

# ALTERNATING CURRENTS SERIES CIRCUITS 

PHYL202
dPhys

## THEORY

AC Current: Current that fluctuates sinusoidally with time at a fixed frequency

AC Voltage: Voltage that fluctuates sinusoidally with time at a fixed frequency

Alternating Current (AC): Flow of electric charge that periodically reverses direction

Resistance: Opposition of a resistance to any current
Capacitive Reactance: Opposition of a capacitor to an alternating current Inductive Reactance: Opposition of an inductor to an alternating current Impedance: AC analog to resistance in a DC circuit, which measures the combined effect of resistance, capacitive reactance and inductive reactance

Phase Angle: Amount by which the voltage and current are out of phase with each other in a circuit

## ALTERNATING CURRENT AND SERIES CIRCUITS

## Resistor in an AC Circuit:

When the resistive load $R$ is connected across the alternating source shown in the figure below, the current flows through it. The alternating current flows in one direction and then in the opposite direction when the polarity is reversed. Also, we know that according to Ohm's Law, potential drop across a resistor is given by

$$
V_{R}=I R .
$$



R

$$
\begin{aligned}
V & =I R & & I=\frac{V_{o}}{R} \sin (\omega t) \\
V_{o} \sin (\omega t) & =I R & & I=I_{o} \sin (\omega t)
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{I} \\
& \mathrm{~V}_{\mathrm{R}}
\end{aligned}
$$

where $I_{o}=\frac{V_{o}}{R}$ is the maximum value of current in the circuit.


Current in a purely resistive circuit is in phase with the applied voltage

## ALTERNATING CURRENT AND SERIES CIRCUITS

## Capacitor in an AC Circuit:

In a purely capacitive circuit, we use the properties of an inductor to show characteristic relations for the circuit. I flows through the following circuit, then by using Kirchoff's law of voltages for closed loops, we get $V=V_{C}$.


$$
V=V_{C}
$$

$$
V_{o} \sin (\omega t)=\frac{Q}{C}
$$

$$
Q=V_{o} C \sin (\omega t)
$$

$$
\begin{aligned}
I & =\frac{d}{d t}(Q) \\
& =V_{o} C \frac{d}{d t} \sin (\omega t)
\end{aligned}
$$

Current in a purely capacitive circuit leads the applied voltage by a phase difference of $\pi / 2$, i.e. when voltage attains its maxima, current attains its minima, but being ahead of voltage

## ALTERNATING CURRENT AND SERIES CIRCUITS

Capacitor in an AC Circuit:
Here, $\mathrm{I}_{0}$ is the peak current during a complete cycle. This current is given by

$$
\frac{V_{o}}{1 / C \omega}
$$

Now, look at the term in the denominator. It acts as resistance in this circuit and is denoted by $X_{C}$ and is known as capacitive reactance. So,

$$
X_{C}=\frac{1}{C \omega} .
$$

## ALTERNATING CURRENT AND SERIES CIRCUITS

## Inductor in an AC Circuit:

In a purely inductive circuit, we use the properties of an inductor to show characteristic relations for the circuit. According to Kirchoff's voltage law for closed loops, we get $\mathrm{V}=\mathrm{V}_{\mathrm{L}}$


$$
V=L \frac{d}{d t}(I) \quad \int d I=\int \frac{V_{o}}{L} \sin (\omega t) d t
$$

$V_{o} \sin (\omega t)=L \frac{d}{d t}(I)$

$$
I=\frac{V_{o}}{L \omega}(-\cos (\omega t))
$$

Current lags the applied potential difference by a phase of $\pi / 2$

$$
d I=\frac{V_{o}}{L} \sin (\omega t) d t \quad=\frac{V_{o}}{L \omega} \sin \left(\omega t-\frac{\pi}{2}\right)
$$

## ALTERNATING CURRENT AND SERIES CIRCUITS

Inductor in an AC Circuit:
Here, $I_{0}$ is the peak current in the circuit. This current is given by

$$
I_{o}=\frac{V_{o}}{L \omega}
$$

Now, look at the term in the denominator. It acts as resistance in this circuit and is denoted by $X_{L}$ and is known as Inductive reactance. So,

$$
X_{L}=L \omega
$$

## RC CIRCUIT

When you connect a capacitor and a resistor in series to an alternating voltage source, the phase of the current through the capacitor will be $90^{\circ}$ ahead of the voltage. If we were to take the current as our reference for the phase, then the voltage across the capacitor will be $90^{\circ}$ behind the current hence the voltage across the resistor. The total voltage across the RC series combination will be equal to the applied voltage:

$$
V_{a p p}=\sqrt{\left(V_{R}^{2}+V_{C}^{2}\right)}
$$

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Labs

## EXPERIMENT

## ALTERNATING CURRENT AND SERIES CIRCUITS

## RC CIRCUIT

- Construct an RC circuit.


