Boğaziçi University

Introductory Phys Labs

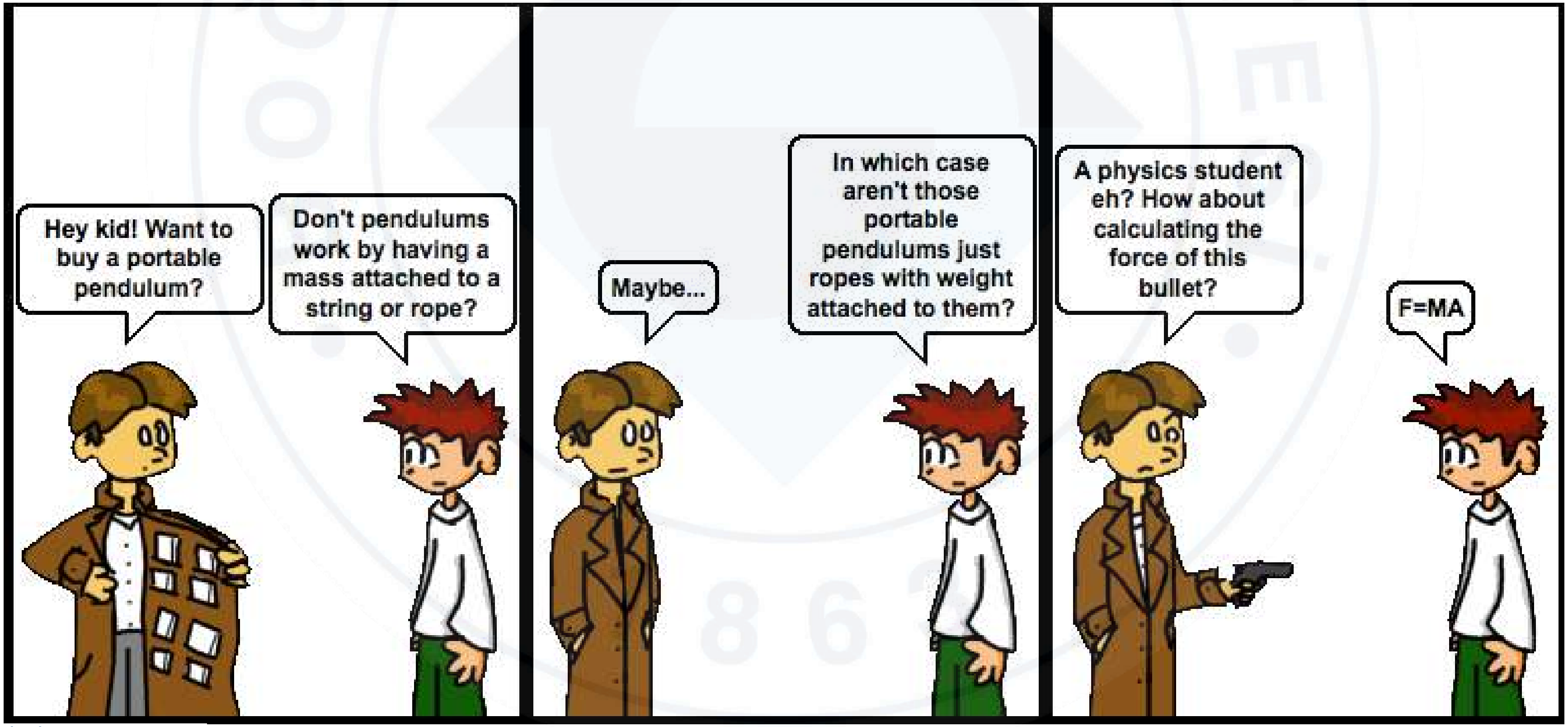
THE SIMPLE PENDULUM

PHYL101

THE SIMPLE PENDULUM

A pendulum is a weight suspended from a pivot so that it can swing freely. The word “pendulum” is new Latin, derived from the Latin “pendulus”, which means “hanging”.

