# Boğaziçi University

# Introductory Phys Labs

# Introduction to Measurement

## for PHYL 201

# Motivation

Suppose you are given the standard acceleration due to gravity as **10**. Does this value by itself make sense?

First of all, we need **units** to define the type of the quantity. Is it **m/s$^2$**, **ft/s$^2$**, or else?

Is **10m/s$^2$** enough? What might be **wrong** with this value?

Does it mean that standard acceleration due to gravity has as many m/s$^2$ as we have fingers?

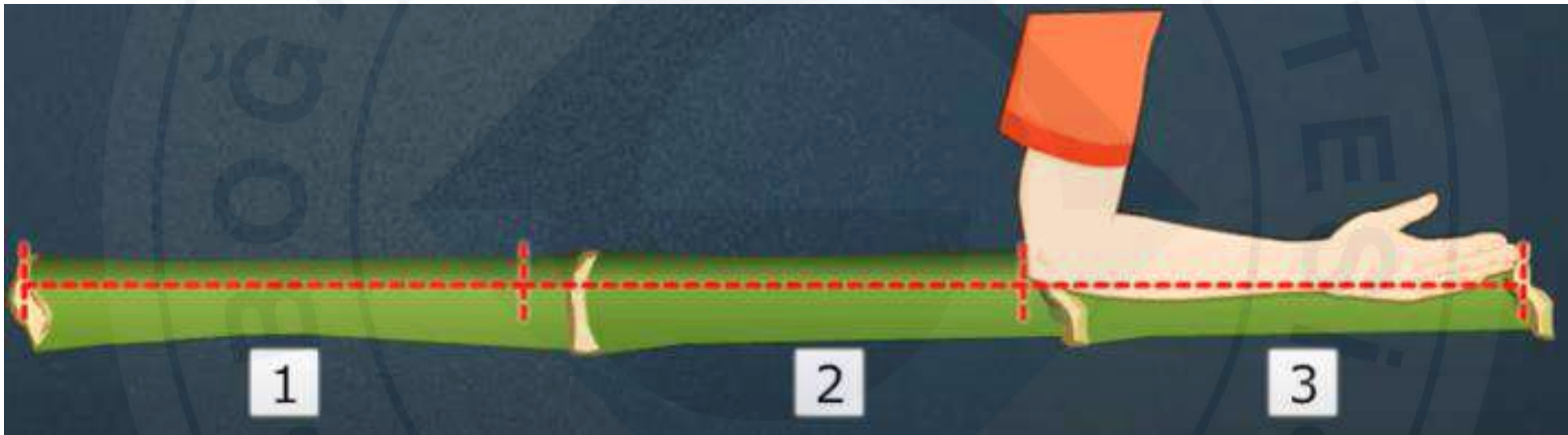So, somehow can we **count** standard acceleration due to gravity?

What is the difference between **counting** and **measurement**, if any?

BOĞAZİÇİ UNIVERSITY
Physics Department

# What is Measurement?

**Measurement** is a process for determining the size of a physical quantity.

We answer "how much?" via measurement.



For example, this bamboo stick is three cubits long.

# What is Measurement?

A cubit is an example of **non-standard** unit of measurement.

Other non standard units include

- **tablespoon for volume,**

- **grain for weight,**

- **and heart rate for time.**

Though these units are simple to use, they lack **precision**.

For scientific experiments, we need better than that. We need fixed, reliable units such as the **meter, gram, and second**.

# Precision and Accuracy

We feel that some measurements are better than others.

Measuring tape is better than human-based units in a length measurement, simply because it is more precise.

Precision is a measure of reproducibility in a measurement.

When precise, repeated measurements give close together results.

Precision is represented by the number of digits given.

For example, 176.3 cm is more precise than 176. cm.
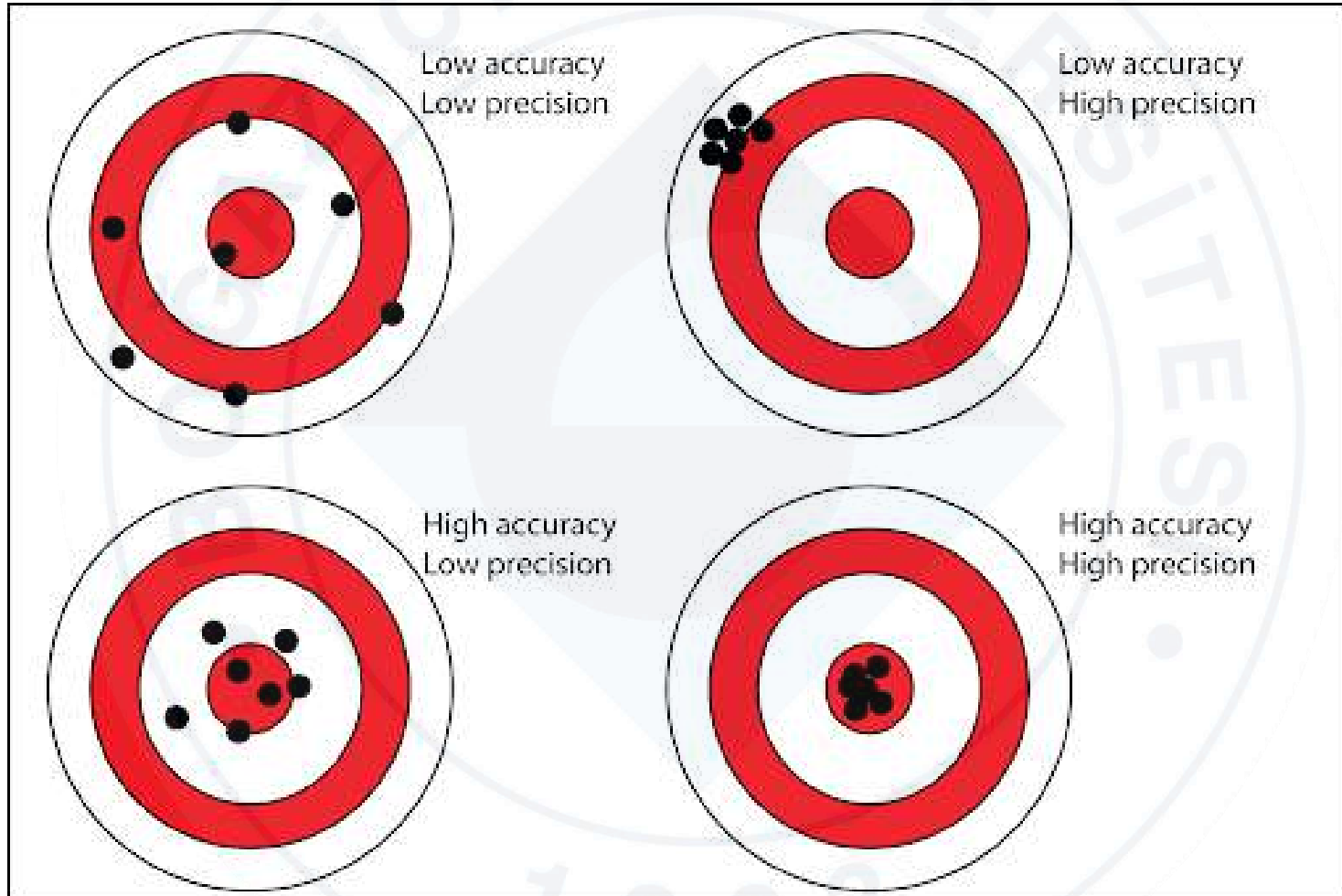
BOĞAZİÇİ UNIVERSITY
Physics Department
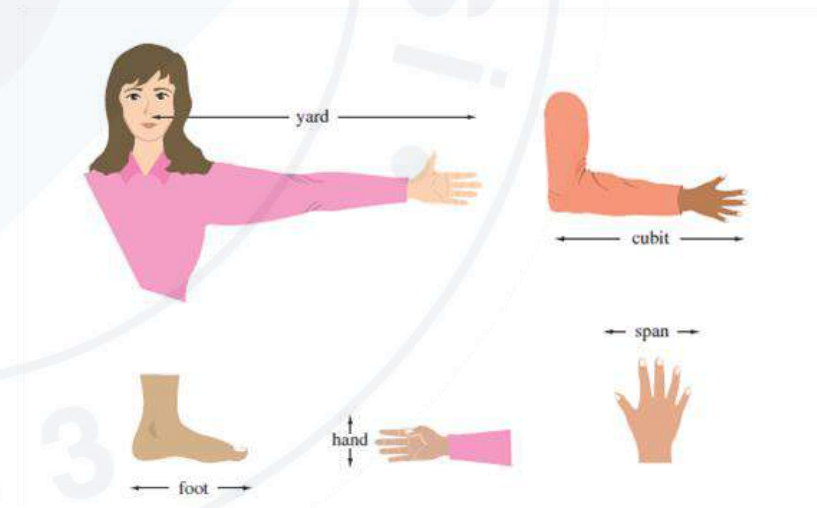
# Precision and Accuracy

However, precision does not mean **accuracy**, that is, closeness to the "true" value.