## ALTERNATING CURRENT AND SERIES CIRCUITS

## RC CIRCUIT

- Measure the voltage across each element.



## ALTERNATING CURRENT AND SERIES CIRCUITS

## RC CIRCUIT

Description / Notation

Value \& Unit

Potential difference
across the resistance $V_{\mathrm{R}}=$

Potential difference across the capacitor $V_{\mathrm{C}}=$

Applied potential $\quad V_{\text {app }}=$

Phasor Diagram of the RC Circuit:
Scaling Factor:

## RC CIRCUIT

- Then draw the phasor diagram by taking the current (i.e. the voltage across the resistor) as the reference.

- Then draw two circles by using compasses with radii equal to $V_{\mathrm{c}}$ from the tip and $V_{\text {app }}$ from the beginning of the phasor corresponding to the voltage across the resistor.


## ALTERNATING CURRENT AND SERIES CIRCUITS

## RC CIRCUIT

- Phasors for $V_{C}$ and $V_{\text {app }}$ will meet each other at the point where the circles intersect. Determine the angle between $V_{C}$ and $V_{R}$.



## RL CIRCUIT

When you connect an inductor and a resistor in series to an alternating voltage source, the current through the inductor will be $90^{\circ}$ behind the voltage. If we were to take the current as our reference for the phase, then the voltage across the inductor will be $90^{\circ}$ ahead of the current hence the voltage across the resistor. The total voltage across the RL series combination will be equal to the applied voltage:

$$
V_{a p p}=\sqrt{V_{R}^{2}+V_{L}^{2}}
$$

## ALTERNATING CURRENT AND SERIES CIRCUITS

## RL CIRCUIT

However, the inductor also has some internal resistance, $R_{\mathrm{L}}$. Because of its internal resistance, the voltage across the inductor will not be exactly $90^{\circ}$ ahead but at angle calculated from:

$$
\tan \theta=\frac{2 \pi j L}{R_{L}}
$$

We should also modify Equation (5) accordingly:

$$
V_{a p p}=I \sqrt{\left(R+R_{L}\right)^{2}+X_{L}^{2}}
$$

## ALTERNATING CURRENT AND SERIES CIRCUITS

RL CIRCUIT

- Construct an RL circuit.



## ALTERNATING CURRENT AND SERIES CIRCUITS



- Measure the voltage across each element.


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## ALTERNATING CURRENT AND SERIES CIRCUITS

## RL CIRCUIT

Description / Notation
Value \& Unit

Current in the circuit $I=$

Potential difference
across the resistance $V_{\mathrm{R}}=$

Potential difference
across the inductor $\quad V_{\mathrm{L}}=$

Applied potential $\quad V_{\mathrm{app}}=$

Phasor Diagram of the RL Circuit:
Scaling factor:

## RL CIRCUIT

- Then draw the phasor diagram by taking the current (i.e. the voltage across the resistor) as the reference.

$$
V_{R}
$$

- Then draw two circles by using compasses with radii equal to $V_{\mathrm{L}}$ from the tip and $V_{\text {app }}$ from the beginning of the phasor corresponding to the voltage across the resistor.


## ALTERNATING CURRENT AND SERIES CIRCUITS

## RL CIRCUIT

- Phasors for $V_{\mathrm{L}}$ and $V_{\text {app }}$ will meet each other at the point where the circles intersect.



## RL CIRCUIT

- Determine the internal resistance of the inductor by measuring the current through the circuit and the horizontal component of $V_{\mathrm{L}}$ from your phasor diagram.