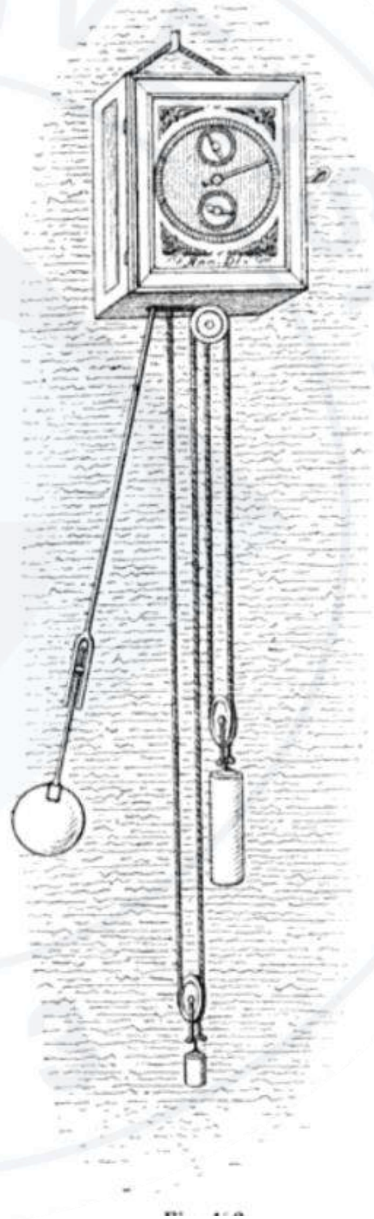
The *simple* part has some additional constraints that makes the pendulum easier to analyze.

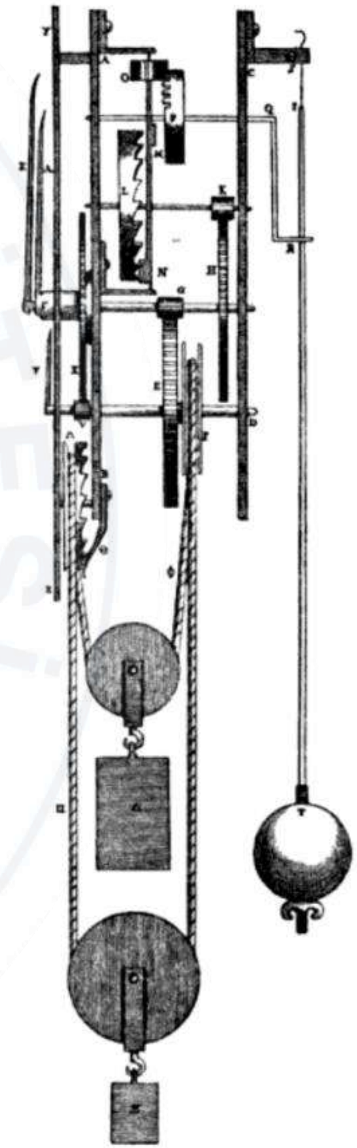


THE SIMPLE PENDULUM

- Around 1602, Galileo Galilei studied pendulum properties after watching a swinging chandelier in the cathedral of Pisa's domed ceiling.
- Using his pulse as a time measurement, he observed the swinging motion has a fixed period.
- Thus pendulums became timekeeping devices.

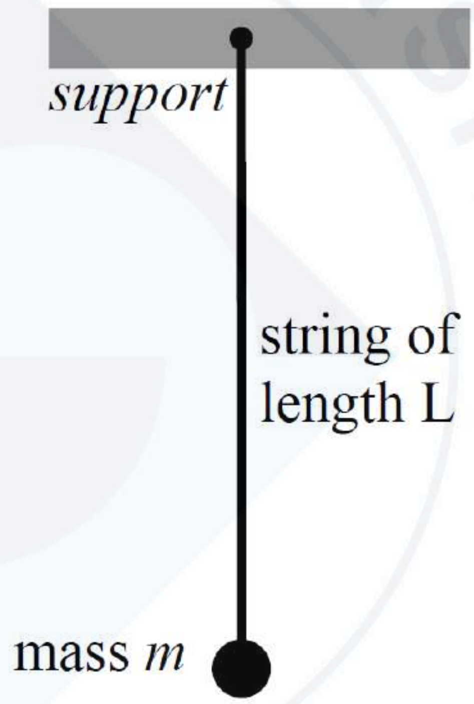
54 years later, first pendulum clock was invented in 1656 by Dutch scientist Christiaan Huygens. A more modern version from 1904 is given below.