Your weight might appear as 314.15 kg on a broken precision balance. Though that value is more precise than 75. kg on a bathroom balance, say, which one do you think is more accurate?

# Precision and Accuracy

**A good measurement is both precise and accurate.**

**For precision, we use "better", more precise equipment.**

**For example, a micrometer is more precise than a vernier scale, which is more precise than a meterstick.**

**A meterstick reads up to 0.1 cm. It measures a barley grain as, say, 1.3 cm.**
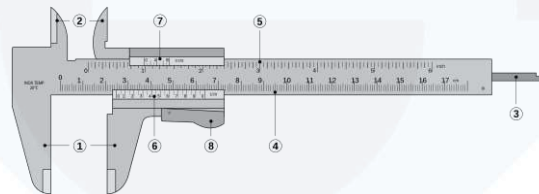
**A vernier scale reads up to 0.01 cm. It measures the same grain as, say, 1.34 cm.**

**A micrometer reads up to 0.001 cm. It measure the same grain as, say, 1.343 cm.**

# Precision and Accuracy

For accuracy though, better equipment is not enough. First of all, we can never be sure if our result is close enough to the "true" value. After all, what is true value?

Say, we measure our height ten times using a measuring tape. These ten measurements will not exactly match.

| Height (cm) |
| --- |
| 173.4 |
| 173.2 |
| 173.6 |
| 173.4 |
| 173.5 |
| 173.5 |
| 173.1 |
| 173.7 |
| 173.2 |
| 173.4 |

So, which one is the true value, if any?

Is it the most frequent reading (the mode)?

Is it the average, or median?

If so, how do we explain when these statistical measures change by different measurements?

It is clear that we can never know what true value is. We can only hope to approach it. For this, we need to take many measurements and take their average. The average of many measurements is our best shot at the "true value", or rather the accepted value.

# Precision and Accuracy

**Although, repeated measurements do not match, they follow a pattern.**

**They seem to distribute about a value. Their distribution is known as normal distribution.**
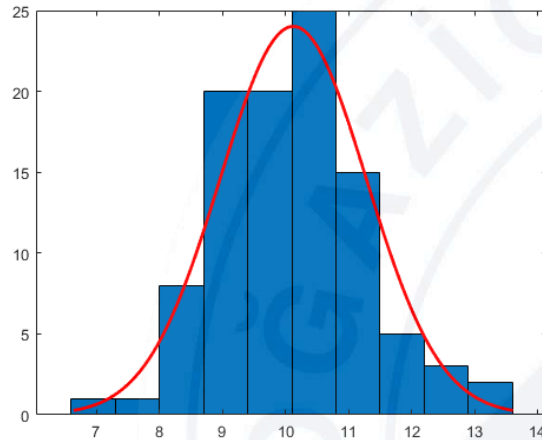
**Repeated measurements form a Gaussian curve (or bell curve).**
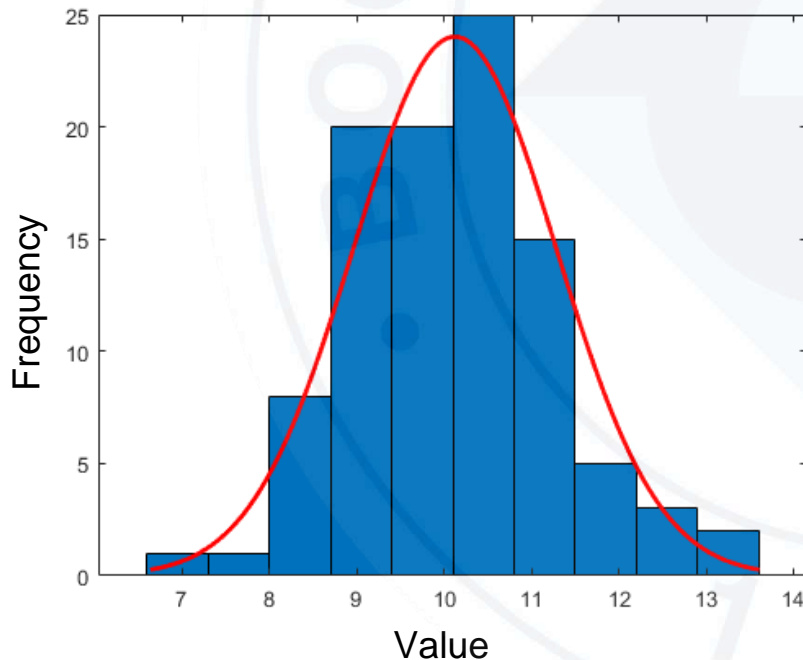


**This histogram shows a distribution of repeated measurements.**

**For example, a value between 10 and 11 is obtained 25 times.**

BOĞAZİÇİ UNIVERSITY
Physics Department

# Precision and Accuracy



Many measurements
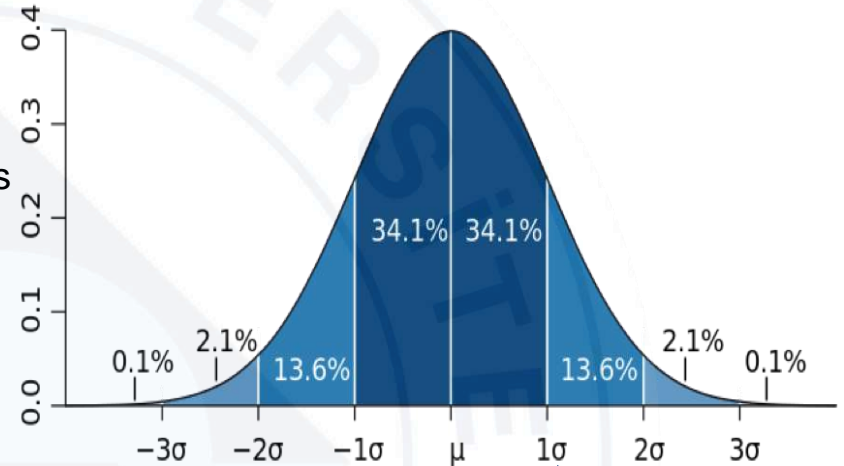
**On the right, we see a normal distribution.**

**Average**   **Standard Deviation**

**What we call "true value" is the average.**

**The idea is that the more measurements we take, the closer we get to the true value.**

**Standard deviation is a measure of spread and it represents error.**

BOĞAZİÇİ UNIVERSITY
Physics Department

# Error

**Error is inherent in measurements.**

**There are many sources of error. In measuring your height, for example,**

- **you may use a faulty measure tape,**
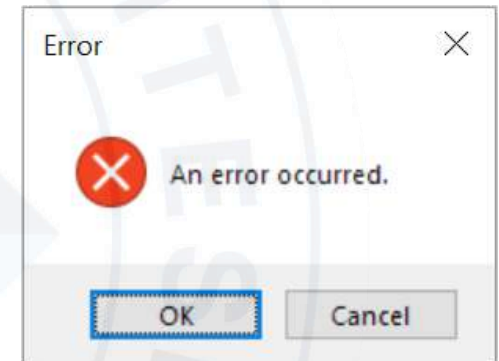
- **measure while lying down,**

- **or your hands may be shaking.**

Error

An error occurred.