## ALTERNATING CURRENT AND SERIES CIRCUITS

## RL CIRCUIT

Description / Symbol Value / Calculation Result

Potential difference due to the internal resistance of the inductor $\quad V_{\mathrm{r}}=$

Internal resistance
of the inductor $r_{\mathrm{L}}=$

## RLC CIRCUIT

When you connect an inductor, a capacitor, and a resistor in series to an alternating voltage source, the current through the inductor and the capacitor will be $90^{\circ}$ behind and ahead of the voltage, respectively. If we were to take the current as our reference for the phase, then the voltage across the inductor and the capacitor will be $90^{\circ}$ ahead of and behind the current (hence the voltage across the resistor), respectively. The total voltage across the RLC series combination will be equal to the applied voltage:

$$
V_{a p p}=\sqrt{\left(V_{L}-V_{C}\right)^{2}+V_{R}^{2}}
$$

## ALTERNATING CURRENT AND SERIES CIRCUITS

## RLC CIRCUIT

However, the inductor also has some internal resistance, $R_{\mathrm{L}}$. Because of its internal resistance, the voltage across the inductor will not be exactly $90^{\circ}$ ahead but at angle calculated from:

$$
\tan \theta=\frac{2 \pi f L}{R_{L}}
$$

Applied Potential:

$$
V_{a p p}=I \sqrt{\left(R+R_{L}\right)^{2}+\left(X_{L}-X_{C}\right)^{2}}=I Z
$$

Total Impedance:

$$
Z=\sqrt{\left(R+R_{L}\right)^{2}+\left(X_{L}-X_{C}\right)^{2}}
$$

## RLC CIRCUIT

Now the phase difference between the current and the voltage is more complex and given by:

$$
\tan \Phi=\frac{\left(X_{L}-X_{C}\right)}{R_{\text {total }}}
$$

This $\Phi$ angle is the angle between the applied voltage and the resulting current phasors. It determines the total average power used in an RLC circuit:

$$
\bar{P}=V_{r m s} I_{r m s} \cos \Phi
$$

We should remember that the values measured by instruments like voltmeters, ammeters, etc. are root-mean-squared values and not the peak values.

## ALTERNATING CURRENT AND SERIES CIRCUITS

## RLC CIRCUIT

- Construct an RLC circuit.


## ALTERNATING CURRENT AND SERIES CIRCUITS

## RLC CIRCUIT

- Measure the voltage across each element.


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## ALTERNATING CURRENT AND SERIES CIRCUITS

## RLC CIRCUIT

- Measure the voltage across each element.



## ALTERNATING CURRENT AND SERIES CIRCUITS

RLC CIRCUIT

Description / Notation
Value \& Unit

Current in the circuit $I=$

Potential difference
across the resistance $V_{\mathrm{R}}=$
Potential difference
across the inductor $\quad V_{\mathrm{L}}=$
Potential difference
across the capacitor $V_{\mathrm{C}}=$

Applied potential $\quad V_{\text {app }}=$

Phasor Diagram of the RLC Circuit:
Scaling factor:

## RLC CIRCUIT

- Then draw the phasor diagram by taking the current (i.e. the voltage across the resistor) as the reference.
- Then draw $\mathrm{V}_{\mathrm{C}} 90^{\circ}$ behind $\mathrm{V}_{\mathrm{R}}$ (or perpendicular in the negative direction) by assuming it is pure capacitance.
- Then draw two circles by using compasses with radii equal to $V_{\mathrm{L}}$ from the tip of the phasor corresponding to the voltage across the capacitor, $\mathrm{V}_{\mathrm{c}}$ and $V_{\text {app }}$ from the beginning of the phasor corresponding to the voltage across the resistor, $\mathrm{V}_{\mathrm{R}}$.
- Phasors for $V_{\mathrm{L}}$ and $V_{\text {app }}$ will meet each other at the point where the circles intersect.


## ALTERNATING CURRENT AND SERIES CIRCUITS

RLC CIRCUIT

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## ALTERNATING CURRENT AND SERIES CIRCUITS

## RLC CIRCUIT

Using the current value and the internal resistance of the inductor determined in the previous step,

- Determine the capacitive and inductive reactances first and
- Then calculate the value of the capacitor and the inductor.


## ALTERNATING CURRENT AND SERIES CIRCUITS

RLC CIRCUIT

Capacitive
reactance $\quad X_{\mathrm{C}}=$

Value of the
capacitor
$C=$

Inductive
reactance $\quad X_{\mathrm{L}}=$

Value of the
inductor $L=$

## ALTERNATING CURRENT AND SERIES CIRCUITS

## RLC CIRCUIT

- Finally determine the phase angle $\Phi$ and the average power dissipated in the RLC circuit.

| Description / Symbol | Value / Calculation <br> (show step by step) |
| :--- | :--- |
| Phase angle $\varphi=\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ |  |

Average dissipated
power $\quad P=$

