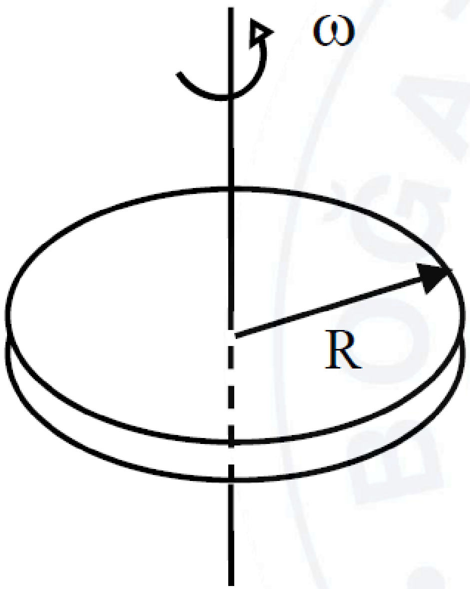
We have managed to find a way to extend our result to rigid objects and the last obstacle in our way is to find a way to divide the object to point-like particles. If we were to choose these particles to have infinitesimally small masses Δm_i and take the limit $\Delta m_i \rightarrow 0$ of the previous sum, we get the integral form;

$$I = \lim_{\Delta m_i \to 0} \sum_i \Delta m_i r_i^2 = \int r^2 \, dm$$

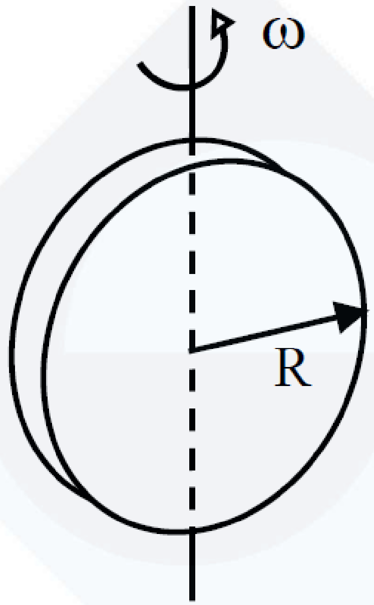
Since you may be unfamiliar with the integration techniques, you are not required to carry out any integrations during this experiment. Nevertheless, you should know that the moment of inertia formulas that are given in the next slide are calculated by this integral.

ROTATIONAL INERTIA - Theoretical background

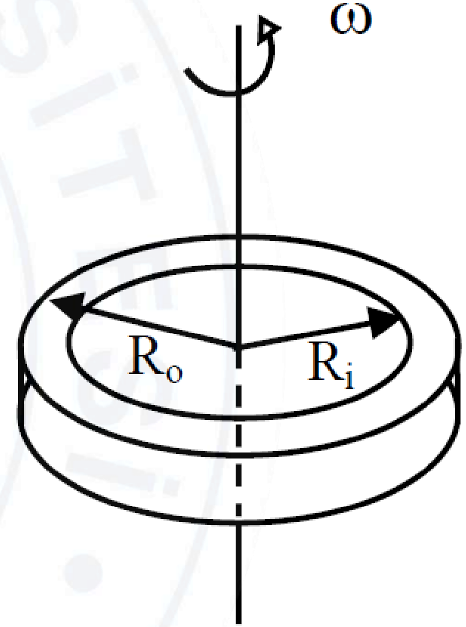
Theoretical rotational inertia formulas that we are going to use during this experiment are given below. Each assumed to have uniformly distributed mass of *M.*