THE SIMPLE PENDULUM – Aim

- **What to measure:** Length L of the pendulum, time t for the pendulum to complete 10 periods.
- **What to calculate :** Period T
- **Experimental findings :** Gravitational acceleration g

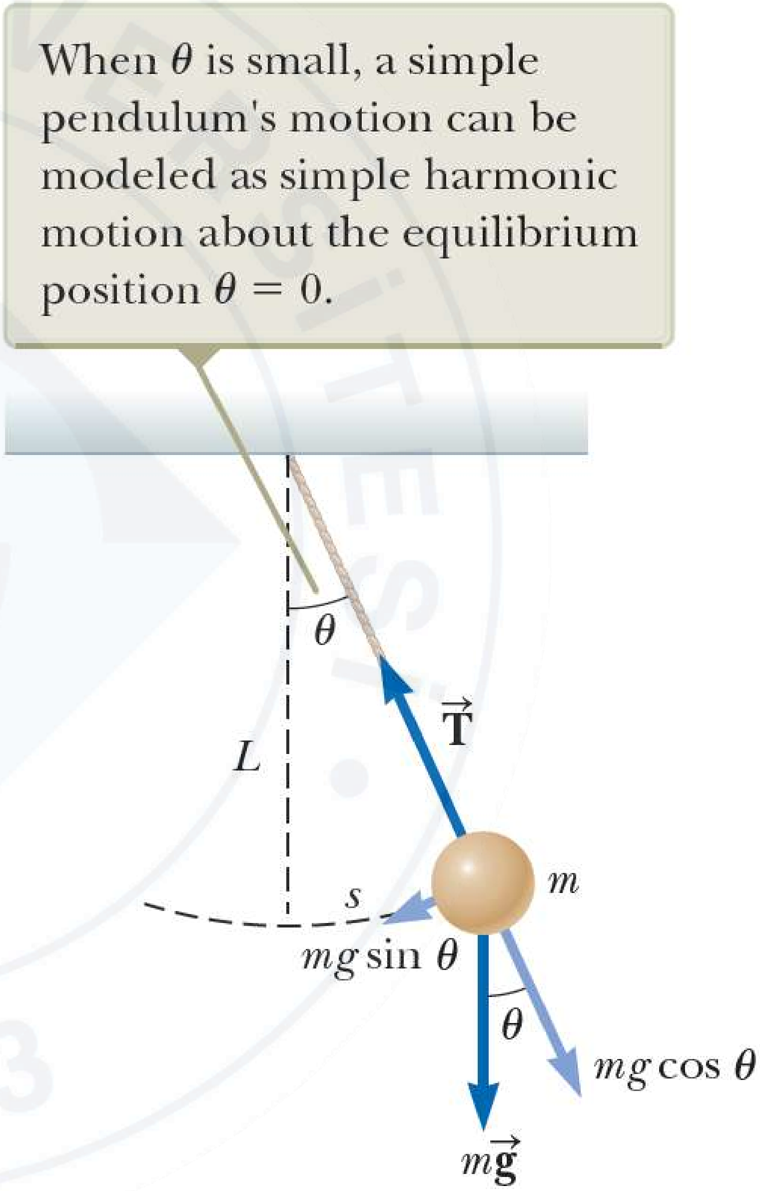


THEORY

As the pendulum oscillates, the pendulum experiences a restoring force with a magnitude of $mgsin(\theta)$. Writing Newton's second law when the pendulum is at θ;