OK    Cancel

**Sources of error are typically classified into three groups: instrumental error, systematic error and random error.**

# Error

**Instrumental error** is when you use bad equipment. To reduce instrumental error, simply use better, more precise measuring instruments.

**Systematic error**, also bias error, is when you follow a **faulty procedure**, use **poorly calibrated equipment**, or **disregard environmental conditions**. To reduce systematic errors, be careful and mindful of your measurement procedure.



- For example, any measurement you take with this ruler will be off by **1.0** cm because of end-point bias. So you should record the length of the pencil not as **11.0** cm. but as **10.0** cm.

**Random error**, also statistical error, is **due to chance**, and we cannot control it.

Random error causes fluctuation in repeated measurements. To reduce random error, we simply take many measurements and then take their average.

- Random error is responsible for **normal distribution**.

BOĞAZİÇİ UNIVERSITY
Physics Department

# Error

As error is inherent in measurement, we need a way to describe error in our results.

One common way of describing error is to introduce an error window using the $\pm$ sign, as in 176.3 $\pm$ 0.1 cm.

Such a result means more than a pure number. It means that if we measure the same length, the result will most likely be within 0.1 cm of 176.3 cm.

As you may guess, 176.3 $\pm$ 0.1 cm is more precise a measurement than, say, 176.3 $\pm$ 0.6 cm.

A bare result such as 176.3 cm lacks its measure of precision and therefore is meaningless.
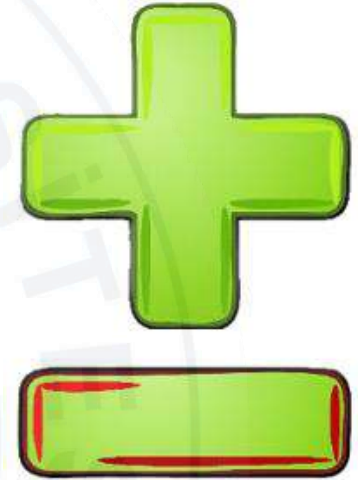
# Significant Figures

However, we assign a meaning to such values as 176.3 cm.

Our convention is that, in a result, when not stated explicitly, the error is in the last digit, and its size is determined by the measuring instrument.

So, 176.3 cm really means 176.3 $\pm$ 0.1 cm.

Consider you step on a digital bathroom scale and read 72.0 in kilograms.

If you report your result as 72. kg, it is implied that your weight is within 1. kg of 72. kg.



Yet, you actually know better than that; your weight is within 0.1 kg of 72.0 kg.

So, the last digit in 72.0 kg is significant. You cannot take it off without changing the meaning of the measured value.

# Significant Figures

**Significant figures** are important because

- **they tell precision,**

- **they help determine the precision of calculated, not directly measured, values such as standard acceleration due to gravity.**

**85.6 cm**

**Suppose you want to find the area a rug covers.**

**You measure the sides, and multiply them.**

**124.2 cm**

**Knowing the errors of side measurements, what can we say about the error in area?**

**We will talk more about significant figures.**

# A Digression on Error

When we say error, we usually mean the precision of a result. For example, 176.3 $\pm$ 0.1 cm has an error of 0.1 cm.

However, error can also indicate accuracy.

When we compare the experimental value, with the true value, their difference is also known as error.

This error shows how accurately we have carried out the experiment.

For example, suppose standard gravity, g, in Istanbul is given as 9.808 m/s$^2$. We accept this value as the true value (TV).

Then we carry out an experiment and determine g as 9.78 m/s$^2$. We call this the experimental value (EV).

The difference between the two values will be called error, as well.

BOĞAZİÇİ UNIVERSITY
Physics Department

# A Digression on Error

Generally, we talk of **percentage error** in this context. It is calculated as

$$\% \text{ Error} = \frac{|TV - EV|}{TV} \times 100$$

where TV and EV denote true value and experimental or evaluated value, respectively.

Suppose now all we have is two experimental values, $EV_1$ and $EV_2$

Then we calculate the error as

$$\% \text{ Error} = \frac{|EV_1 - EV_2|}{(EV_1 + EV_2)/2} \times 100$$

BOĞAZİÇİ UNIVERSITY
Physics Department

# Recording Measurement

**In general, to what precision we should record our measurement is dictated by the measuring instrument.**

**For example, a typical ruler has a resolution of 1 mm. Therefore, a typical ruler scale enables us to read up to 1 mm, no more, no less.**



**How should we record the length of this pencil?**

**Both 10 cm and 10. cm are incorrect. Our result must echo the resolution of the instrument: 10.0 cm.**

# Recording Measurement

**Digital readings work the same way as analog readings.**

**What does this stopwatch read?**

**It is 303.00 seconds.**

**Although there is no difference between 303.00 and 303. to a calculator, these two values are quite different from the standpoint of physics. In other words, the last two zeroes are significant.**

**So, make sure you record up to the resolution of your measuring instrument. Otherwise you risk misperception.**

# Two Rules of Thumb

**Consider the following measurement.**
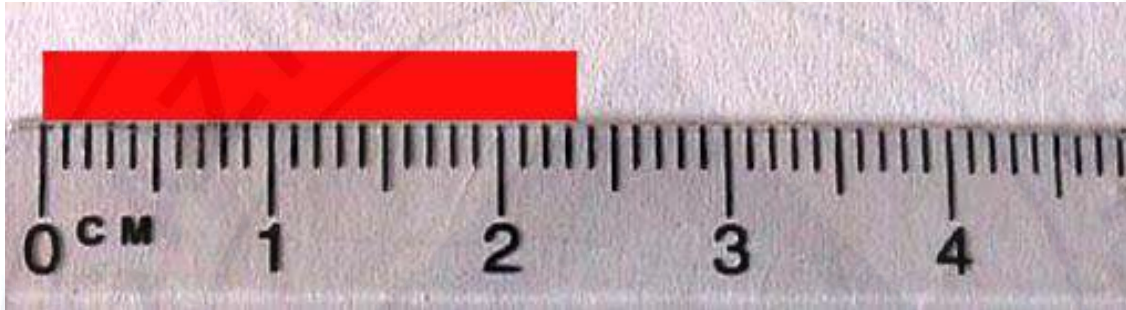


The red strip stretches from **0.0** to somewhere in between **2.3** and **2.4**. In such cases, as a rule of thumb, we should record **the nearest tick**, in this case **2.3 cm**.

**Another issue is about the precision of given values of lab equipment.**

Say, you are given a washer of **100 g**, or a resistor of **50 Ω**.

**!** If you do not measure these yourself and there is no indication that these are measured and given to you, you should accept and record these values **as they are**.

In this case, all 0's are significant, and these values are no more precise than their last digit. So they are **not** 100.0 g or 50.00 Ω, but **100. g** and **50. Ω**.