Disk about its CM



Disk about its diameter



Ring

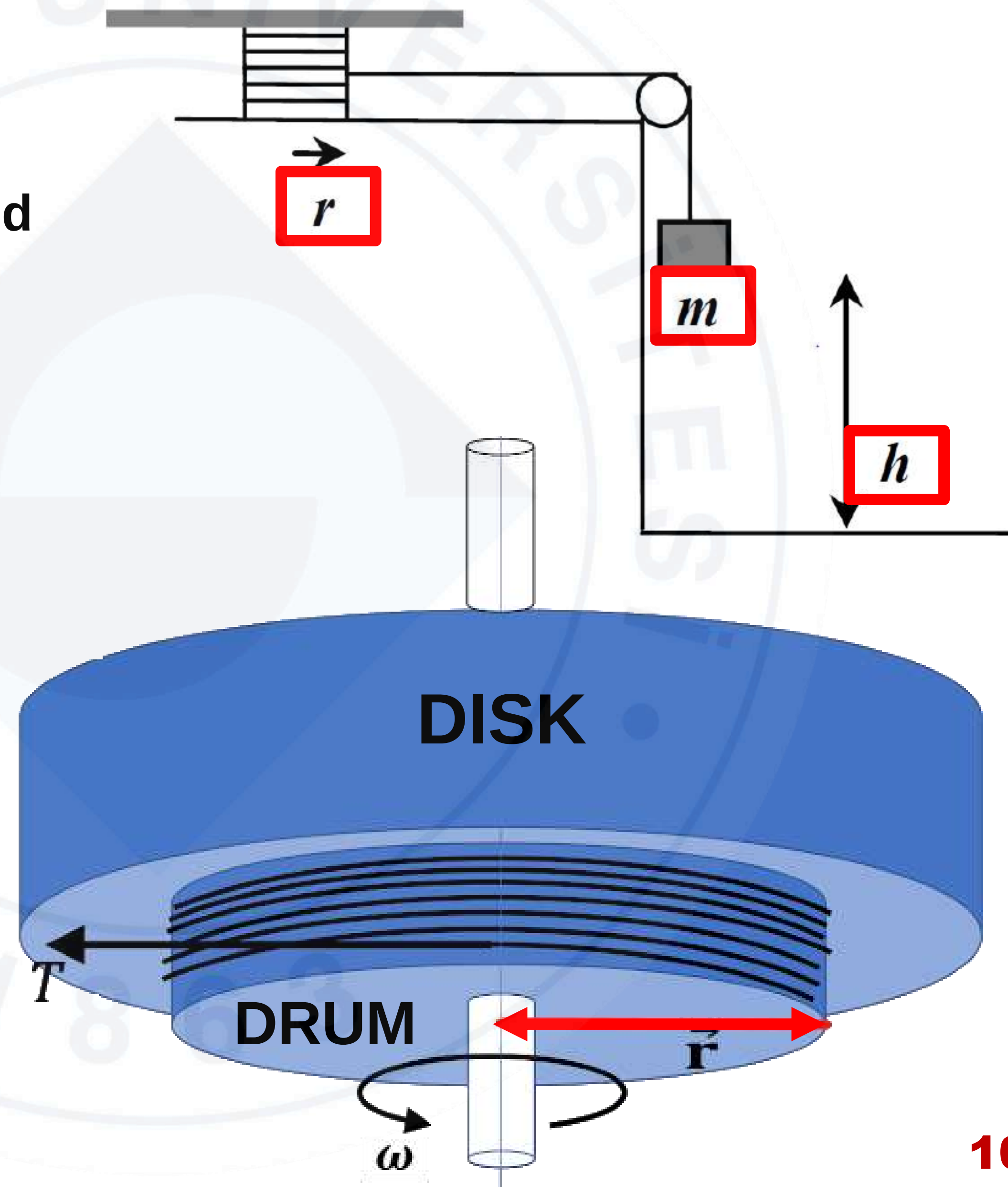
$$I_{DISK}^{CM} = \frac{1}{2} MR^2$$

$$I_{DISK}^{Diameter} = \frac{1}{4} MR^2$$

$$I_{RING} = \frac{1}{2} M\left(R_i^2 + R_o^2\right)$$

As given in the sketch on the right side, rope is wound around the drum and the mass *m* *is* released from a height of *h*. Here, *r* is the radius of the drum that we wound our rope around. Drum and the object on top can rotate freely. A 3D side view is given below.

Here, *T* is the tension throughout the rope and ω is the angular speed.

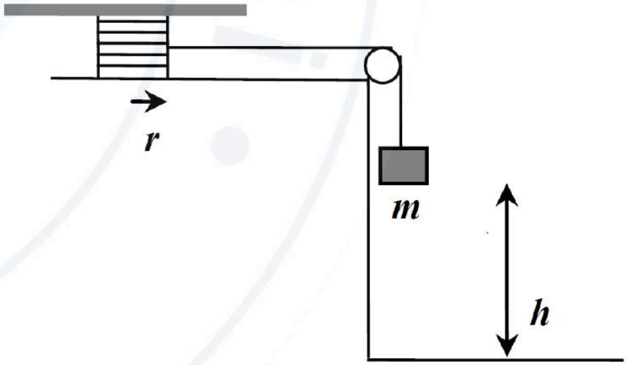


What to measure: Radius of the drum r, height of mass holder from the floor h, mass on the mass holder m, time for descent t for ***experimental calculations***. Mass of the disk M_{disk}, mass of the ring M_{ring}, radius of the disk R_{disk}, inner and outer radius of the ring R_{i-ring} and R_{o-ring} for ***theoretical calculations*** given in slide 10.

What to calculate : Combined moment of inertia for 3 configurations

Experimental findings : Moment of inertia of I_{DISK}^{CM}, $I_{DISK}^{Diameter}$, I_{RING} and I_{DRUM}

Theoretical findings : Moment of inertia of I_{DISK}^{CM}, $I_{DISK}^{Diameter}$ and I_{RING}



ROTATIONAL INERTIA – Theory

By conservation of energy, the potential energy lost by the hanging mass will be converted to kinetic energy throughout the system. Drum and the object that is placed on top will gain rotational kinetic energy while hanged mass m will gain translational kinetic energy. Let us denote final velocity v of masses just before they hit the ground and the final angular velocity of the rotating part ω. So, we get;

$$E_{Pot.} = E_{Kin.} = K_T + K_R$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Note that I is combination of rotational inertia of the drum and the object that is placed on top of it. We can use $v = r\omega$ to replace ω ;

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \quad \Rightarrow \quad I = mr^2\left(\frac{2gh}{v^2} - 1\right)$$

In this experiment, we won't be measuring v but instead we will measure time t of descent of mass m.

Since the mass experiences a free fall from rest with constant acceleration a, we have;

$$h = \frac{1}{2}at^2$$

The mass m starts from rest so the final velocity is $v = at$. Replacing into the equation above and leave v alone;

$$h = \frac{1}{2}\frac{v}{t}t^2 = \frac{1}{2}vt \Rightarrow \quad v = \frac{2h}{t}$$

Substituting this into the moment of inertia equation in the previous slide, we get;

$$\boxed{I = mr^2\left(\frac{gt^2}{2h} - 1\right)}$$

Hence, we will measure m, r, t, h and use $g = 9.81\ m/s^2$ to calculate experimental value of I for 3 different configurations. Note that this formula does not contain any properties of the object that is rotated such as mass and shape, which makes it perfect to find moment of inertia of objects with arbitrary shapes.