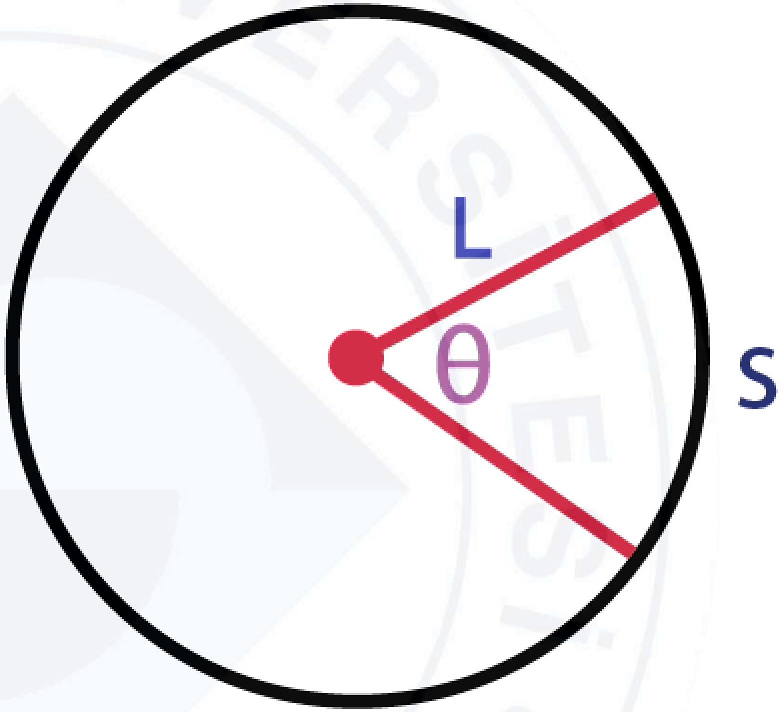
$$F = ma(\theta) = -mgsin(\theta)$$

$$a(\theta) = -gsin(\theta)$$



Recall the arc length s of a circle with radius r with central angle θ.

$$s = L\theta$$



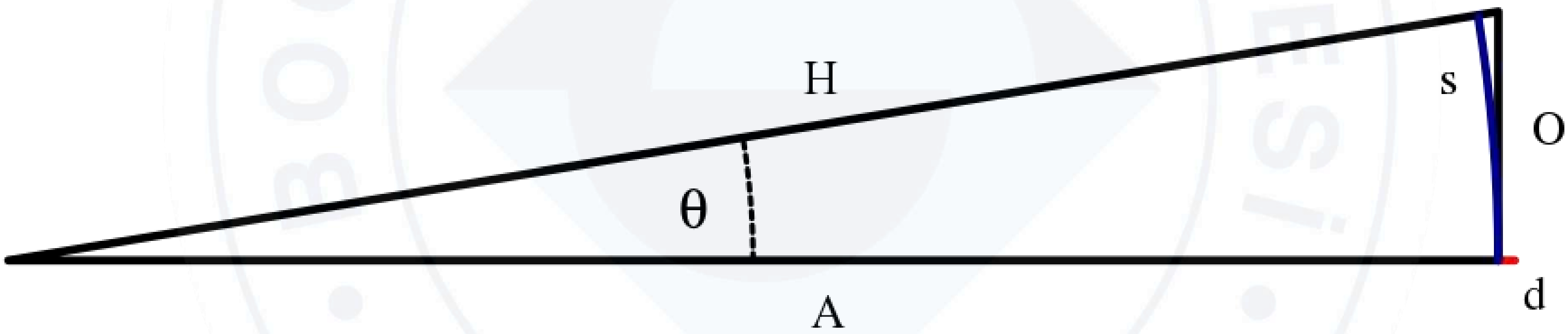
We take second time derivative of this equation. Since L is constant we get;

$$a = L\alpha$$

Here, α is the angular acceleration. Let us switch a with $L\alpha$ in the previous equation.

We get; $$\alpha(\theta) = -\frac{g\sin(\theta)}{L}$$

For small angles, $sin(\theta) \cong \theta$ is a good approximation. We can see a geometric justification below;



$$\sin\theta = \frac{O}{H} \approx \frac{s}{A} = \frac{A\theta}{A} = \theta$$

We had;

$$\alpha(\theta) = -\frac{g\sin(\theta)}{L}$$

Using $\sin(\theta) \cong \theta$ and writing α as second time derivative of θ;

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta \cong 0$$

This is a second order differential equation and its solution for θ is;

$$\theta(t) = \theta_0 \cos(\sqrt{\frac{g}{L}}t)$$

Since cosine function is periodic with 2π , inside of the cosine should be equal to 1 period T of the pendulum. Thus;

$$\sqrt{\frac{g}{L}}T = 2\pi \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{L}{g}}$$

If we leave g alone in the previous equation, we will get;