# Few Common Mistakes

**Measurements done with a single instrument do not have to share the same number of significant figures.**
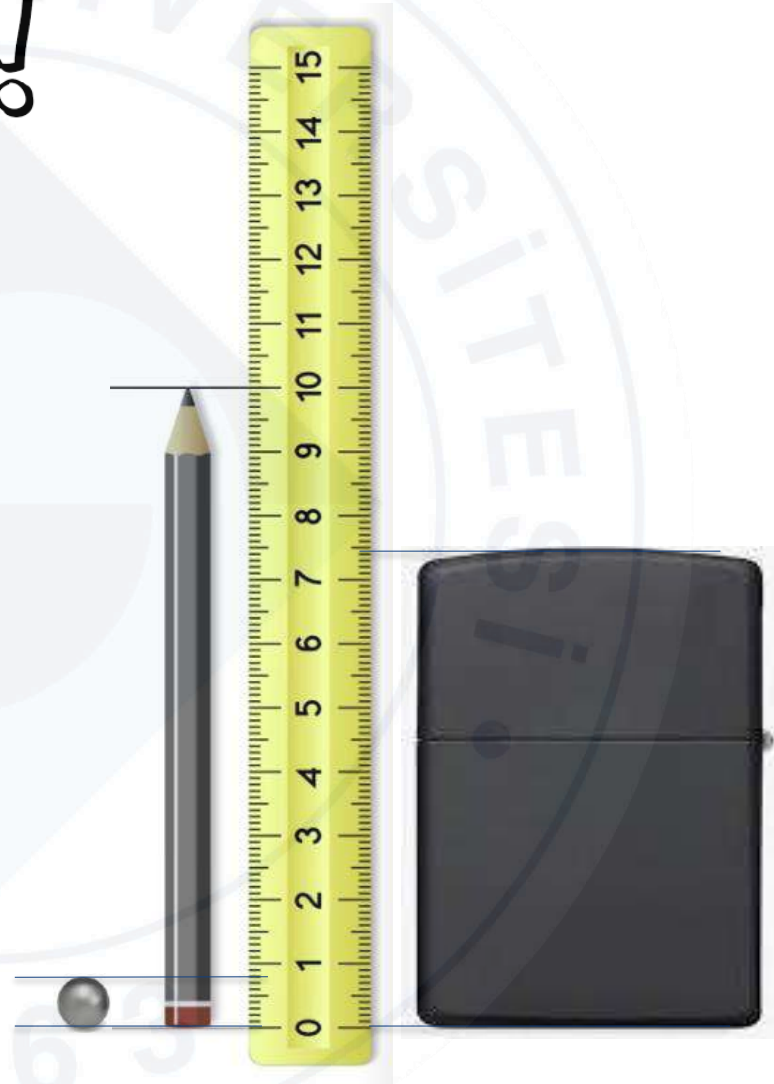
**For example, consider the three measurements shown on the right.**

**The pencil is 10.0 cm. It has 3 s.f.**

**The lighter is 7.4 cm. It has 2 s.f.**

**The steel ball is 0.8 cm. It has 1 s.f**

# Few Common Mistakes

**Consider this stack of coins.**

**A stack of 37 coins is 12.1 cm tall.**

**Dividing 12.1 cm by 37, we get 0.327 cm.**

**If we measured a single coin, we could measure only up to 0.1 cm precision; we would get 0.3 cm.**

**But measuring many at once and dividing by the total number gives more significant figures.**

**We will see more about significant figure arithmetic later on.**

BOĞAZİÇİ UNIVERSITY
Physics Department

- **Physical values are dimensionful, so they require units.**

- **Error is inherent in measurements. Physical values without error are meaningless.**

- **Rigor in measurement corrects systematic error, and repetition reduces random error. Multiple measurements done carefully is the best we can do about error.**

- **We imply error with significant figures. Typically, error is in the last digit.**

- **Precision of a measurement is dictated by the measuring instrument. The precision of a measured value should be the same as the resolution of the instrument. For example, you can read on a regular meterstick 176.3 cm, but not 176. cm or 176.30 cm.**

# Significant Figures

# Significant Figures

**Significant figures** are the 'meaningful' digits that account for the precision of measurements or values derived from those measurements.

**Benefiting from these figures, we can be consistent while reading measurements in devices and noting the values in an experiment.**

We find these significant figures to show the values properly.

- We count the number of significant figures in measurements, since they will determine the precision or the number of significant figures of calculated values.

So, how can we find and count these significant figures?

BOĞAZİÇİ UNIVERSITY
Physics Department

# Rules for Counting Significant Figures (sf)

## Nonzero integers are always significant

**If a digit has a nonzero value, we can say that the digit has a meaningful contribution**

**to the measurement.**

3456. cm        4 sf

12.97 cm        4 sf

1.11278 cm        6 sf

# Rules for Counting Significant Figures (sf)

## Leading zeros are always insignificant

If a measurement begins with zeros, we can say that those zeros do not contribute to the precision of that measurement and they can be written in a more scientific notation.

0.0034 ms    2 sf    ➡    $3.4 \times 10^{-3}$ ms

0.123 ms    3 sf    ➡    $1.23 \times 10^{-1}$ ms

# Rules for Counting Significant Figures (sf)

## Zeros between nonzero integers are always significant

**Since there will be at least a nonzero value after those zeros and**

**the last nonzero value will determine the precision of the measurement,**

**we can say that zeros in the middle are significant.**

0.003004 kg     4 sf     ➡     $3.004 \times 10^{-3}$ kg

102.003 g     6 sf

# Rules for Counting Significant Figures (sf)

## Trailing zeros are significant if a decimal point exists

- **If zeros exist after the decimal, it must be significant since it indicates the precision on that digit.**

- **However, if there is no decimal and you know that *this is a measurement* (poorly reported one), trailing zeros will be insignificant.**

1.600 A          4 sf

1200 mA          2 sf          ➡          $1.2 \times 10^3$ mA

# Significant Figures

- **Operations with values that have no significant figures do not change the significant figure of the result, i.e., averaging.**

$$\frac{11.\ s + 33.\ s}{2} = 22.\ s \qquad \text{not 20 s}$$

- **Rounding**

→ **Round numbers to 2 sf**

37.2793 → 37.
37.9793 → 38.
37.5793 → 38.
38.5793 → 38.

- **Handling 0's in measurement devices while reporting**

**0 at the end is also significant since the device can measure that digit**

**Example: A ruler with units of cm**

… 127 128 129 130 131 132 133 …
3 sf  3 sf  3 sf  ???  3 sf  3 sf  3 sf
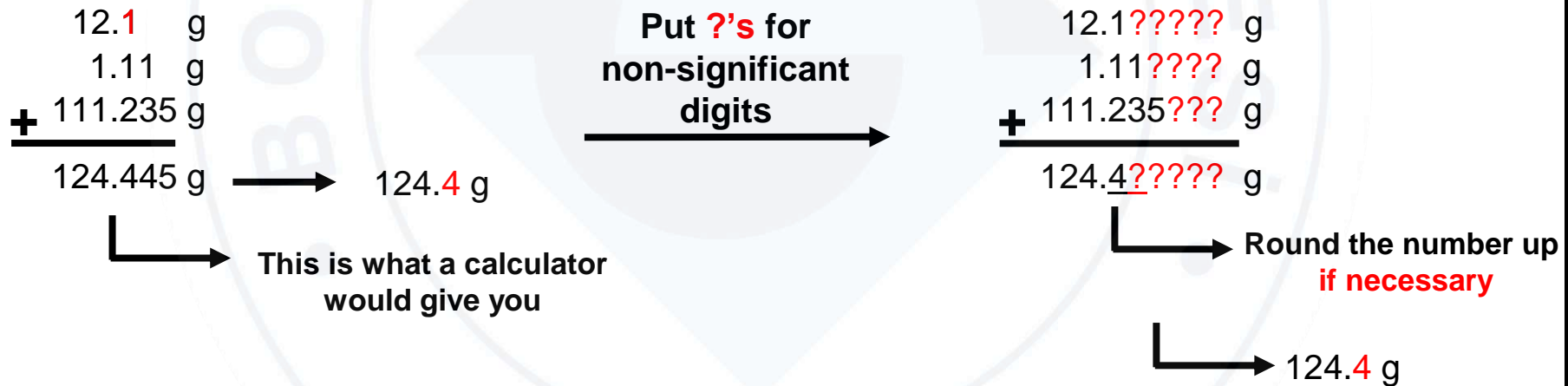
130 cm

13.0 x 10$^1$ cm
1.30 x 10$^2$ cm
130. cm

→ Also report the other measurements like **127** and **128** with a decimal point as **127.** and **128.** so as not to confuse them with integers.