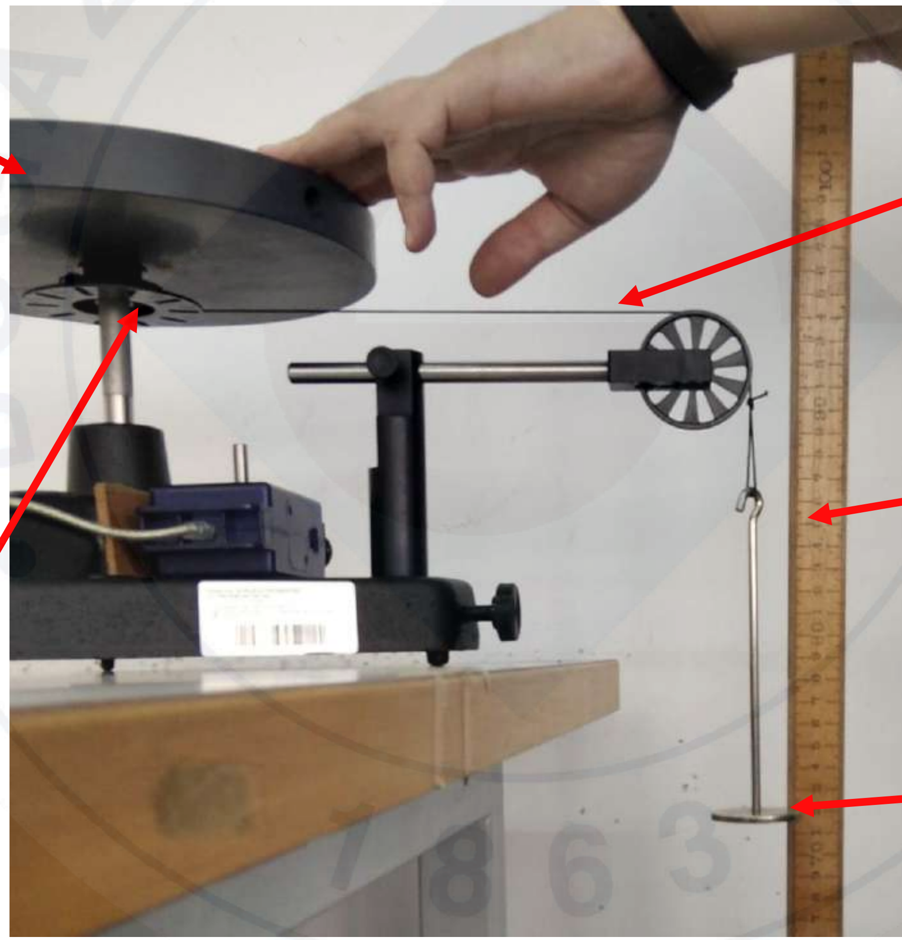
APPARATUS

ROTATIONAL INERTIA - Apparatus

During this experiment, we are going to calculate moment of inertia of a disk and a ring using the setup below. Then, we will compare our results with theoretical calculations of these objects.

Disk is placed on top of the ***drum*** and will rotate around an axis that goes through its center of mass ***CM***

This is the ***drum*** on which we will place the ***disk*** and the ***ring***



The rope is wound around the ***drum*** and a mass is hanged to the end of it

Ruler is here to measure the height of the masses from the ground ***h***

The ***mass holder*** is hanged at the end of the rope and it does not have any additional weights stacked up on it in this picture. Total mass here will be called ***m***

ROTATIONAL INERTIA - Apparatus

These are sample videos. Do not use these to take measurements.

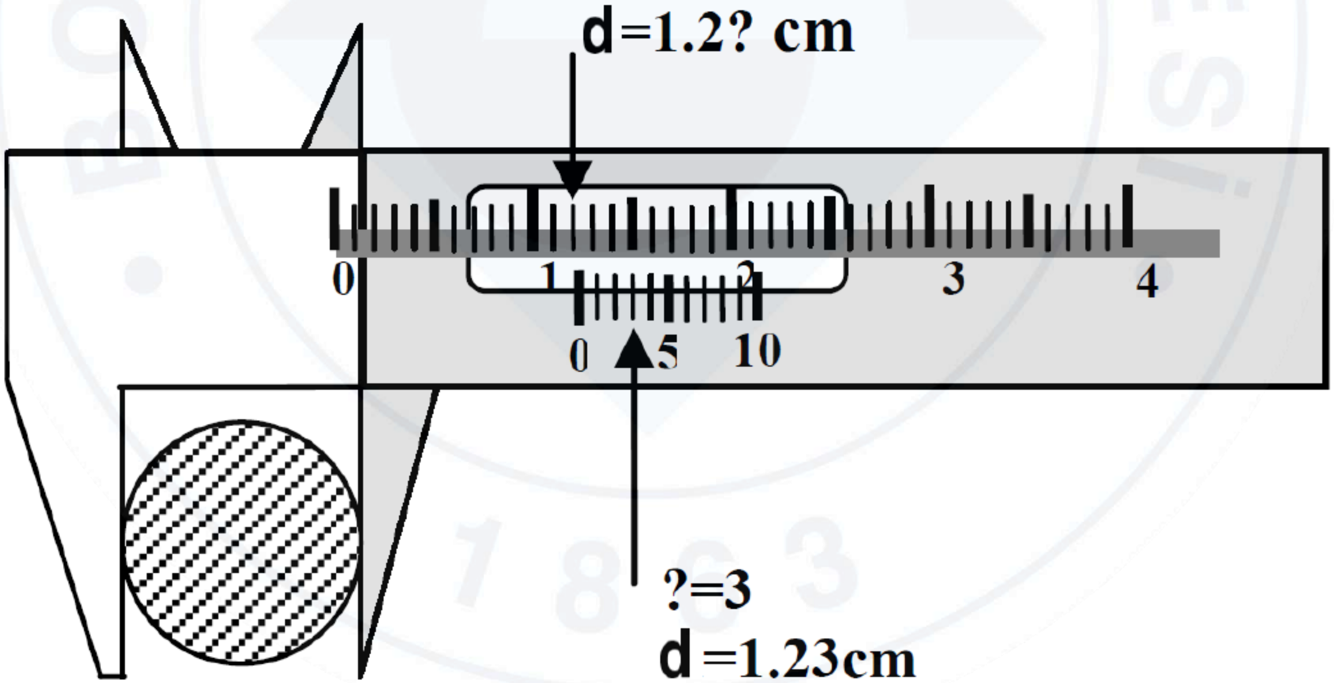
Disk about its CM

Disk about its diameter

Ring

Suppose we have a reading of diameter *d* as given in the figure. Extension of 0 bar on the lower scale points to ruler on top gives us a reading with precision up to 0.1 cm. In this example, this line points to somewhere between 1.2 cm and 1.3 cm. To find the next digit, we check where the lines on both upper and lower scales coincide. In our example, 3 on the lower coincides with 1.5 cm on the upper scale so our final reading of *d* is;