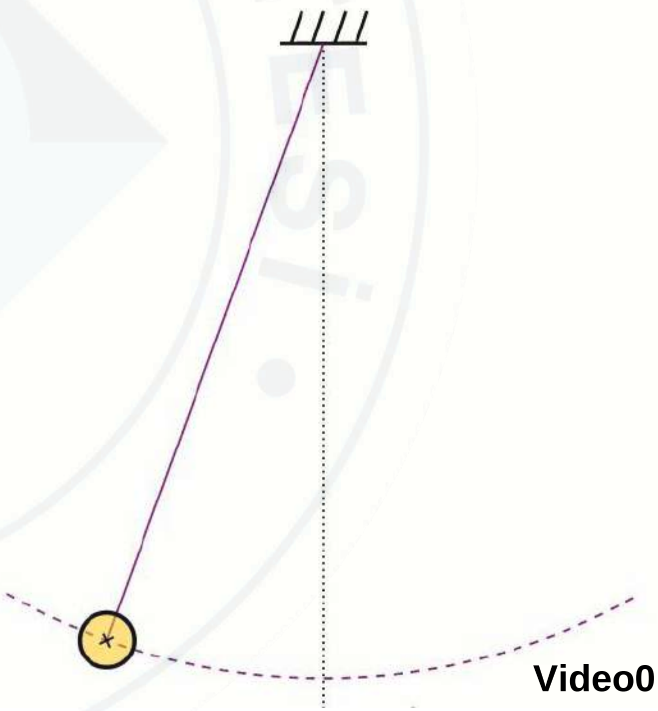
$$g = 4\pi^2 \frac{L}{T^2}$$

Note that the precision and accuracy of the gravitational constant g is determined by the precision and accuracy of length L and period T. It does not depend on mass m.

After finding the experimental value of gravitational acceleration g, we will compare it to the true value;

$$g_{tv} = 9.81 m/s^2 = 981 cm/s^2$$

Simulation of 1 period



Video0

BOĞAZİÇİ UNIVERSITY
Physics Department

APPARATUS

THE SIMPLE PENDULUM – Apparatus

It can move up and down

Makes the Length shorter or longer

Fixed point (support)