BOĞAZİÇİ UNIVERSITY
Physics Department

# Calculations with Significant Figures

# Calculations with Significant Figures

## Addition and Subtraction (A&S)

In addition and subtraction, the precision of a result is determined by the last significant digit of the least precise value used within the operation.

12.1    g
1.11    g
+ 111.235 g
_____
124.445 g  →  124.4 g

This is what a calculator would give you

**Put ?'s for non-significant digits** →

12.1????? g
1.11???? g
+ 111.235??? g
_____
124.4????? g

**Round the number up if necessary**

→ 124.4 g

**Note**: As seen in this example, the result has 4 significant figures, even though none of the terms added has 4 significant figures.

# Calculations with Significant Figures

## Addition and Subtraction (A&S)

In addition and subtraction, the precision of a result is determined by the last significant digit of the least precise value used within the operation.

**More Examples:**

$3 \times 10^1$ mg

$$\begin{array}{r} 30 \text{ mg} \\ + \quad 142.1 \text{ mg} \\ \hline 172.1 \text{ mg} \end{array}$$

**"0" in "30" is not significant**

170  g

$3.0 \times 10^1$ mg

$$\begin{array}{r} 30. \text{ mg} \\ + \quad 142.1 \text{ mg} \\ \hline 172.1 \text{ mg} \end{array}$$

**"0" in "30." is significant**

172  g

$$\begin{array}{r} 20 \text{ mg} \\ 30. \text{ mg} \\ + \quad 142.0 \text{ mg} \\ \hline 192.0 \text{ mg} \end{array}$$

?    190   g

BOĞAZİÇİ UNIVERSITY
Physics Department

## Addition and Subtraction (A&S)

➤ **An Exception: If you are instructed to make some number of independent measurement of a quantity and just find the average value, report the average with the precision of the measurement with the least s.f.**

**If you round off your result by regarding the s.f. of the sum, realize that the precision can increase indefinately when you increase the number of measurements.**

| | | |
|---|---|---|
| 6.21 | m | |
| 6.18 | m | |
| 6.25 | m | } 5 |
| 6.21 | m | |
| + 6.23 | m | |
| 31.08 | m | |

$$\frac{31.08 \text{ m}}{5} = 6.2\cancel{16} \text{ m}$$
$$= 6.22 \text{ m}$$

| | | |
|---|---|---|
| 6.22 | m | |
| 6.19 | m | |
| 6.21 | m | } 20 |
| ⋮ | | |
| + 6.25 | m | |
| 124.15 | m | |

$$\frac{124.15 \text{ m}}{20} = 6.20\cancel{75} \text{ m}$$
$$= 6.21 \text{ m}$$

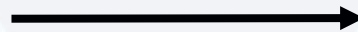| | | |
|---|---|---|
| 6.19 | m | |
| 6.24 | m | |
| 6.22 | m | } 200 |
| ⋮ | | |
| + 6.17 | m | |
| 1241.67 | m | |

$$\frac{1241.67 \text{ m}}{200} = 6.20\cancel{835} \text{ m}$$
$$= 6.21 \text{ m}$$

# Calculations with Significant Figures

**Multiplication and Division (M&D) (also valid for powers and roots)**

**The number of significant figures in the result is equal to the least significant figure of a measurement within the operation.**

12.1 cm ➡ 3 sf
✕  4.0 cm ➡ 2 sf
48.4 cm² ➡ 48 cm²

12.1? cm
✕  4.0? cm
.????
0.000
+ 48.4?
48.???? cm²

➡ 48 cm²

BOĞAZİÇİ UNIVERSITY
Physics Department

# Calculations with Significant Figures

## Logarithm

*mantissa*

$$\text{Log } (1.71 \times 10^1) \quad = 1.233$$

*characteristic*

**The part of a logarithm after the decimal point is called *mantissa*, while the part before the decimal point is called *characteristic*.**

**When taking the logarithm of a number, the mantissa of the logarithm should have the same number of significant figures as the number whose log is being taken.**

**The characteristic of a logarithm indicates the exponent when the number is expressed as a power of ten.**

$$\log 2.53 = \log (2.53 \times 10^0) = 0.403$$

$$\log 25.3 = \log (2.53 \times 10^1) = 1.403$$

$$\log 253 = \log (2.53 \times 10^2) = 2.403$$

$$\log 253.3 = \log (2.533 \times 10^2) = 2.4036$$

$$\log 253.34 = \log (2.5334 \times 10^2) = 2.40370$$

**BOĞAZİÇİ UNIVERSITY**
Physics Department

# Calculations with Significant Figures

## Mixed Operations

The previous rules for A&S and M&D still apply, but order of operations should additionally be taken into account. It is as follows:

**Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction**
**(Please Excuse My Dear Aunt Sally)**

While applying rules according to the order of operations, keep a track of what the limiting significant figure is in each step without rounding in intermediate steps.
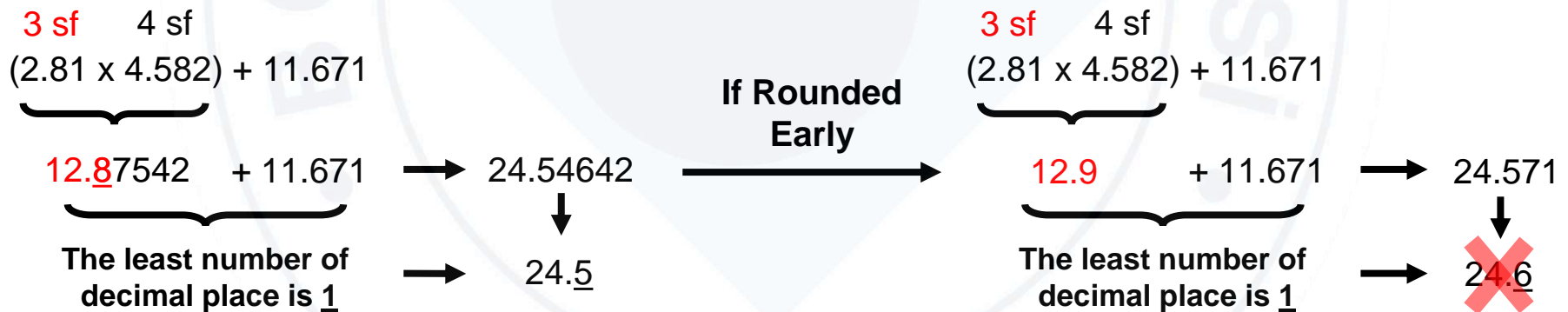


**Keep in mind that you should always round off your result at the end of your calculations.**

# Dimensions and Units

# Dimensions and Units

A dimension is a measure of a physical quantity without numerical values, while a unit is the way of assigning a value to that dimension.

Whereas a physical quantity has one type of dimension, it may have more than one unit.

| DIMENSION | VALUE |
| --- | --- |
| Length | 7 meters = 700 centimeters = $7 \times 10^{-3}$ kilometer |
| Mass | 5 kilograms = $5 \times 10^{3}$ grams |
| Time | 1 hour = 60 minutes = 3600 seconds |

# Dimensions and Units

- **In physics, there are 7 base or fundamental quantities.**

- **The rest of the physical quantities are called derived quantities and expressed in terms of the base quantities.**

- **The following table shows these 7 base quantities. The orange ones are the base quantities you will encounter in the introductory physics labs.**

| Base Quantity | Dimension | International System of Units (SI Units) | Unit Symbol |
|---|---|---|---|
| Length | L | Meter | m |
| Mass | M | Kilogram | kg |
| Time | T | Second | s |
| Electric current | I | Ampere | A |
| Absolute Temperature | Θ | Kelvin | K |
| Amount of substance | N | Mole | mol |
| Luminous intensity | J | Candela | cd |

# Dimensions and Units

**Dimensional analysis is the way of expressing the dimension of a derived quantity as a product of the dimensions of the base quantities.**