$$1.2cm + 0.03cm = 1.23cm$$

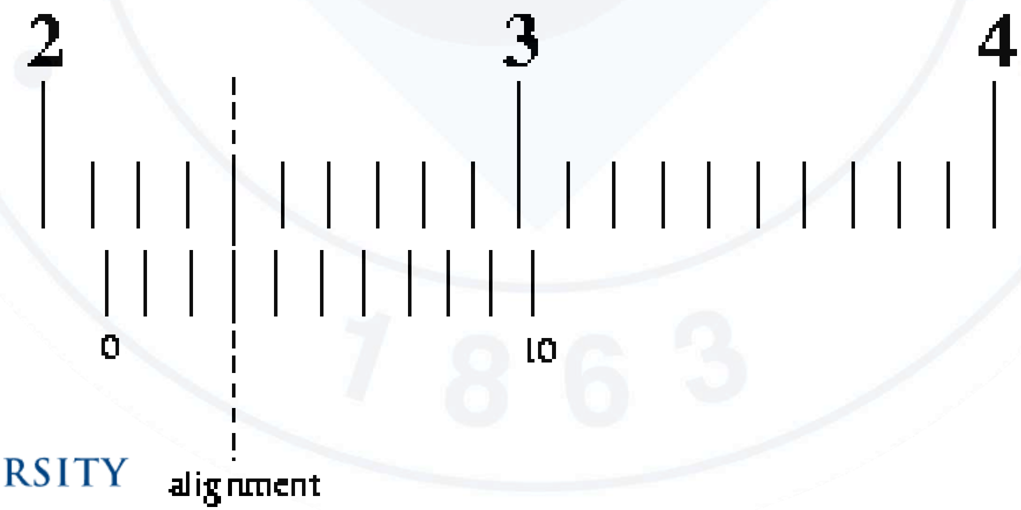


Let us look at more examples on how to read a vernier caliper.

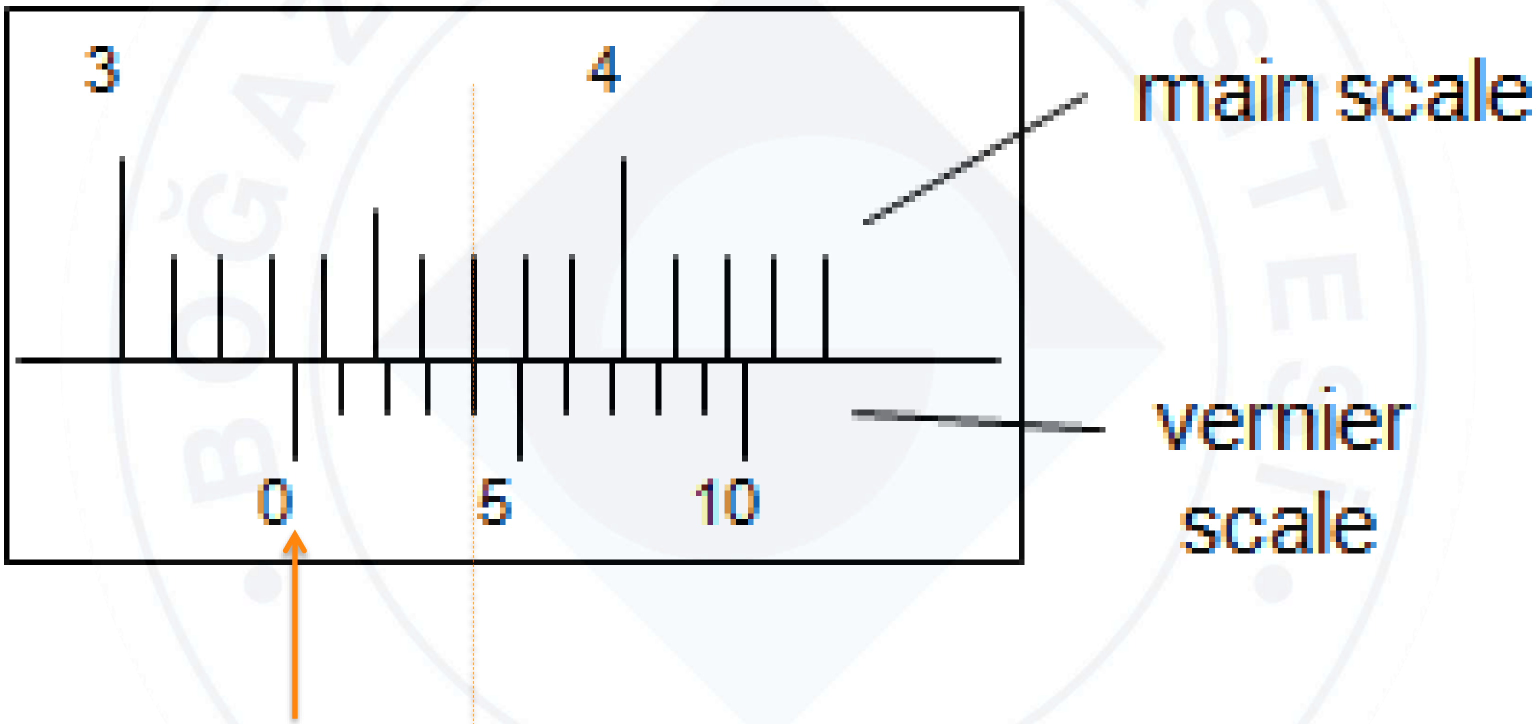
How to measure with vernier calipers:

If a vernier caliper output a measurement reading of 2.13 cm, this means that:

- **The main scale contributes the main number(s) and one decimal place to the reading (E.g. 2.1 cm, whereby 2 is the main number and 0.1 is the one decimal place number)**
- **The vernier scale contributes the second decimal place to the reading (E.g. 0.03 cm). Look at the image below and look closely for an alignment of the scale lines of the main scale and vernier scale. The aligned line corresponds to 3.**



Example:

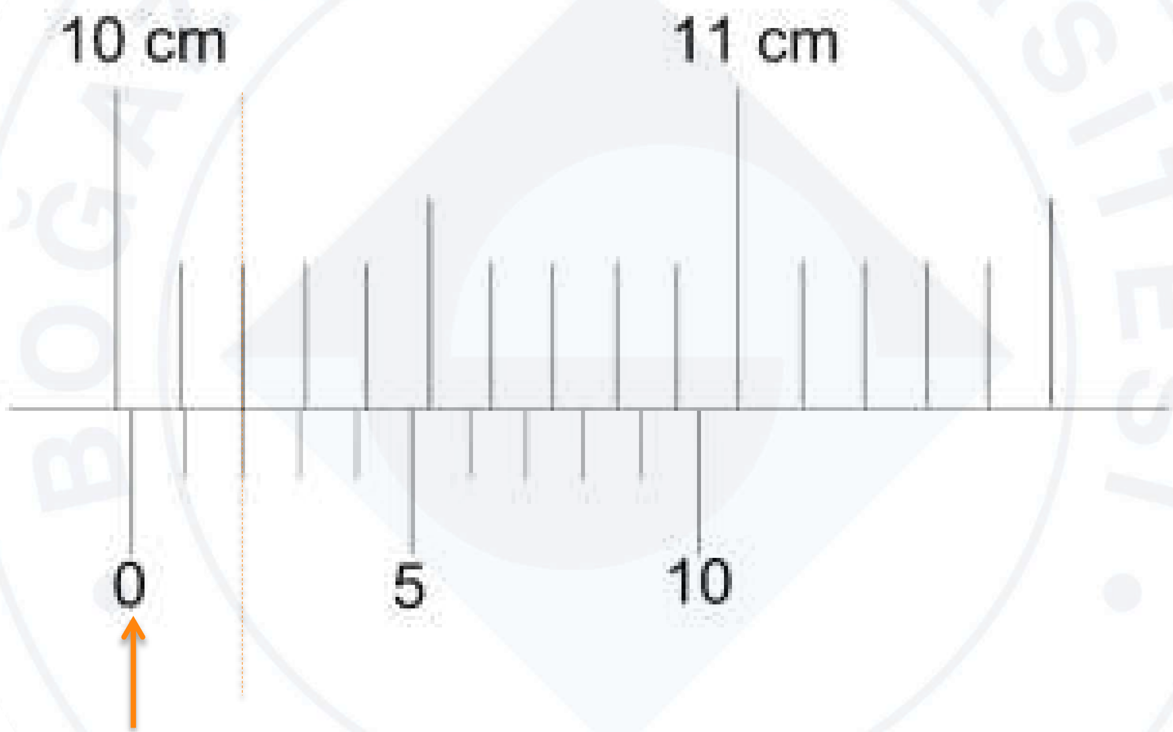


Main number: 3
One decimal: 3
3.3

The aligned line corresponds to 4.
0.04

RESULT: 3.34 cm

Example:

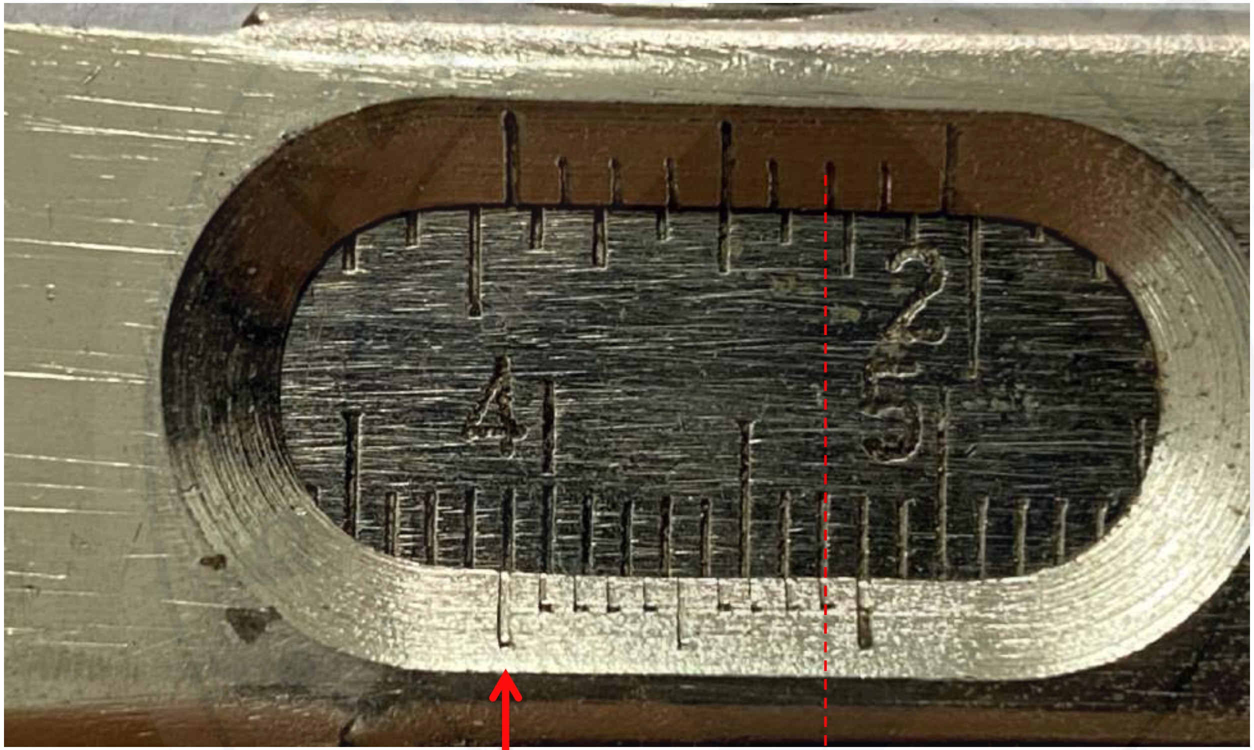


Main number: 10
One decimal: 0
10.0

The aligned line corresponds to 2.
0.02

RESULT: 10.02 cm

Special Case: Reading of 3.89 cm, 3.90 cm and 3.91cm.

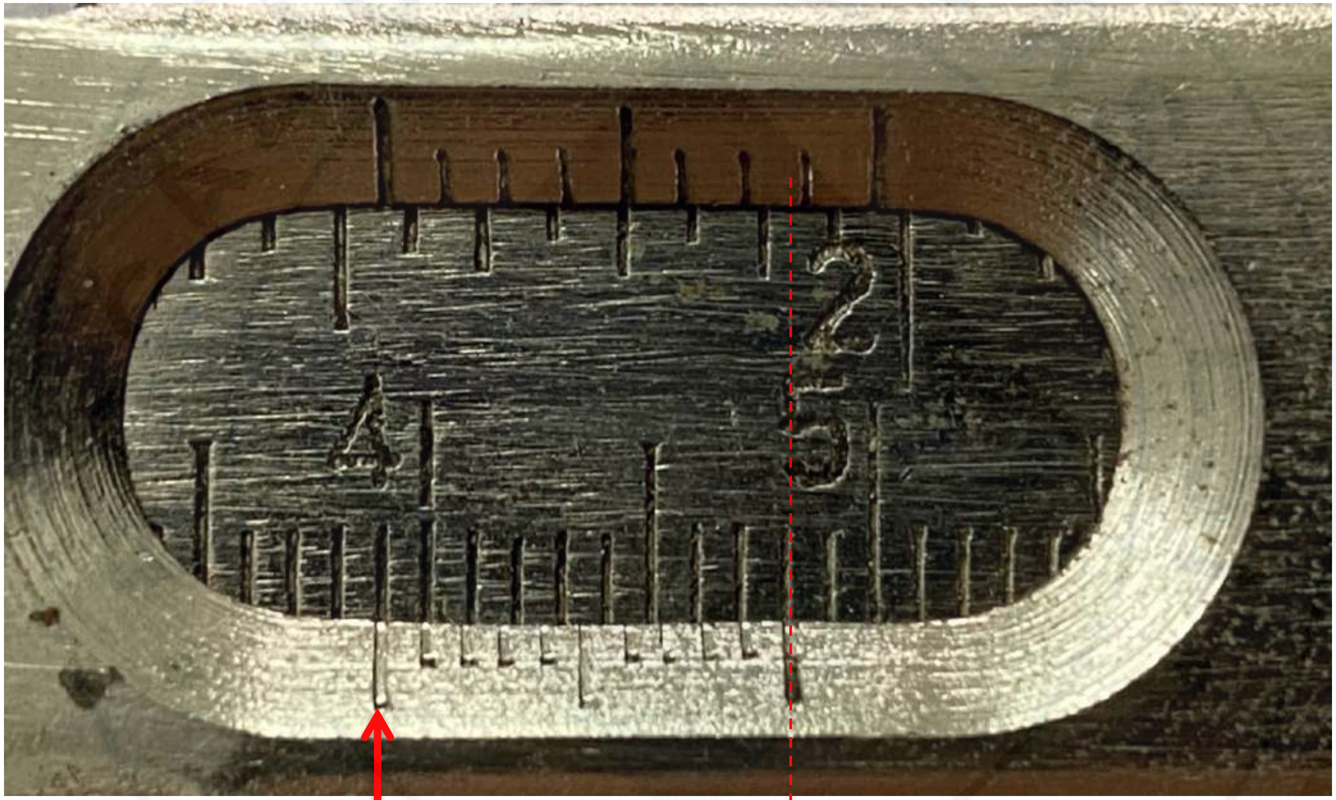


Main number: 3
One decimal: 8
3.8

The aligned line corresponds to 9.
0.09

RESULT:
3.89 cm

Special Case: Reading of 3.89 cm, 3.90 cm and 3.91cm.