Length of the pendulum L

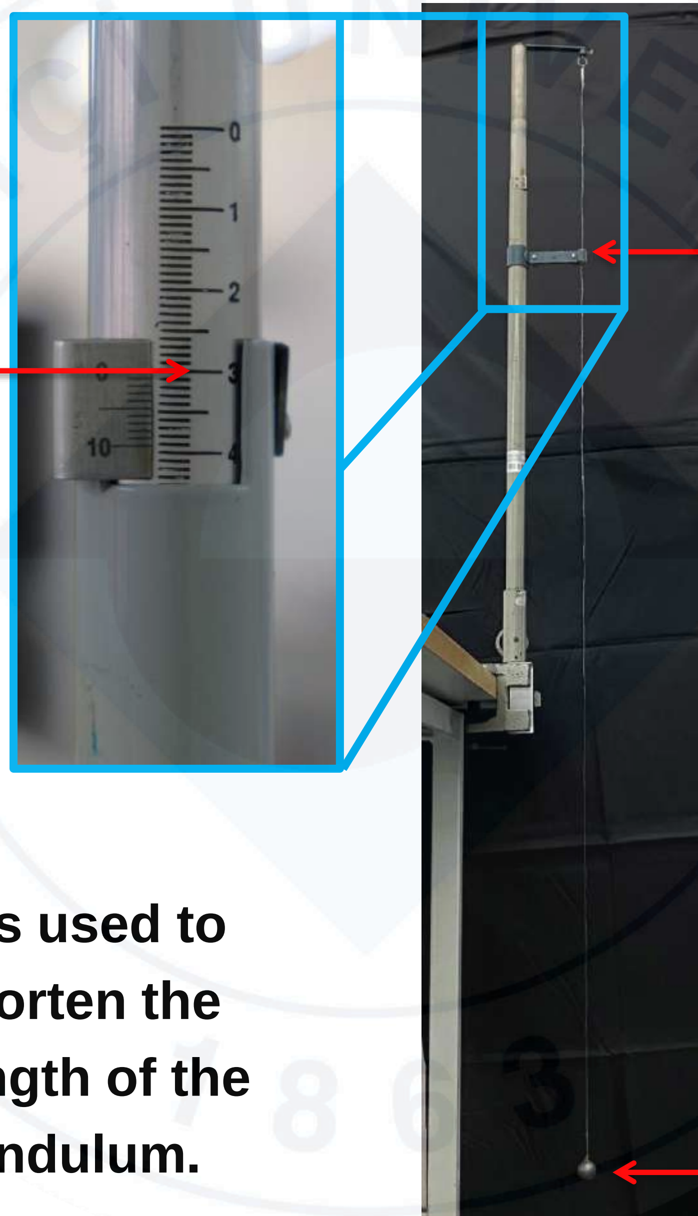
Center of Metal Ball

Conditions for Simple Pendulum

- **Point mass at the end**
- **Long string is massless and does not stretch**
- **Small oscillations $\sim 10^o$**

THE SIMPLE PENDULUM – Apparatus

This is where we read the Vernier.



It is used to shorten the length of the pendulum.

EXPERIMENT

On page 39 of your lab book, you will write down the true value of gravitational acceleration $g_{tv} = 9.81\ m/s^2$ and number of oscillations you are to record which will be 10.

Description	Symbol		Value & Unit
Acceleration due to gravity	g_{TV}	=	....
Number of Oscillations	N	=	....

THE SIMPLE PENDULUM – Experiment

PhysLabs

For the first measurement L_1, you will be shown the initial length of the pendulum in the DataVideos as given in these samples. Do <u>not</u> record these!

For L_1 measurement, Vernier is set to zero.

$$L_1 = 152.2cm - 3.6cm = 148.6cm$$

Then, you will record your L_1 to the first row of the first column.

Length of Pendulum L (unit)	*10 periods* t (unit)	*One Period* T (unit)
# of Significant Figures : s.f.	*# of Significant Figures :* s.f.	*# of Significant Figures :* s.f.
L_1		

THE SIMPLE PENDULUM – Experiment

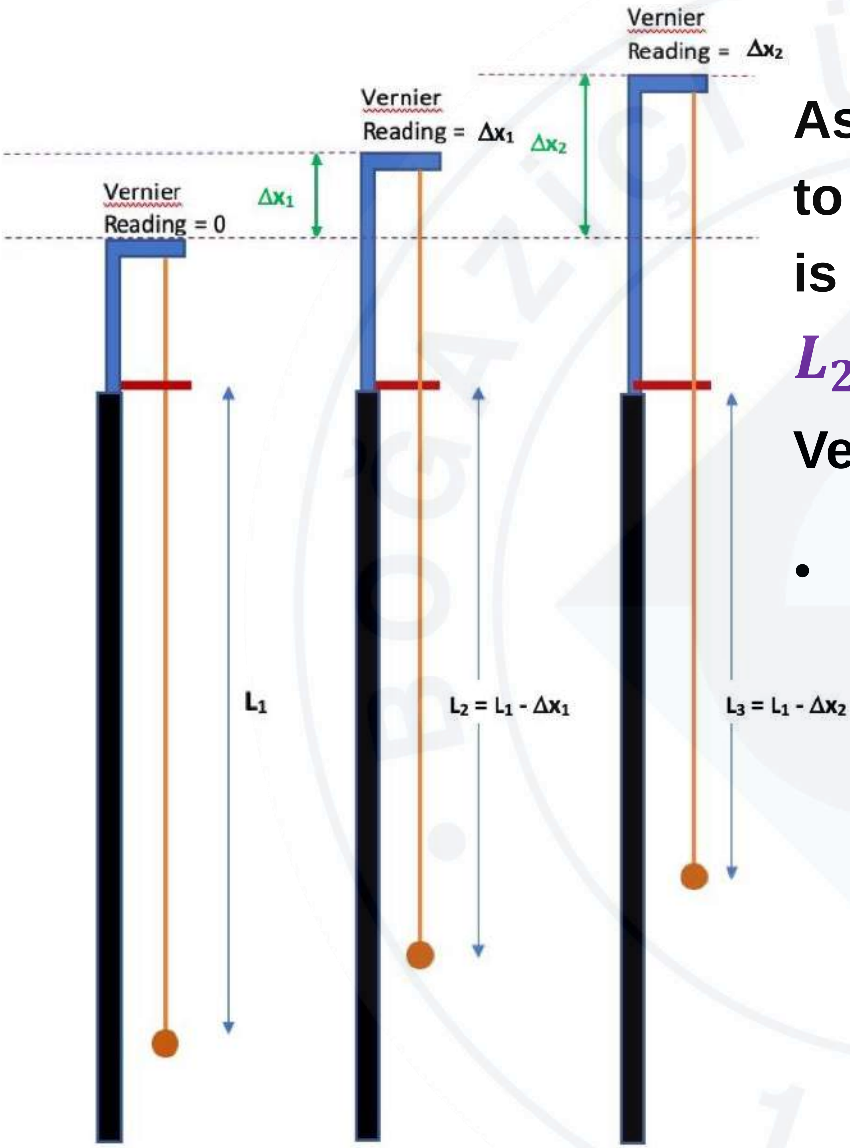
Right after the length measurement is done in the Lab, determine the time for 10 oscillations of the pendulum with that length.