**Examples:**

| DERIVED QUANTITY | DIMENSION |
|:---:|:---:|
| Velocity | Length / Time = L / T = $L\,T^{-1}$ |
| Frequency | 1 / Time = 1 / T = $T^{-1}$ |
| Density | Mass / (Length)$^3$ = M / $L^3$ = $M\,L^{-3}$ |

**More examples:**

**Dimension of force**

**To find its dimension what we need is a force formula. We know that**

**F = m a where F, m and a are force, mass and acceleration respectively. So,**

$$[F] = M \times L / T^2 = M \, L \, T^{-2} \, .$$

BOĞAZİÇİ UNIVERSITY
Physics Department

# Dimensional Analysis

**More examples:**

**Dimension of energy**

**Although it does not matter which energy formula is chosen, let's take kinetic energy formula into consideration first. Then we can also consider the potential energy and can check that their dimensions are really the same as stated.**

**The kinetic energy is given by $E_k = \frac{1}{2} m v^2$ , so the dimension of energy is**

$$[E] = M \times ( L / T )^2 = M L^2 T^{-2} .$$

**The potential energy is given by $E_p = m g h$ , so the dimension of energy is**

$$[E] = M \times L/T^2 \times L = M L^2 T^{-2} .$$

BOĞAZİÇİ UNIVERSITY
Physics Department

# Dimensional Analysis

**Why is the dimensional analysis important?**

**Let's say you are confused with the formula for the period of simple harmonic motion and can't remember its correct form. Was it $T=2\pi\sqrt{g/L}$ or $T=2\pi\sqrt{L/g}$?**

**How to figure it out?**

**We know that**

**the period T must have the dimension of time that is T ,**

**the length of the pendulum L has the dimension of length that is L ,**

**the gravitational acceleration g has the dimension of acceleration that is L T$^{-2}$**

**and $2\pi$ is a dimensionless constant. So,**

$[T] = [2\pi\sqrt{g/L}] = (L^{1/2}\ T^{-1})\ /\ L^{1/2} = T^{-1}$ 　　　$[T] = [2\pi\sqrt{L/g}] = L^{1/2}\ /\ (L^{1/2}\ T^{-1}) = T$

　　　　　　**incorrect** 　　　　　　　　　　　　　　　　**correct**

# Unit Systems

## MKS

**M for meter**

**K for kilogram**

**S for second**

## CGS

**C for centimeter**

**G for gram**

**S for second**

## Examples:

**Force:**

5 Newton = 5 kg m s$^{-2}$

$5 \times 10^5$ dyne = $5 \times 10^5$ g cm s$^{-2}$

**Energy:**

7 Joule = 7 kg m$^2$ s$^{-2}$

$7 \times 10^7$ erg = $7 \times 10^7$ g cm$^2$ s$^{-2}$

**BOĞAZİÇİ UNIVERSITY**
Physics Department

# Plotting a Graph

# Plotting a Graph

**Graph plotting enables us to see what kind of relation exists between two physical quantities in an experimental setup. So, in order to see that relation we need to**

- **collect data by doing the experiment,**

- **mark data on a graph paper,**

- **draw best fit line or curve based on the trend presented by data points,**

- **analyse the line or the curve.**

# Collecting Data

**Let's say in order to find the velocity of a car (i.e. the relation between the position and the time),**

- **we have done an experiment**

- **and collected the data for the position of a car at successive time intervals as in the table below.**

| Time: t (second) | Position: x (meter) |
|---|---|
| 0.00 | 1.2 |
| 0.10 | 1.6 |
| 0.20 | 1.9 |
| 0.30 | 1.8 |
| 0.40 | 2.5 |
| 0.50 | 2.9 |
| 0.60 | 3.4 |
| 0.70 | 3.6 |

# Marking Data Points

**Since we will draw position vs time graph, we should**

- **label the axes and write the units accordingly,**

- **scale the axes based on the data range and the resolution of the data points (while the position data has 0.1 meter resolution, the time data has 0.01 second resolution in our case),**

- **mark the data points.**

BOĞAZİÇİ UNIVERSITY
Physics Department
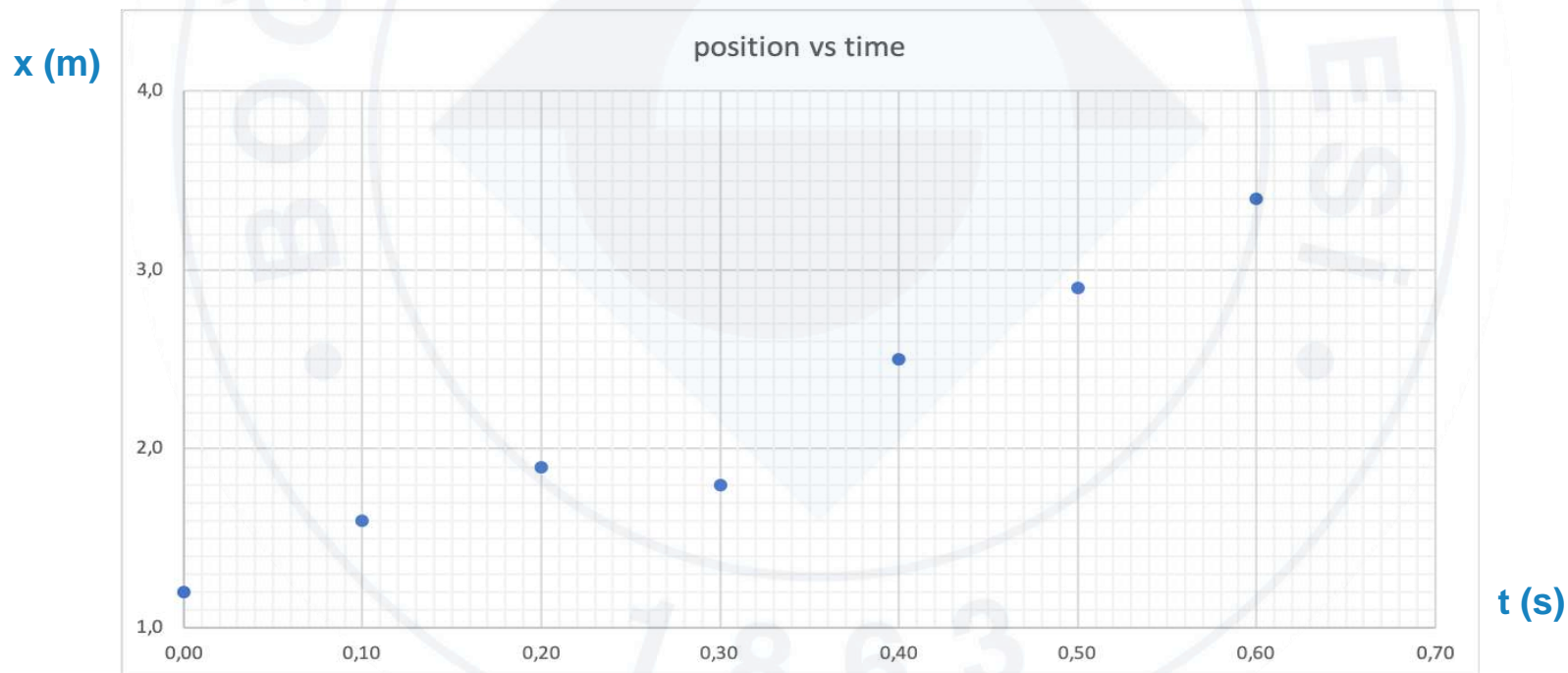
# Drawing Best-fit Line

**How to draw the best-fit line**

- **Before drawing the line or the curve, we should consider the distribution of the data points to decide the trend between two physical quantities.**

- **In our case, based on the data it is obvious that there is a linear relation betwen the position and the time.**

- **So, we should draw a best-fit line as below, not a curve!**



**Keep in your mind that since the experimental error is always present and inevitable, our best-fit line does not necessarily have to pass through all data points.**