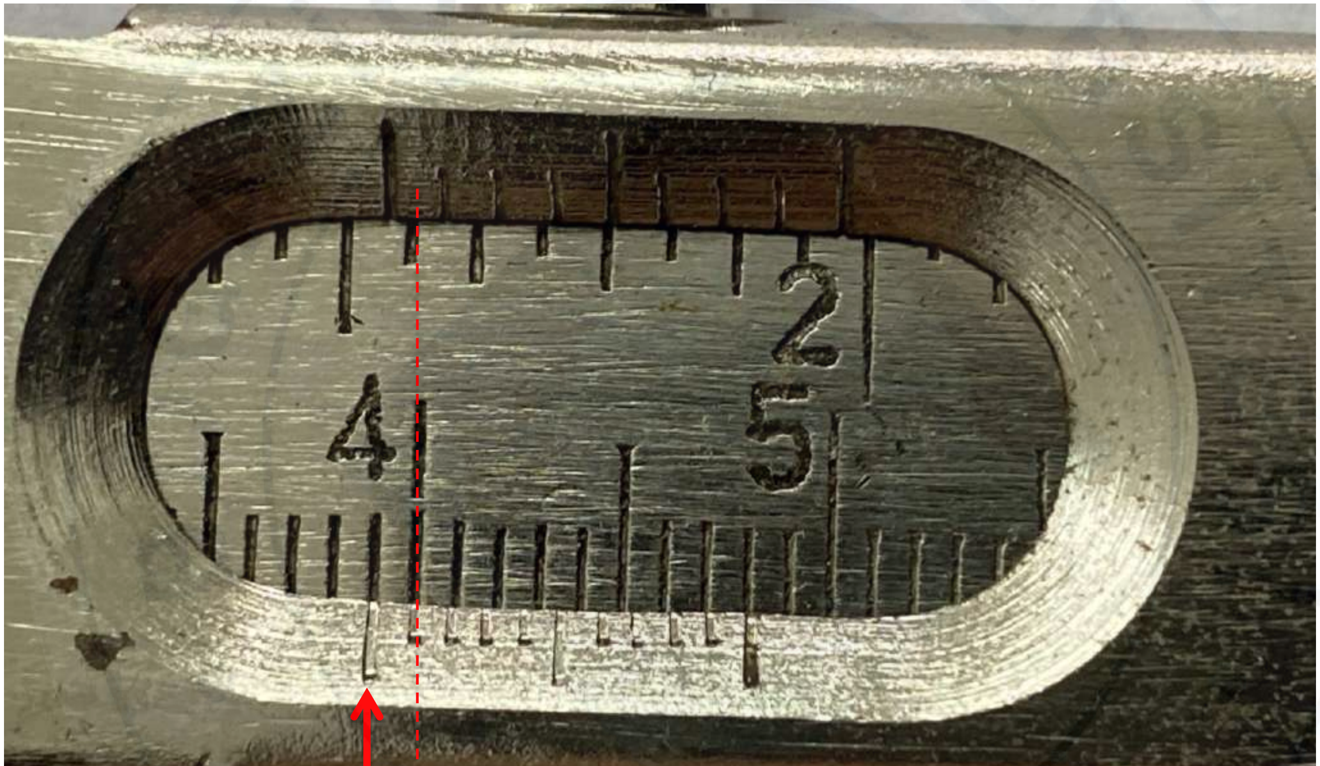


Main number: 3
One decimal: 9
3.9

The aligned line corresponds to 10.
0.00

RESULT:
3.90 cm

Special Case: Reading of 3.89 cm, 3.90 cm and 3.91cm.



Main number: 3
One decimal: 9
3.9

The aligned line corresponds to 1.
0.01

RESULT:
3.91 cm

EXPERIMENT

ROTATIONAL INERTIA – Experiment

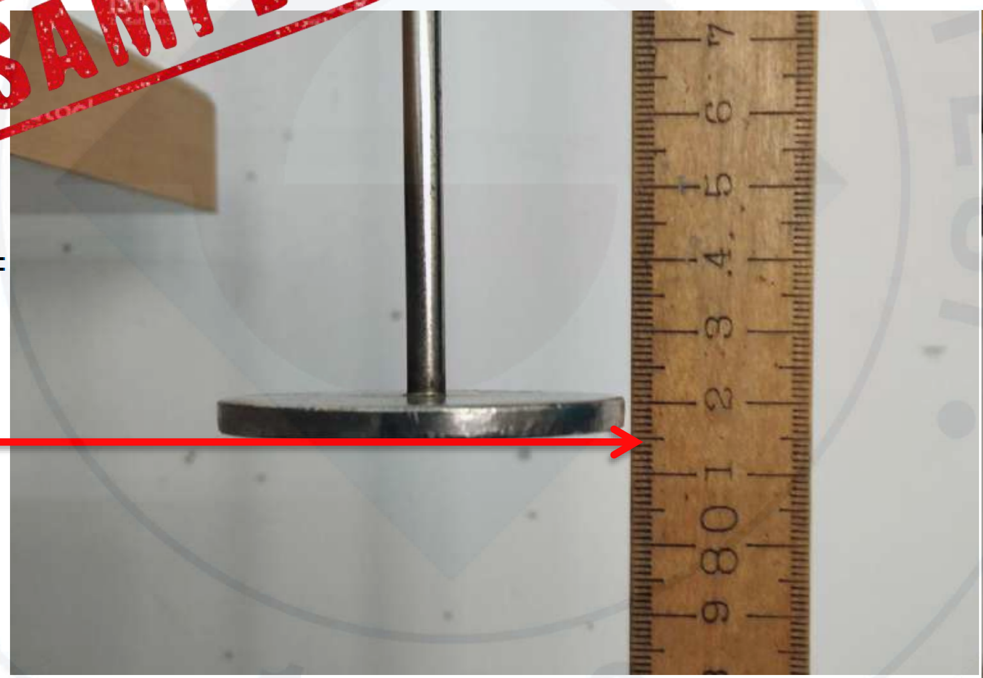
The following images are just to give you an idea of what to expect to find in the Lab. Do <u>not</u> use the values you see in this presentation. You will find similar content to these pictures in the Lab. This is on page 95 of your lab book.

SAMPLE

Description / Symbol			Value & Unit
Diameter of the drum	d	$=$	 
Radius of the drum	r	$=$	**Will be derived from *d*** . . . .

Read height *h* of the mass holder from the floor from the bottom of mass hanger and record to page 95 of your lab book.

Description / Symbol	Value & Unit
Height of mass holder from the floor h =	. .



Read height *h* from here

Next, record the mass on the mass holder *m* to page 95 of your lab book.

Description / Symbol		Value & Unit
Mass on the mass holder	$m^* =$	

You will use the same *m* for all configs.

Precision: 1g

PART 1
Rotational Inertia of Disk

- About CM
- About Diameter

PhysLabs

Time the descent of mass *m* when disk is rotated about its center of mass CM and about its diameter. There will be two sections in the DataVideo for each configuration and two *t* values will be measured from these. Then, record them to page 95 of your lab book.