Using the stopwatch of your cellphones, time *t* for 10 oscillations as shown on the left.

PhysLabs

Then, you will record your t_1 to the first row of the second column.

Length of Pendulum *L* (unit)	*10 periods* *t* (unit)	*One Period* *T* (unit)
# of Significant Figures : s.f.	*# of Significant Figures :* s.f.	*# of Significant Figures :* s.f.
L_1	t_1	



As the blue part that the string is attached to moves upward, the length of the string is shortened by the same amount. For L_2, L_3, L_4 and L_5, you will only need the Vernier reading Δx.

- **The following four L measurements will be derived from Vernier reading Δx;**
  - Vernier Reading: Δx_1 , $L_2 = L_1 - \Delta x_1$
  - Vernier Reading: Δx_2 , $L_3 = L_1 - \Delta x_2$
  - and so on..

PhysLabs

For the L measurements after L_1, here you have a clip of increasing Vernier reading. Here, $\Delta x_i = 3.0cm, x_f = 6.0\ cm$.

Video3

At the end of the increment, you will see a close-up shot of the final Vernier reading just like the one given below. So, length of the pendulum for this measurement is $L = L_1 - 6.0\text{cm}$. Just after, you will measure the oscillations with this L.

As you move on to different pendulum lengths, you will change the Vernier reading for that measurement and from that you will calculate L from it. Then you will measure t.

Length of Pendulum L (unit)	*10 periods* t (unit)	*One Period* T (unit)
# of Significant Figures : s.f.	*# of Significant Figures :* s.f.	*# of Significant Figures :* s.f.
L_1	t_1	
$L_2 = L_1 - \Delta x_1$	t_2	
$L_3 = L_1 - \Delta x_2$	t_3	
$L_4 = L_1 - \Delta x_3$	t_4	
$L_5 = L_1 - \Delta x_4$	t_5	

THE SIMPLE PENDULUM – Experiment

After measuring first two columns, finish the table by filling the third column. Please do not forget to fill units and significant figures.

Length of Pendulum L (unit)	*10 periods* t (unit)	*One Period* T (unit)
# of Significant Figures : s.f.	*# of Significant Figures :* s.f.	*# of Significant Figures :* s.f.
L_1	t_1	$T_1 = t_1/N$
$L_2 = L_1 - \Delta x_1$	t_2	$T_2 = t_2/N$
$L_3 = L_1 - \Delta x_2$	t_3	$T_3 = t_3/N$
$L_4 = L_1 - \Delta x_3$	t_4	$T_4 = t_4/N$
$L_5 = L_1 - \Delta x_4$	t_5	$T_5 = t_5/N$

On page 41 of your lab books, you are going to calculate 5 gravitational acceleration $g_1, g_2, \ldots$values from the data you have recorded to the previous table using the formula we have derived;

$$g_i = 4\pi^2 \frac{L_i}{T_i^2}$$

Symbol		Calculations (show each step)	Result & Unit
g_1	=	....	....
g_2	=	....	....

For the last part will be to take average of the $g_1, g_2, \ldots$ and record it as $g_{average}$. Then, you will use the formula given below to calculate the percent deviation. Finally, show the dimensional analysis of g.

$$\% \, Deviation \; for \; g = \frac{|g_{TV} - g_{average}|}{g_{TV}} \times 100$$

g_{average} = ...

% Deviation for g: ....

....

Show the dimensional analysis for g: ....