# Drawing Best-fit Line

## Wrong way of drawing the best-fit line

Do not connect your data points by making zigzag as below !!! Instead, use a ruler to draw a straight line.
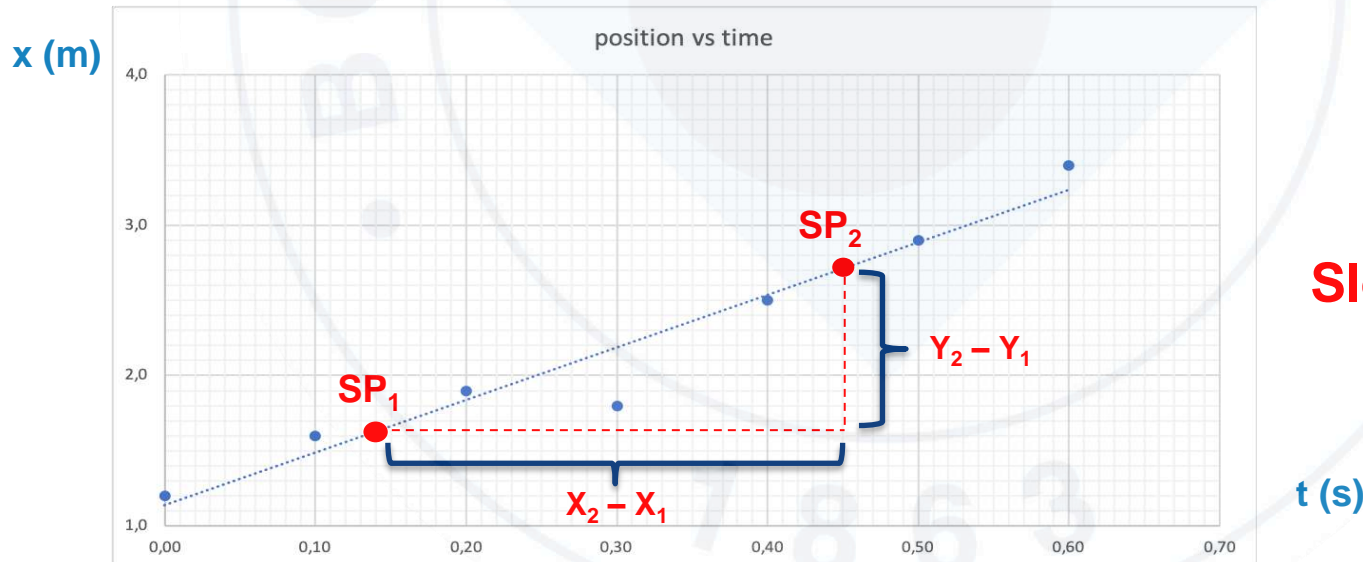


BOĞAZİÇİ UNIVERSITY
Physics Department
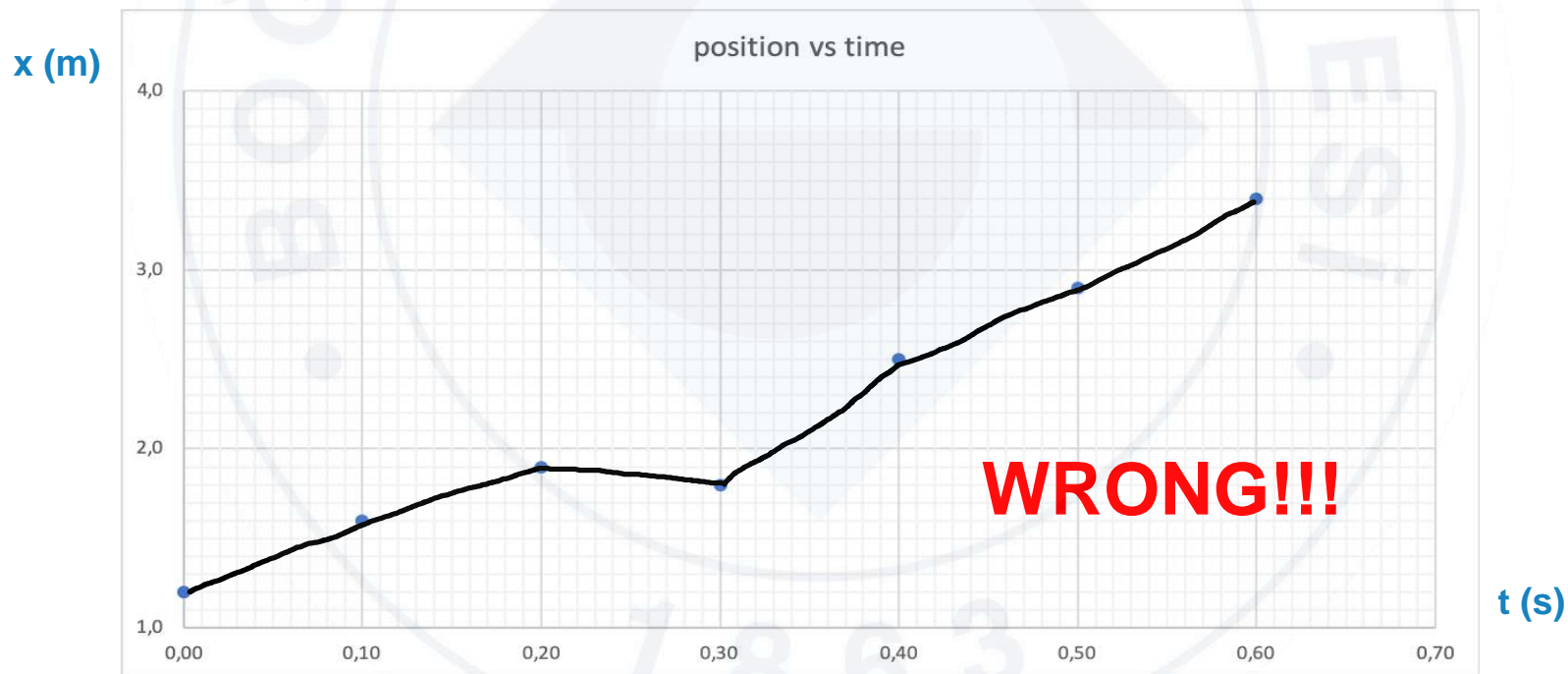
# Analysing the Best-fit Line : Slope

Once we draw the best-fit line, we accept that the most accurate position values for every time coordinate lie along that line. Thus, we should forget about the data points and focus on the best-fit line.

Since we are looking for the velocity of the car which is, by definition, the change in the position in unit time, we should calculate the slope of the best-fit line which is equal to the velocity in our case.
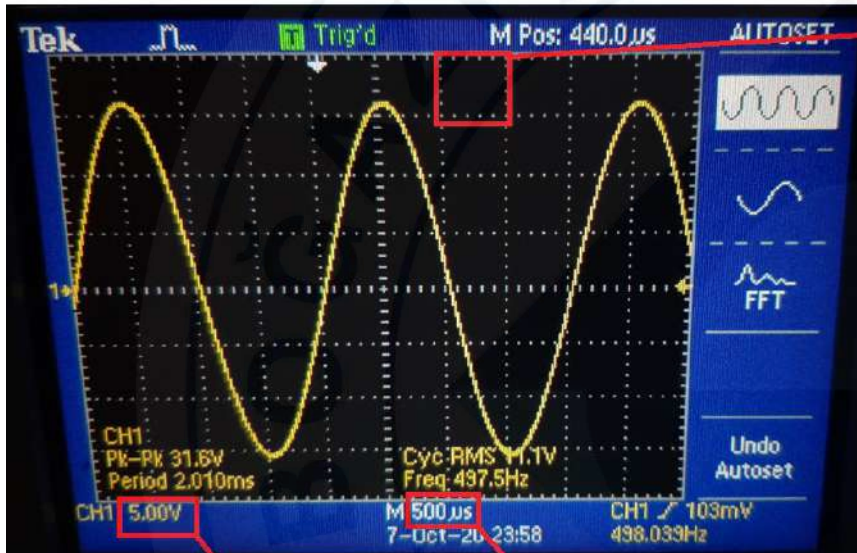
- **Select two slope points on the best-fit line.**

- **Determine the X and Y coordinates of these two slope points.**



$$\text{Slope} = \frac{|Y2 - Y1|}{|X2 - X1|}$$

# APPENDIX

## for PHYL201

## AMMETER (with non-linear scale)

## MICROMETER



| Sleeve value | 7 |
| Thimble value | 0.37 |
| Reading value | 7.37(mm) |

**1 Turn**

||

**100 in Thimble Value**

||

**1 in Sleeve Value**

||

**1.00 mm**

**?    3.15 mm**
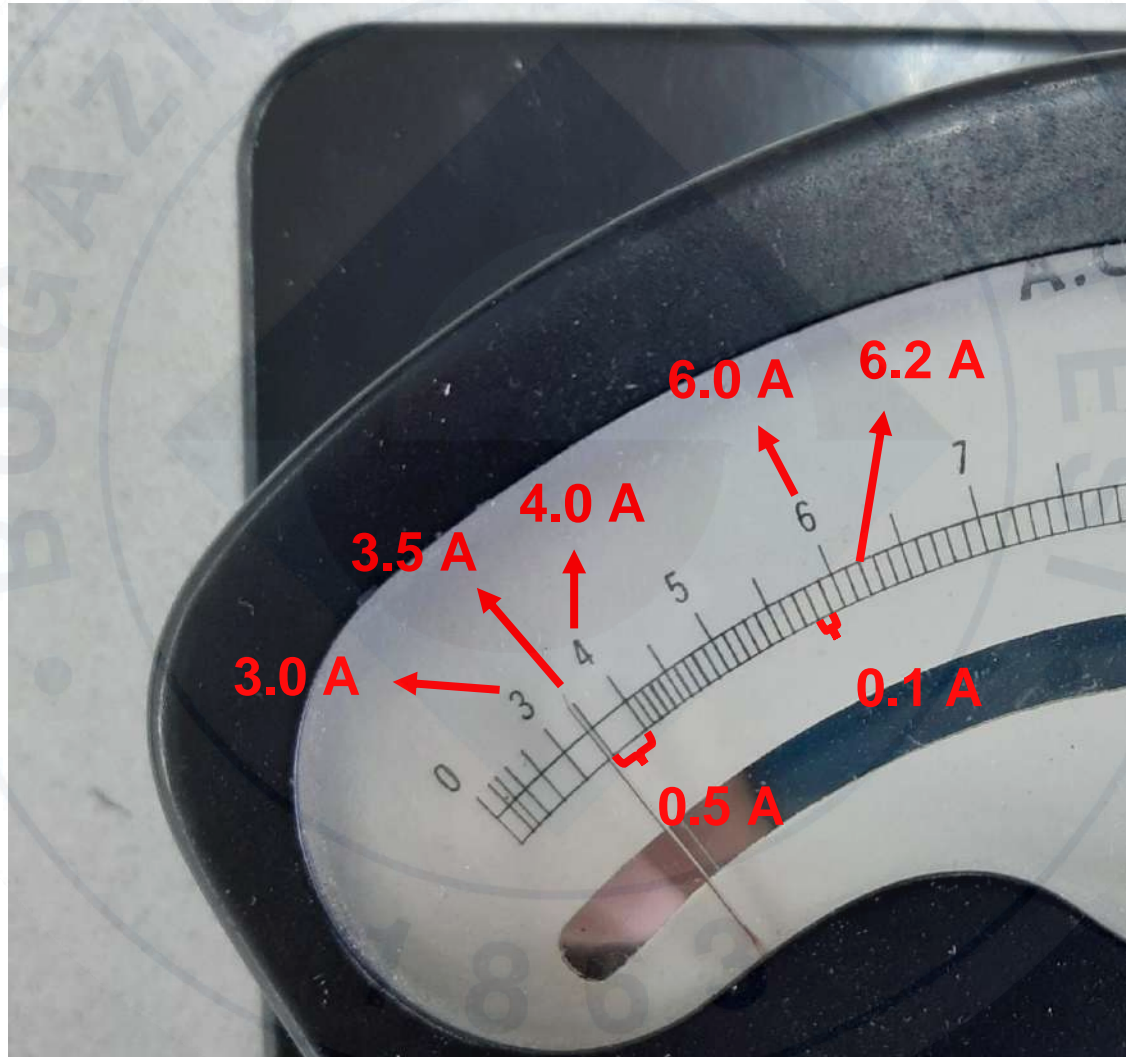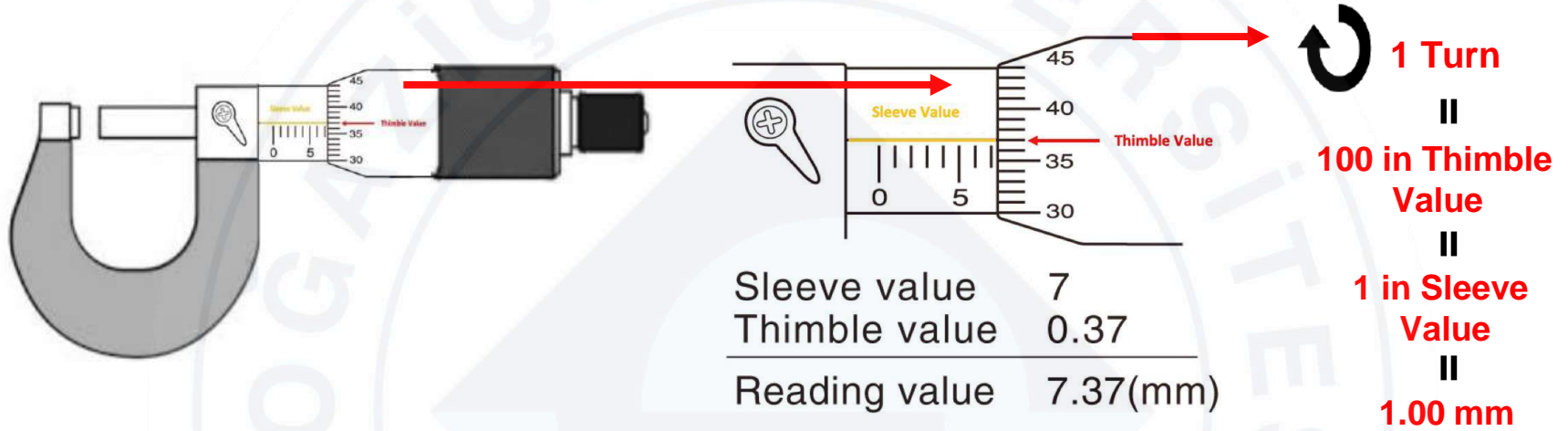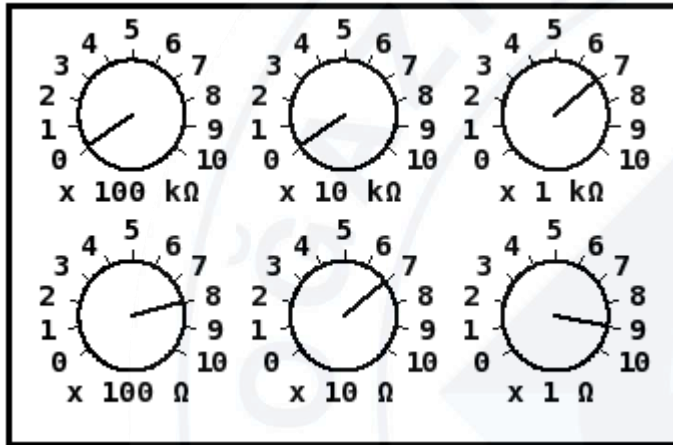
# RESISTANCE BOX



7879. Ω      4 sf

In a resistance box or analog devices like this, precision of a measurement is determined by the smallest measurable digit on the device if not explicitly stated otherwise.

?       3050. Ω      4 sf