Part 1: Rotational Inertia of Disk

			ABOUT CM			ABOUT DIAMETER
Time for descent	t_1	=	.	$t`_1$	=	. . . .
Time for descent	t_2	=	.	$t`_2$	=	. . . .
Average time for descent	t_{ave}	=	**Average of t_1 and t_2**	$t`_{ave}$	=	**Avg. of t_1' and t_2'**

ROTATIONAL INERTIA – Experiment

You will be using your phones to measure time of descent *t* using chronometer of your cell phones. An example is given on the right.

Do not use the time of descent value given here. This is just to show you how the time the descent.

PART 2
Rotational Inertia of Ring

Ring placed on top of the disk rotating about its CM

Adding the ring on top, we do the same as previous slide and record them to page 97 of your lab book.

Part 2: Rotational Inertia of the Ring

Time for descent -1 $t``_1$ = ...

Time for descent -2 $t``_2$ = ...

Average time for descent $t``_{ave}$ = ...

Average of t_1'' and t_2''

You will be shown mass of the disk which you will record to page 97 of your lab book.

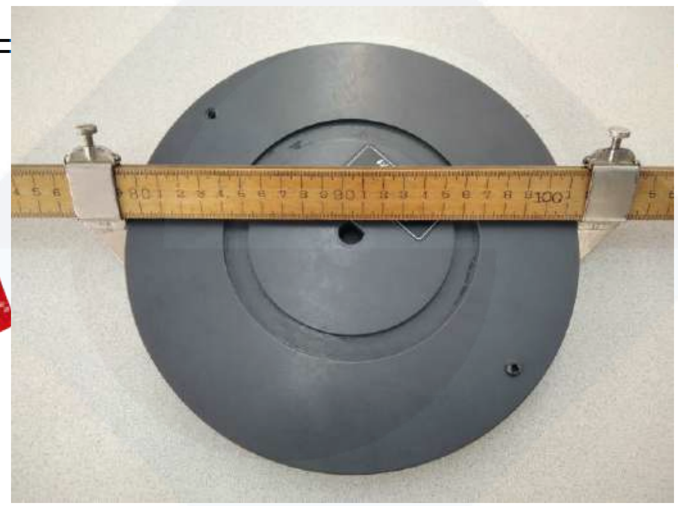
Description / Symbol		Value & Unit
Mass of the Disk M_{disk}	=	. . . . . .

Reading is in grams

PhysLabs

You will be shown radius of the disk which you will record to page 97 of your lab book.

Description / Symbol	Value & Unit
Diameter of the Disk D_{disk} =	
Radius of the Disk R_{disk} =	**Will be derived from** D_{disk}



BOĞAZİÇİ UNIVERSITY
Physics Department

Continuing to page 97 of your lab book, you will record mass measurement of the ring.

Description / Symbol	Value & Unit
Mass of the Ring M_{ring} =	

Reading is in grams

ROTATIONAL INERTIA – Experiment

Continuing to page 97 of your lab book, you will record inner and outer diameter measurements of the ring.

Inner Diameter of the Ring $D_{\text{i-ring}}$ = . . .

Outer Diameter of the Ring $D_{\text{o-ring}}$ = . . .

Inner Radius of the Ring $R_{\text{i-ring}}$ = **Will be derived from D_{i-ring}** . . . .

Outer Radius of the Ring $R_{\text{o-ring}}$ = **Will be derived from D_{o-ring}** . . . .

PhysLabs

On page 99 of your lab book, we will calculate moment of inertia of 3 different configurations by plugging in our measurements we recorded to pages 95 and 97 into the equation we have found;

$$\boxed{I = mr^2\left(\frac{gt^2}{2h} - 1\right)}$$

use $g = 9.81\, m/s^2$

Description / Symbol	Calculations (show each step)	Result
Rotational Inertia of the drum + Disk about its CM $I^{CM}_{drum+disk} =$	....	....
	....	
Rotational Inertia of the drum + Disk about its DIAMETER $I^{DIAMETER}_{drum+disk} =$	....	....
	....	
Rotational Inertia of the drum + the Ring + disk $I_{drum+ring+disk} =$	....	....
	....	

Next on page 99 of your lab book, we will calculate moment of inertia of $I_{DISK}^{CM}, I_{DISK}^{Diameter}, I_{RING}$ and I_{DRUM}. There are 4 unknowns, but we only get 3 equations from our measurements. We borrow one more equation from our theoretical calculations;

$$I_{DISK}^{CM} = 2I_{DISK}^{Diameter}$$

So, you can solve this set of equations and fill the next part in your lab books.

Please show your work fully on a clean A4 (or a blank page from your lab book) and add to your lab report.

Rotational Inertia of the DISK about its CM I_{DISK}^{CM} = ...

Rotational Inertia of the DISK about its DIAMETER $I_{DISK}^{diameter}$ = ...

Rotational Inertia of the RING I_{RING} = ...

Rotational Inertia of the DRUM I_{DRUM} = ...

On page 101 of your lab book, we will calculate moment of inertia of I_{DISK}^{CM}, $I_{DISK}^{Diameter}$ and I_{RING} theoretically by the equations given in slide 8 or page 92 of your lab books.

Theoretical Values for *I*:

I_{DISK}^{CM} = ...

$I_{DISK}^{diameter}$ = ...

I_{RING} = ...

PhysLabs

At last, you will compare your experimental results with the theoretical results by calculating percent error of I for each case;

$$\Delta I = \frac{|I_{th} - I_{exp}|}{I_{th}} \times 100$$

Finalize your report by showing dimensional analysis of rotational inertia.

% Error for Rotational Inertia:

ΔI^{CM}_{DISK} :

$\Delta I^{diameter}_{DISK}$:

ΔI_{RING} :

Show the Dimensional Analysis of Rotational Inertia, I: